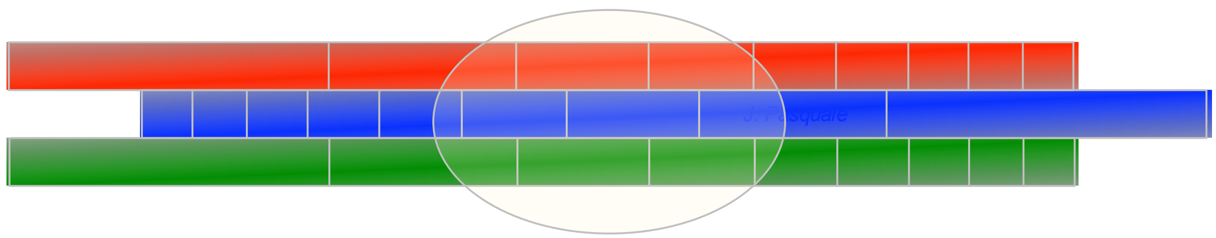
The Slide Rule

Calculating by Mind and Hand

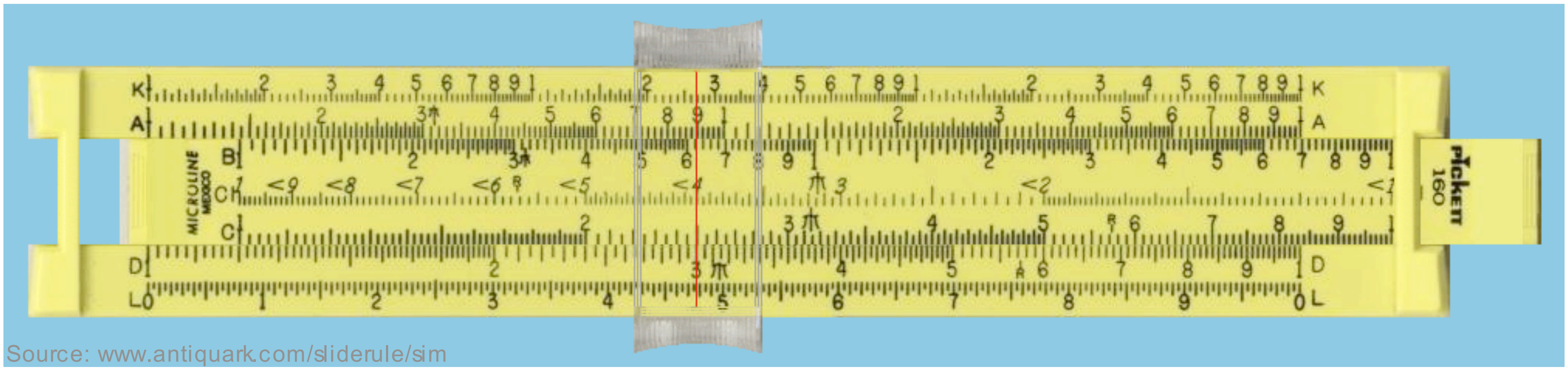


Joe Pasquale

Department of Computer Science and Engineering

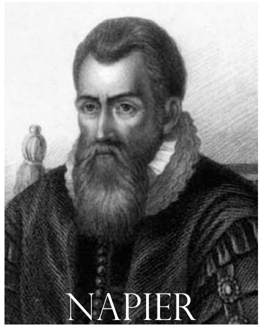
University of California, San Diego

What is a Slide Rule?

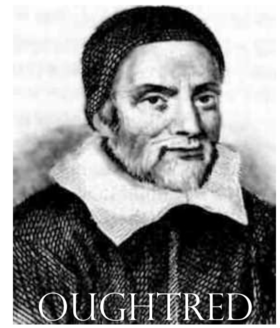


Source: www.antiquark.com/sliderule/sim

- Analog calculator – by mind and hand
- Scales on body and slide, with cursor
- $x \times y$, $x \div y$, $1/x$, x^2, $\sqrt{x}$, x^3, $\sqrt[3]{x}$, x^y, $x^{1/y}$, …
- 10^x, log x, e^x, ln x, sin/tan, sinh/tanh, …

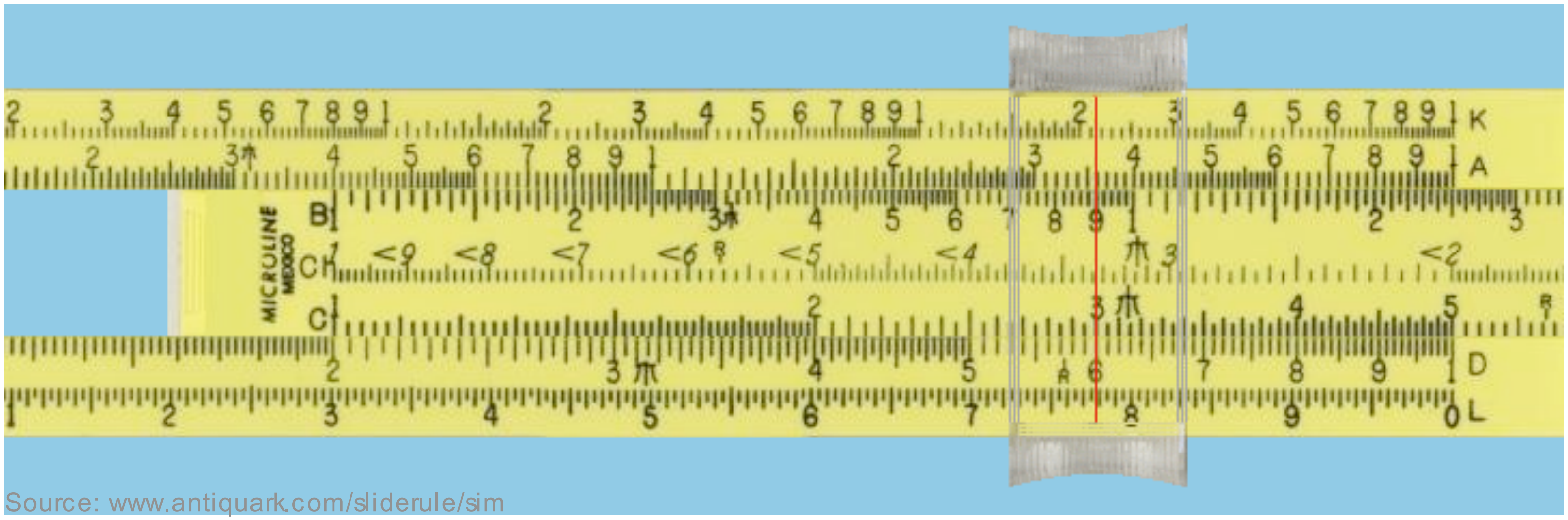


History



- 1614 Napier — logarithms
- 1617 Briggs — common logarithms
- 1620 Gunter — logarithmic scale
- 1630 Oughtred — slide rule
- 1850 Mannheim — standardized scales
- 1891 Cox — duplex slide rule
- 1972 HP — electronic calculator

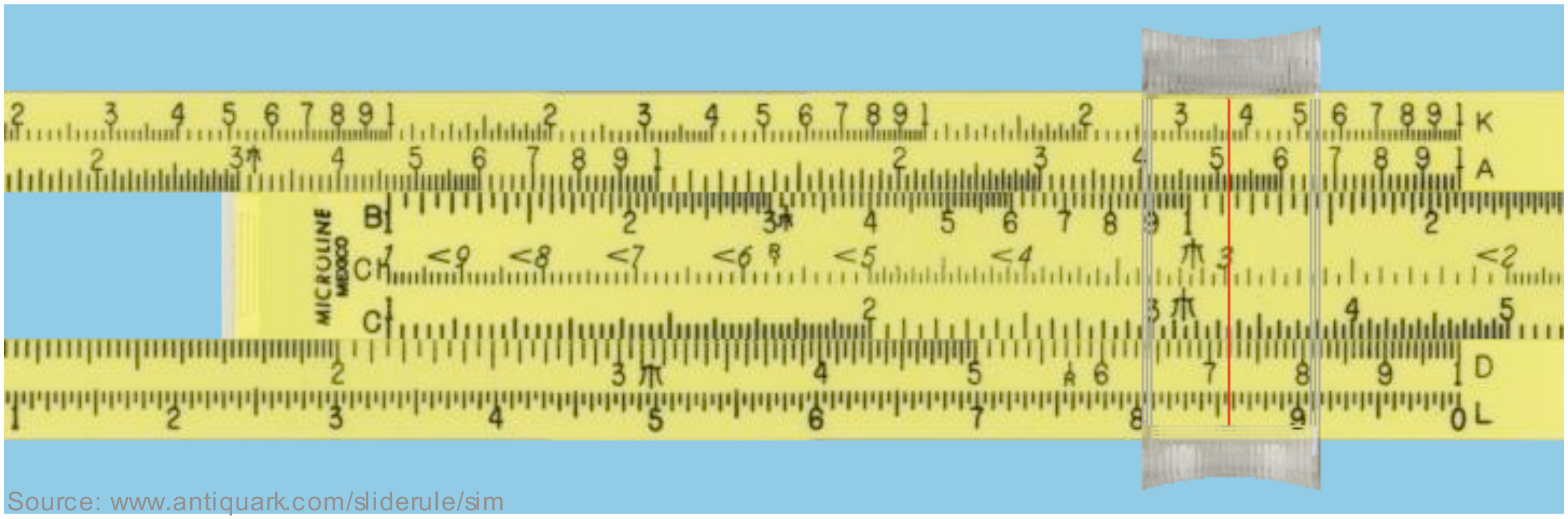
Multiplication: 2 × 3



Source: www.antiquark.com/sliderule/sim

- 1/C above 2/D
- Cursor above 3/C
- Read 6/D

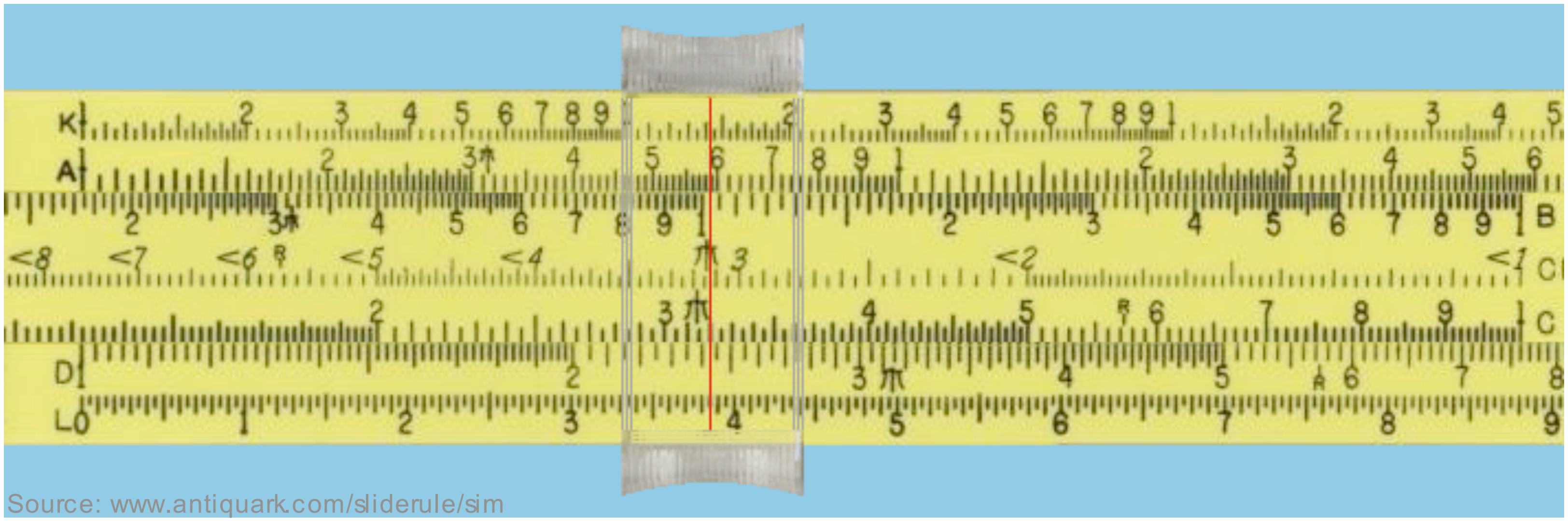
Multiplication: 2.15 × 3.35



Source: www.antiquark.com/sliderule/sim

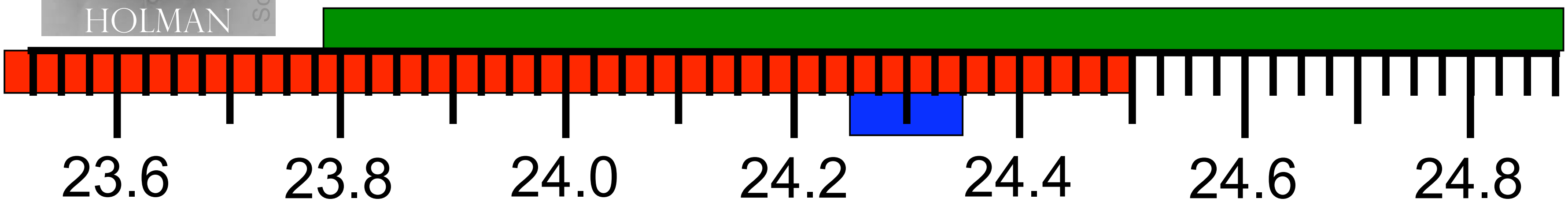
- 1/C above 2.15/D
- Cursor above 3.35/C
- Read 7.20/D (why 7.20 and not 7.2?)

Multiplication: 76 × 0.32



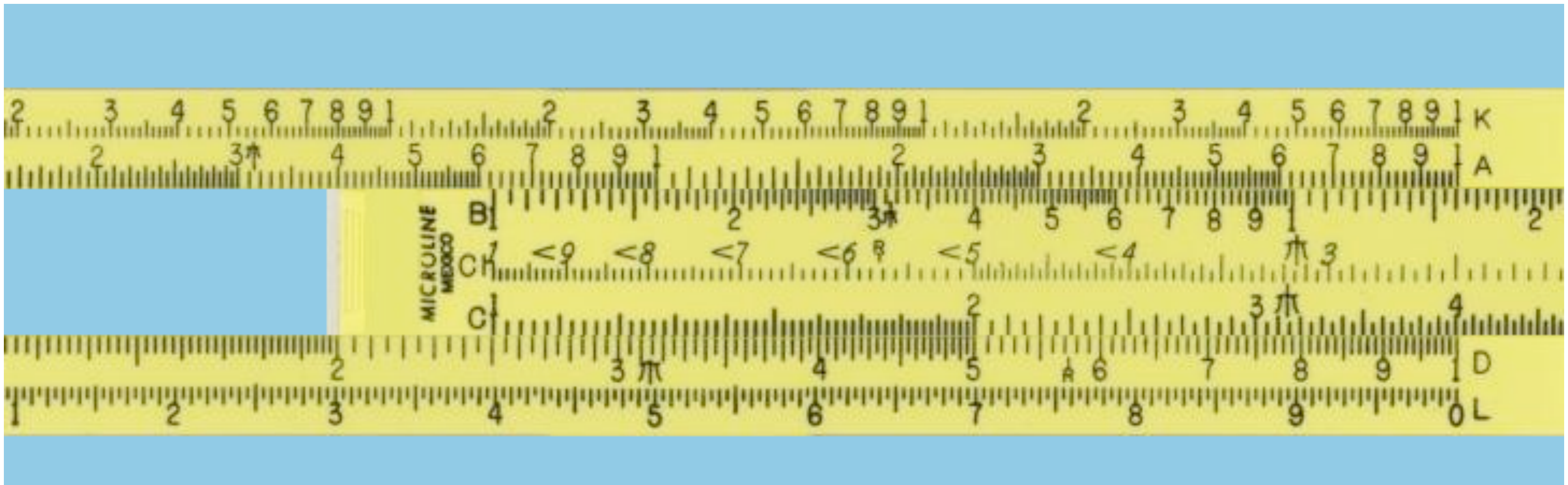
- 1/C above 7.6/D – use right index of C
- Cursor above 3.2/C, read 2.4/D
- Correct for decimal point: 24

Significant Digits



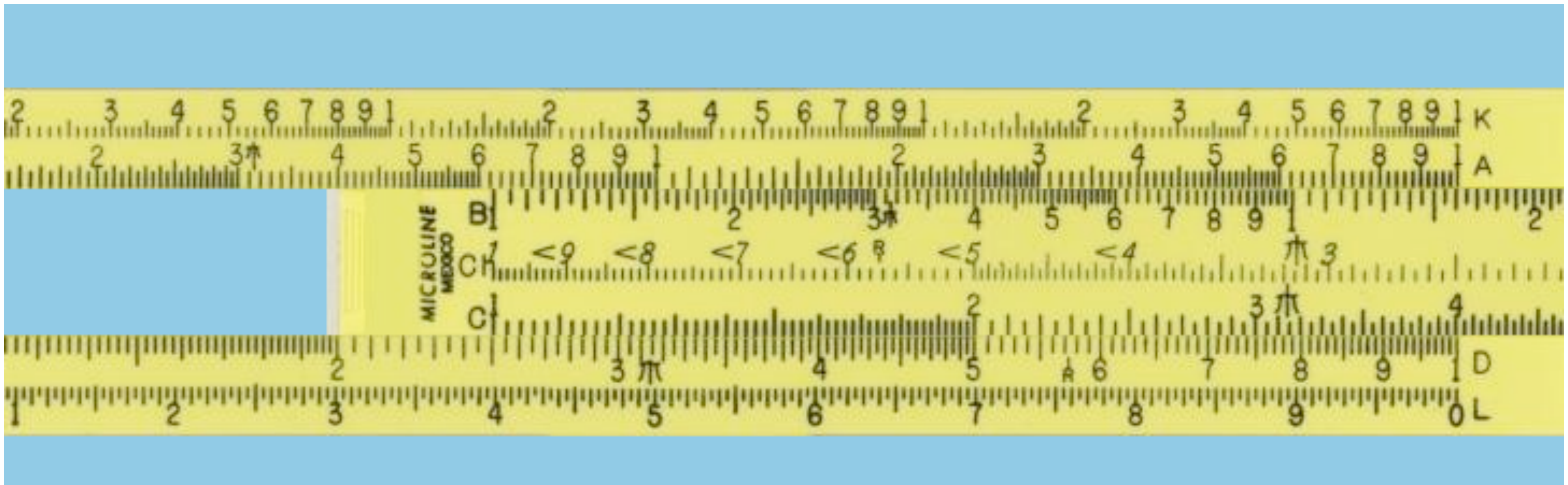
- 76 × .32 ≈ 24.32 [23.7825 – 24.8625]
  - 75.5 × .315 = 23.7825
  - 76.5 × .325 = 24.8625
- 76 × .32 ≈ 24 [23.5 – 24.5] 2 SD
- 76 × .32 ≈ 24.3 [24.25 – 24.35] 3 SD

Division: 5 ÷ 2


Source: http://www.antiquark.com/sliderule/sim/

- 2/C above 5/D
- Read 2.5/D under 1/C

Division: 60 ÷ 0.24

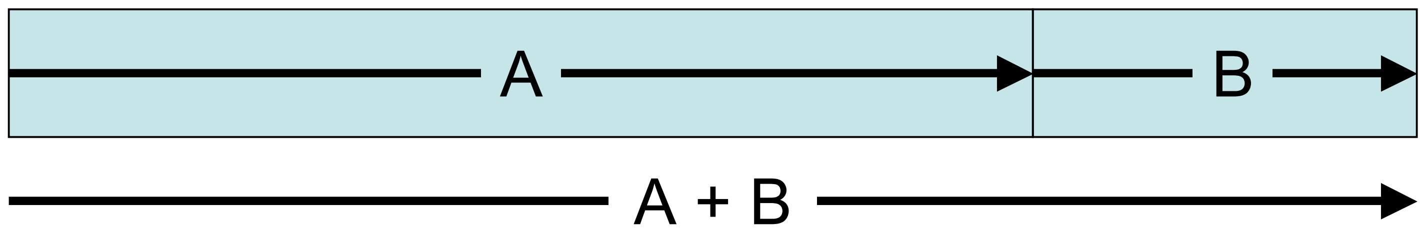


Source: http://www.antiquark.com/sliderule/sim

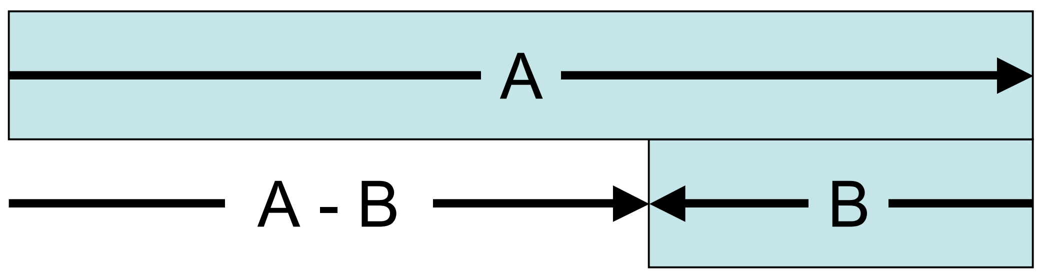
- 2.4/C above 6/D
- Read 2.5/D under 1/C
- Correct decimal point: 250

Algebra of Lengths

- Length(A + B) = Length(A) + Length(B)



- Length(A - B) = Length(A) - Length(B)



Calculating Power

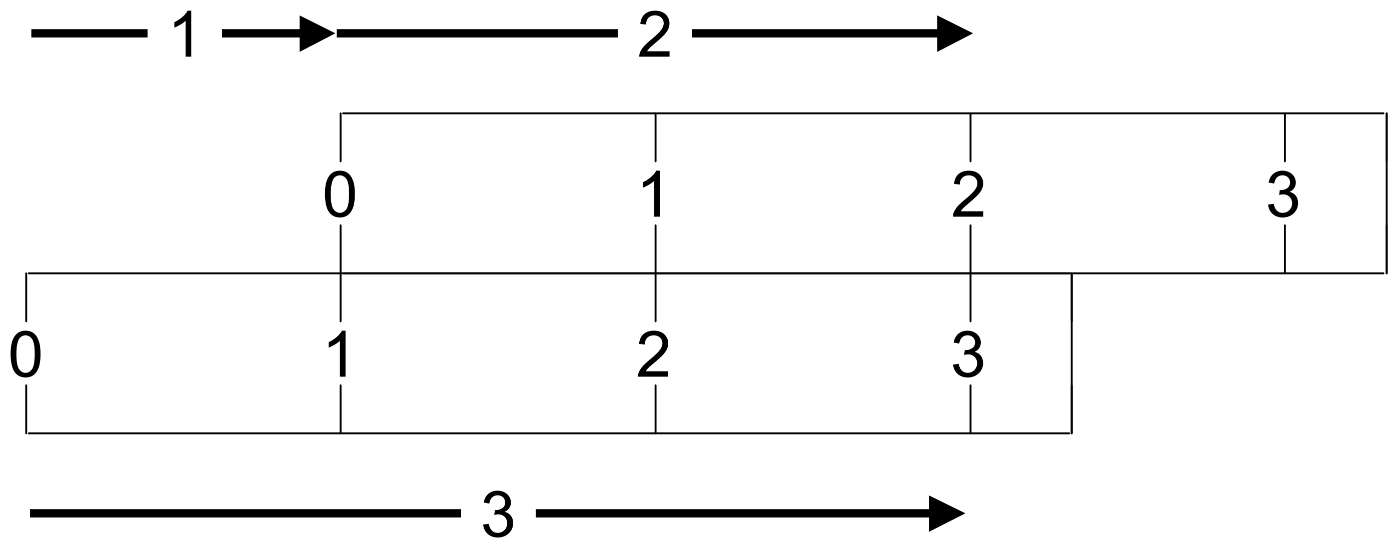
- Any operation expressible in the form

  A + B = C or **A - B = C**

  can be implemented with a slide rule

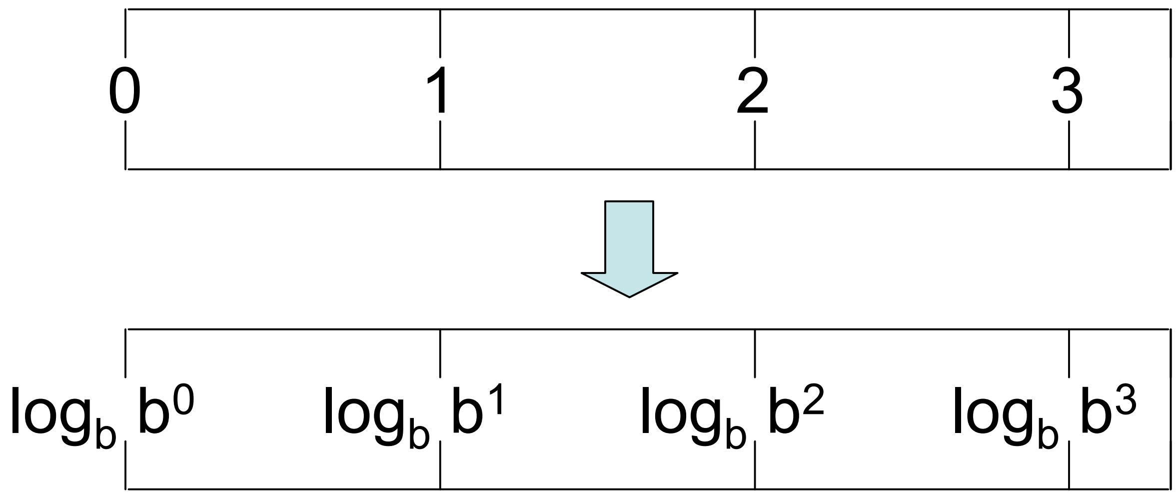
- $x \times y = z \rightarrow \log x + \log y = \log z$
- $x \div y = z \rightarrow \log x - \log y = \log z$
- $x^y = z \rightarrow \log \log x + \log y = \log \log z$

Addition: Adding Lengths



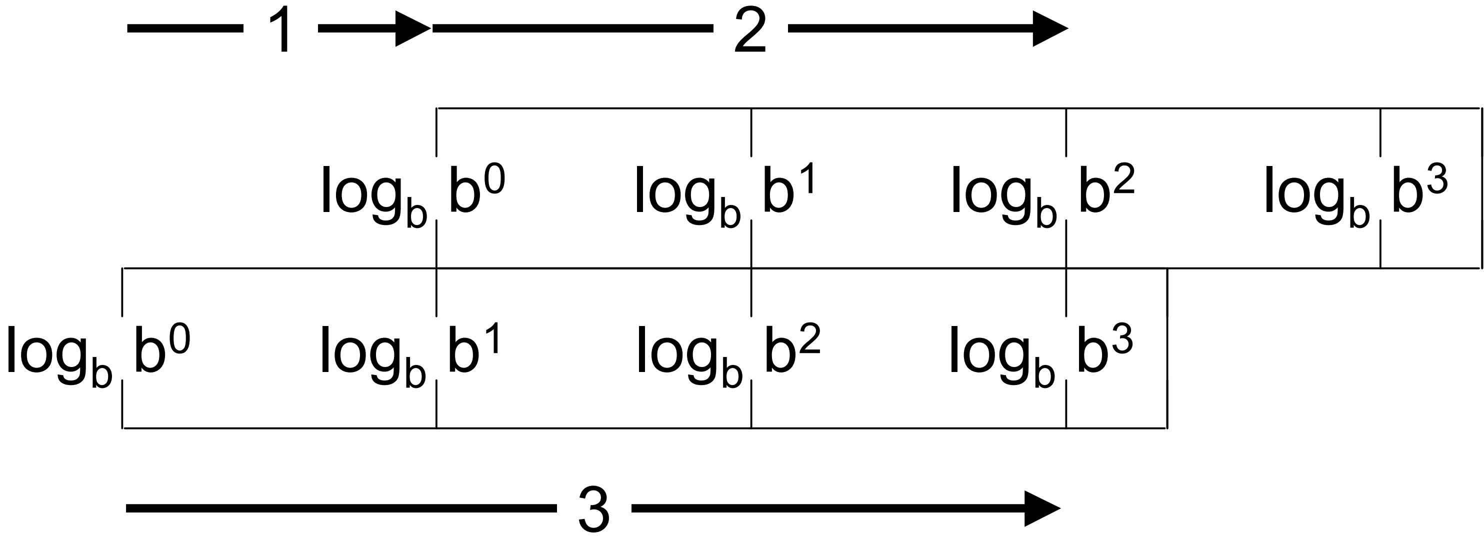
- Example: 1 + 2 = 3
- L(1) + L(2) = L(1+2) = L(3)

Re-Label Scale Indices



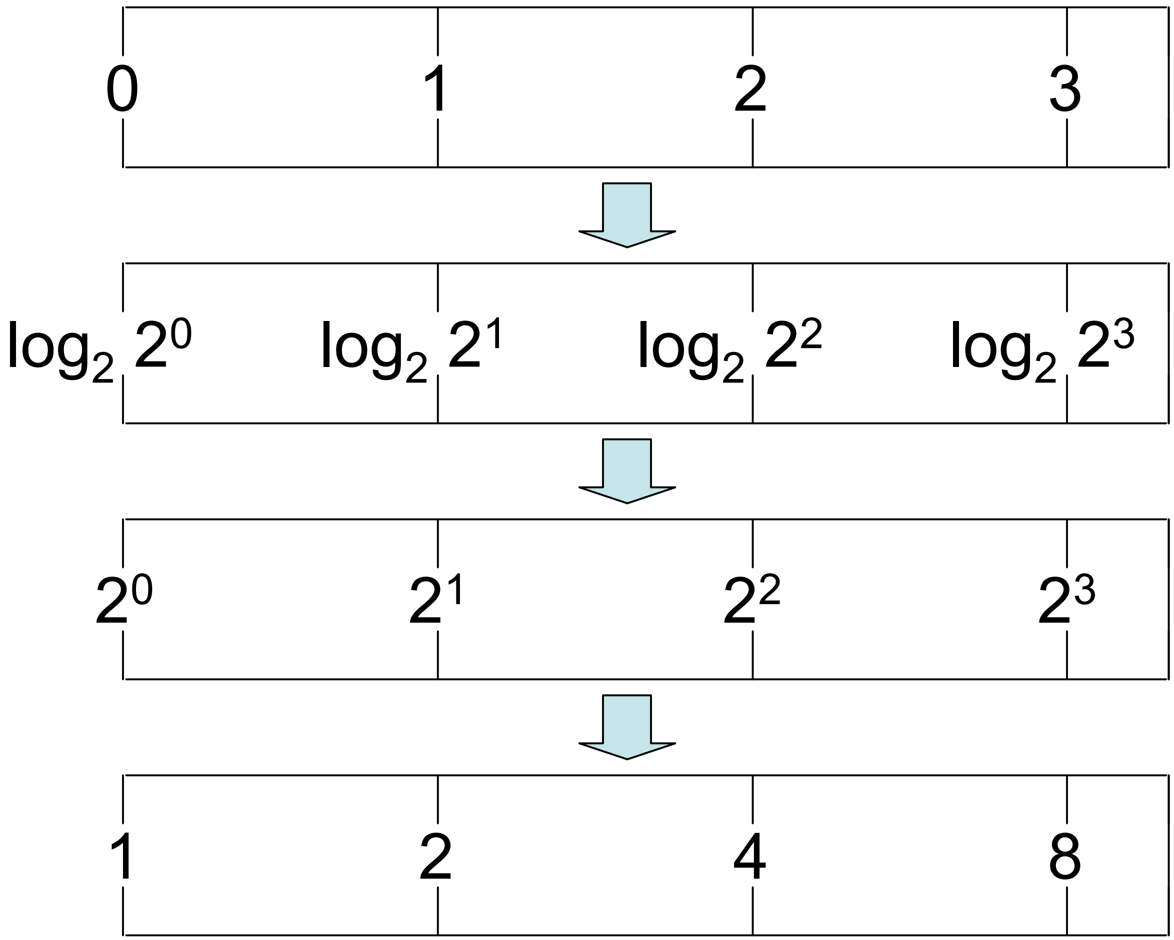
- $x = \log_b b^x$, for any x, for any b
- $0 = \log_b b^0$, $1 = \log_b b^1$, $2 = \log_b b^2$, …

Multiplication: Length ≡ Log

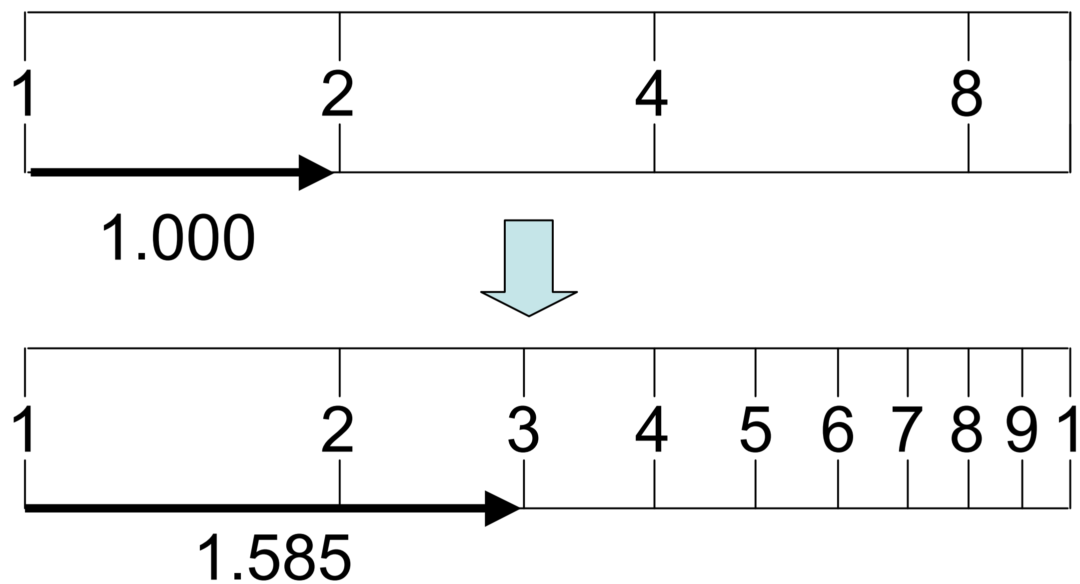


- $\log_b b^1 + \log_b b^2 = \log_b b^3$
- $b^1 \times b^2 = b^3$

Re-label Scale Indices

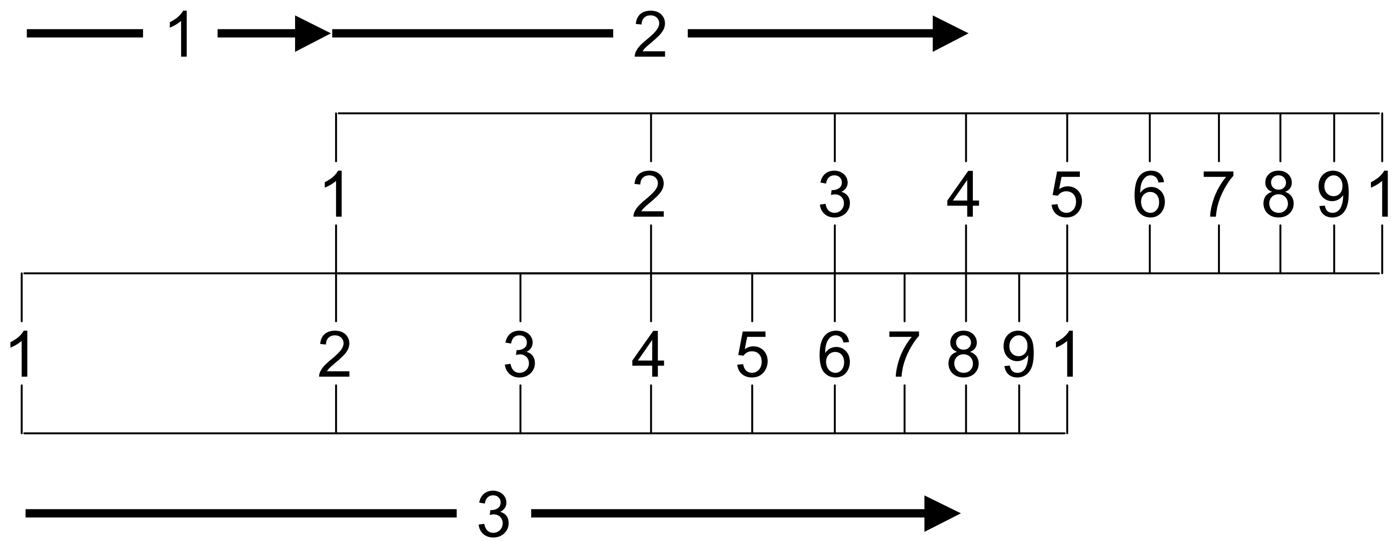


Add Intermediate Labels



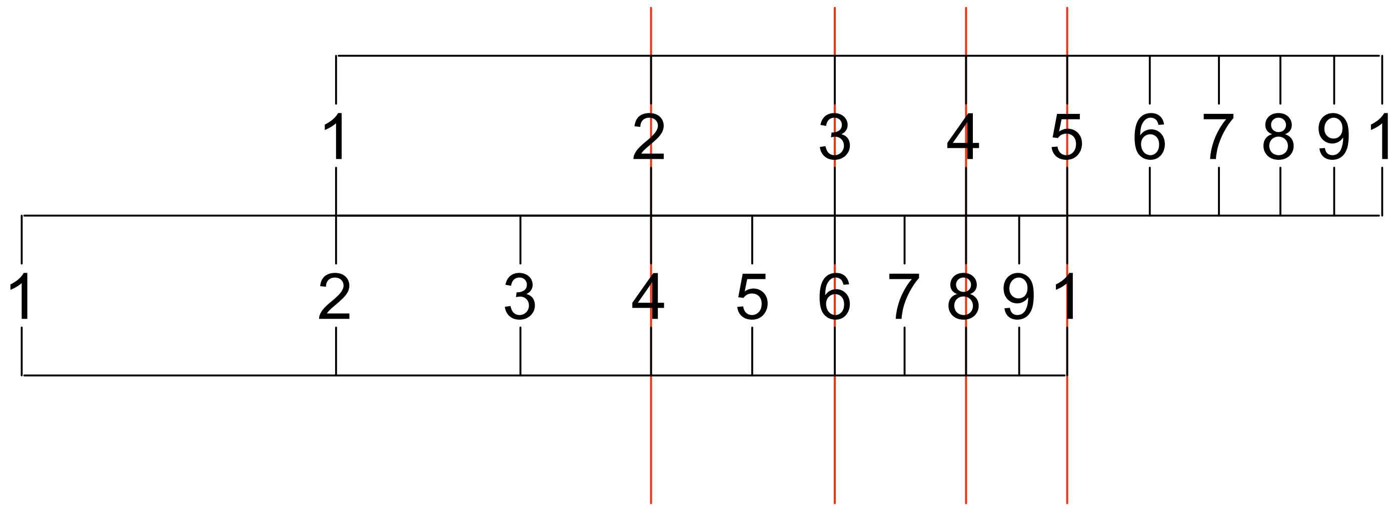
- “x” is located at log x / log 2
- “3” is located at log 3 / log 2 ≈ 1.585

To Multiply, Add Exponents



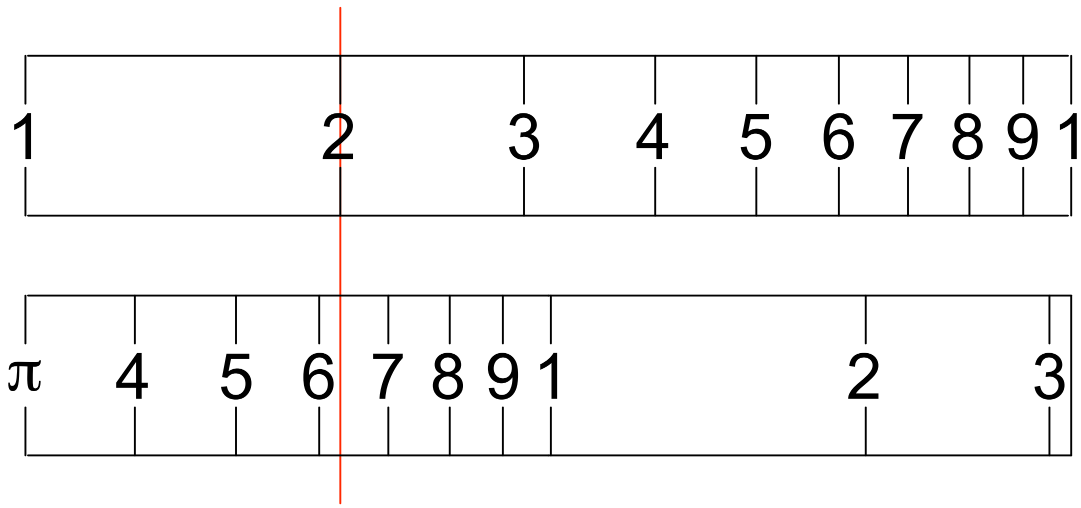
- $2^1 \times 2^2 = 2^{1+2} = 2^3$
- $2 \times 4 = 8$

Multiplication and Division



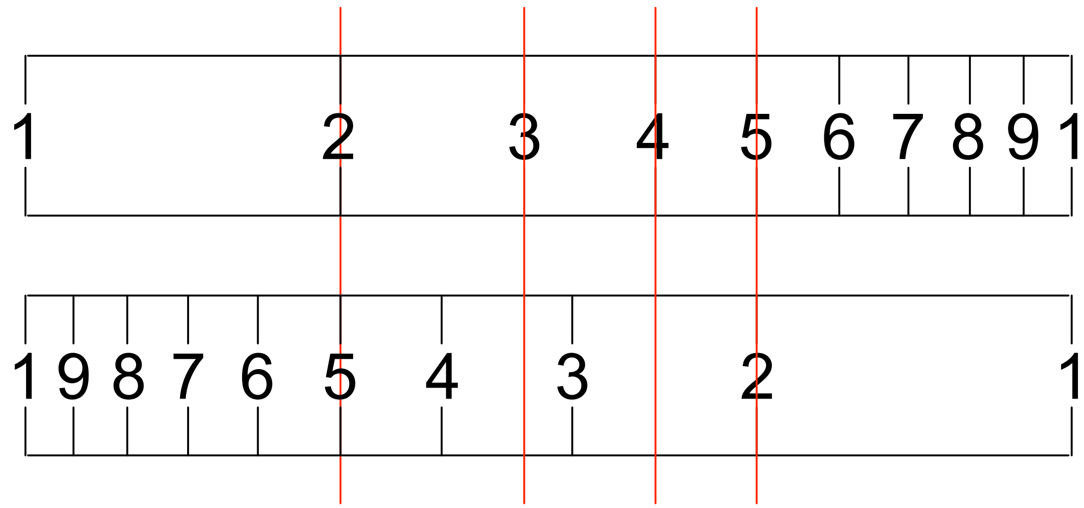
- $2 \times 2 = 4$, $2 \times 3 = 6$, $2 \times 4 = 8$, $2 \times 5 = 10$
- $4 \div 2 = 2$, $6 \div 3 = 2$, $8 \div 4 = 2$, $10 \div 5 = 2$

Multipliers Shift Scales



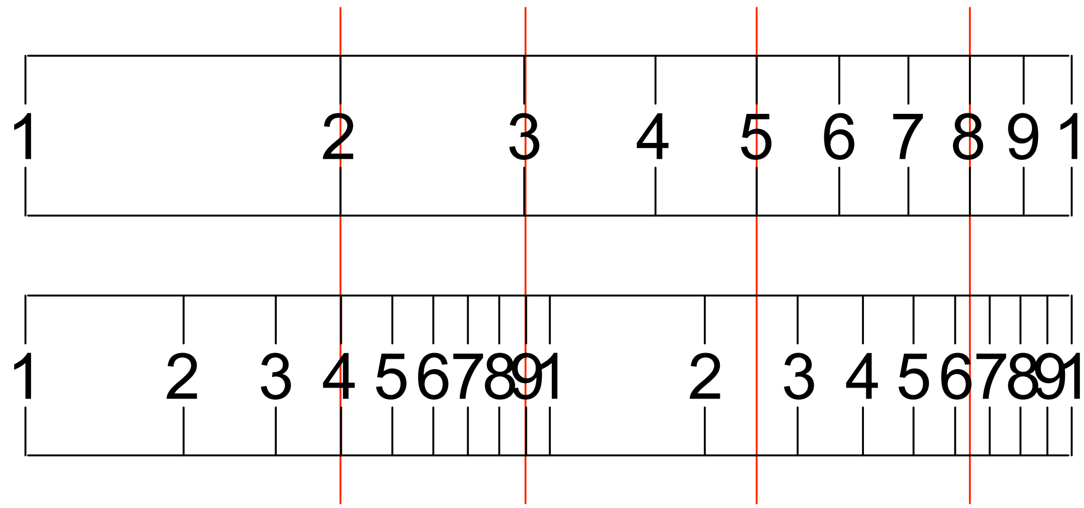
- Multiplication by π, shift scale to left
- $2 \times \pi \approx 6.28$

Reciprocals Invert Scales



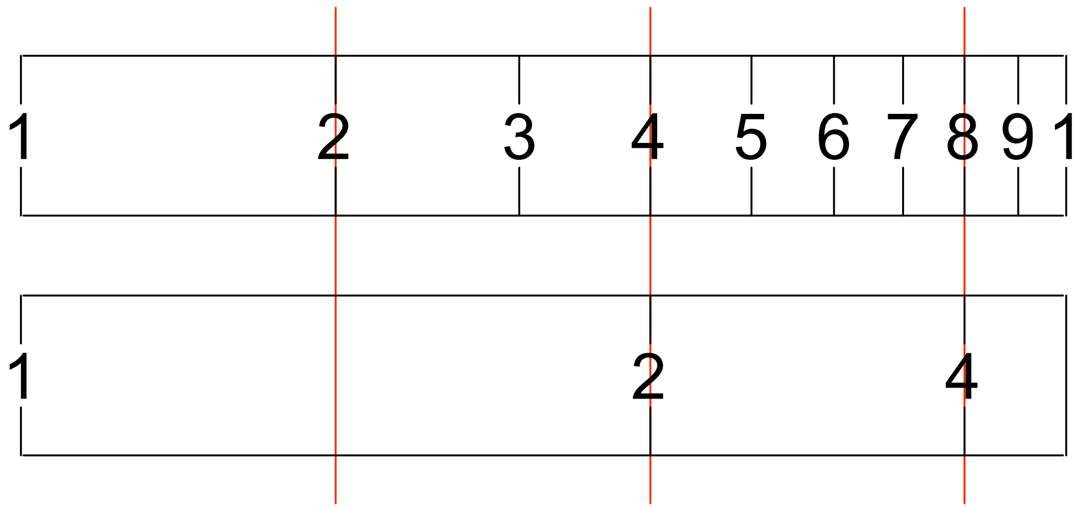
- Reciprocal: scale inverted horizontally
- 1/2 = .5, 1/3 ≈ .33, 1/4 = .25, 1/5 = .2

Powers Compress Scales



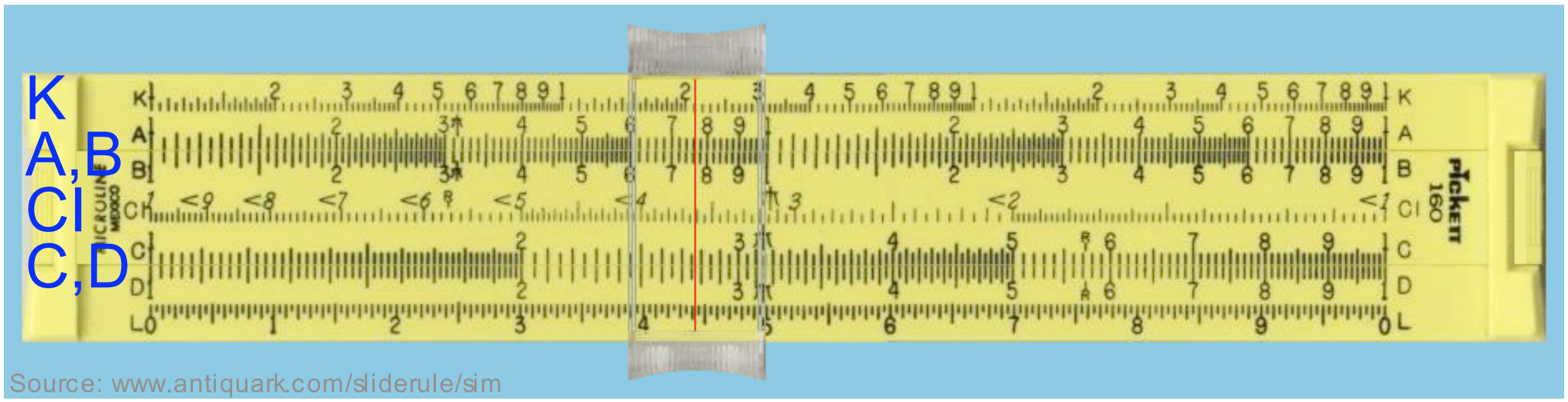
- Square: compress scale by factor of 2
- $2^2 = 4$, $3^2 = 9$, $5^2 = 25$, $8^2 = 64$

Roots Expand Scales



- Square root: expand scale by factor of 2
- $\sqrt{2} \approx 1.41$, $\sqrt{4} = 2$, $\sqrt{9} = 3$

Looking at a Real Slide Rule

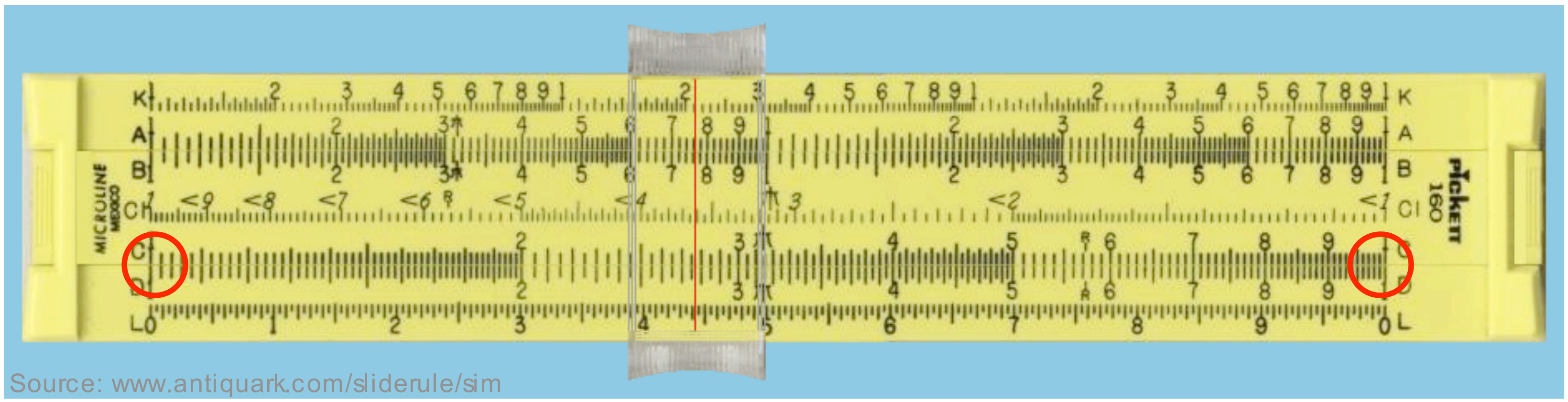


- C, D *reference scales*
- CI reciprocal of C – inversion
- A, B square of C, D – 2x compression
- K cube of C, D – 3x compression

Precision

- Depends on physical length
- 10 inch rule: 3-4 digits
- Ways to increase precision
  - Increase physical length
  - Wrap scale around rule to increase length
  - Magnify the area of focus

Precision — Relative Error



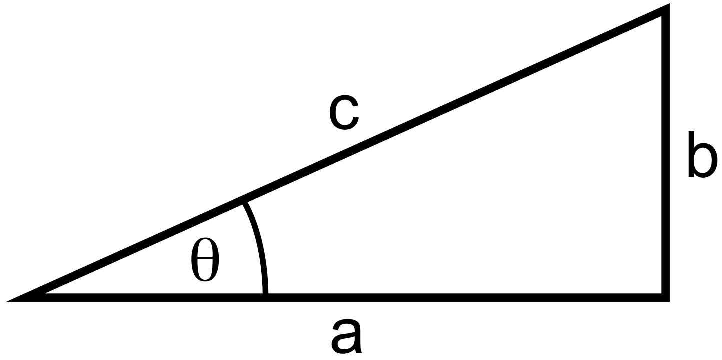
Source: www.antiquark.com/sliderule/sim

- Compare physical distances at extremes
  - Distance (1.00, 1.01) ≈ Distance (9.9, 10)
  - (1.01-1.00)/1.00 = 1%, (10-9.9)/10 = 1%
- Relative error uniform across log scale

Precision vs. Accuracy

- $2 \times 3 = 6$
  - accurate, not precise
- $2.00 \times 3.00 = 6.01$
  - more precise, less accurate
- Are 2 and 2.00 located at same place?
  - Does it matter? Why?

Trigonometry

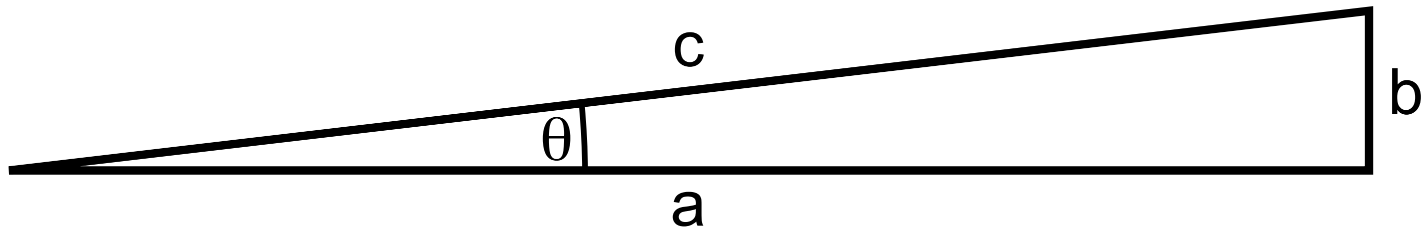


- Recall sin θ = b/c, cos θ = a/c, tan θ = b/a
- Scales for sin θ and tan θ
- To calculate cos θ, use sin 90-θ

Sin and Tan Scale Ranges

- Sin scale: 5.74 – 90.0 degrees
  - sin 5.74 ≈ 0.1, cos 84.26 ≈ 0.1
  - sin 90 = 1.0, cos 0 = 1.0
- Tan scale: 5.71 – 45 – 84.3 degrees
  - tan 5.71 ≈ 0.1
  - tan 45 = 1.0
  - tan 84.3 ≈ 10

$\sin\theta \approx \tan\theta$, for small θ



- $\sin\theta = b/c$, $\tan\theta = b/a$
- For small θ
  - $a \approx c$, therefore $\sin\theta \approx \tan\theta$
  - Use ST scale for $\theta < 5.74$

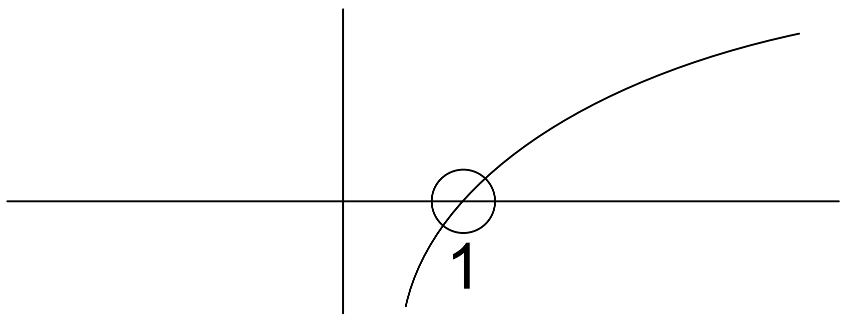
Calculating Arbitrary Powers x^y

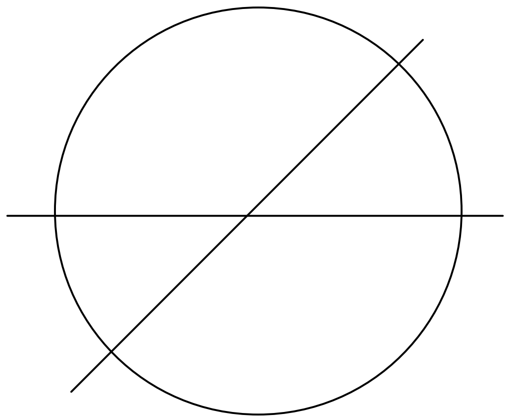
- x^y can be calculated as A + B = C

$$x^y \rightarrow \log x^y = y \log x$$
$$\rightarrow \log \log x^y = \log y + \log \log x$$
$$\rightarrow \underbrace{\log \log x}_{A} + \underbrace{\log y}_{B} = \underbrace{\log \log x^y}_{C}$$

- Note that A and C are same scales: LL
- LL scales devised by Roget in 1815

ln 1+x ≈ x for small x





- Near x = 1, ln 1+x ≈ x (linear)
- log 1 = 0

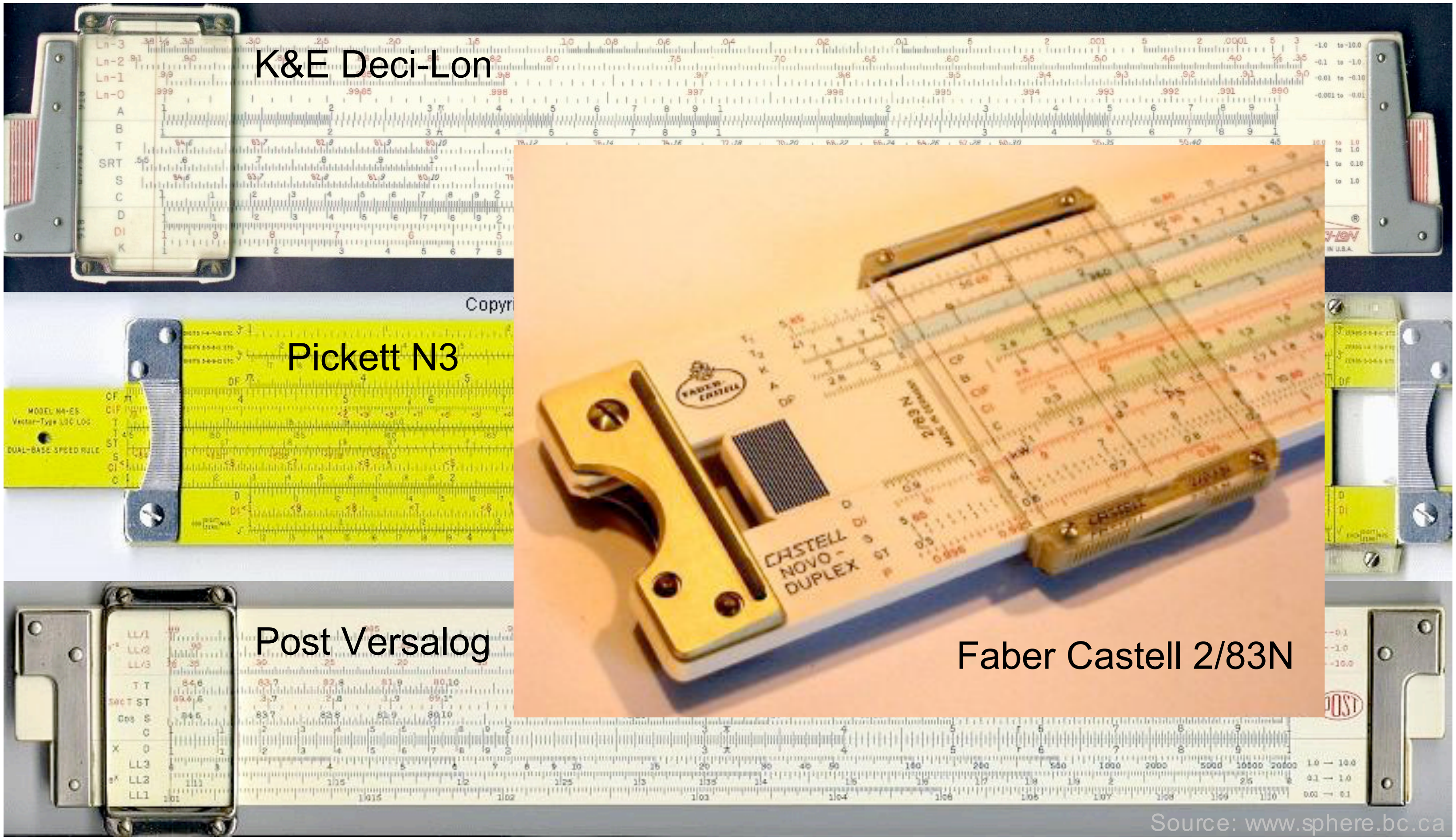
How Were Scales Built?

- The Gilligan's Island Slide Rule Problem
  - You are stranded on an island
  - You, "the professor," must save the crew
  - You decide to build a slide rule
- How do you determine graduations for …
  - a log scale, log log scale, sin scale, tan scale
- Arithmetic + geometry, *no calculators*

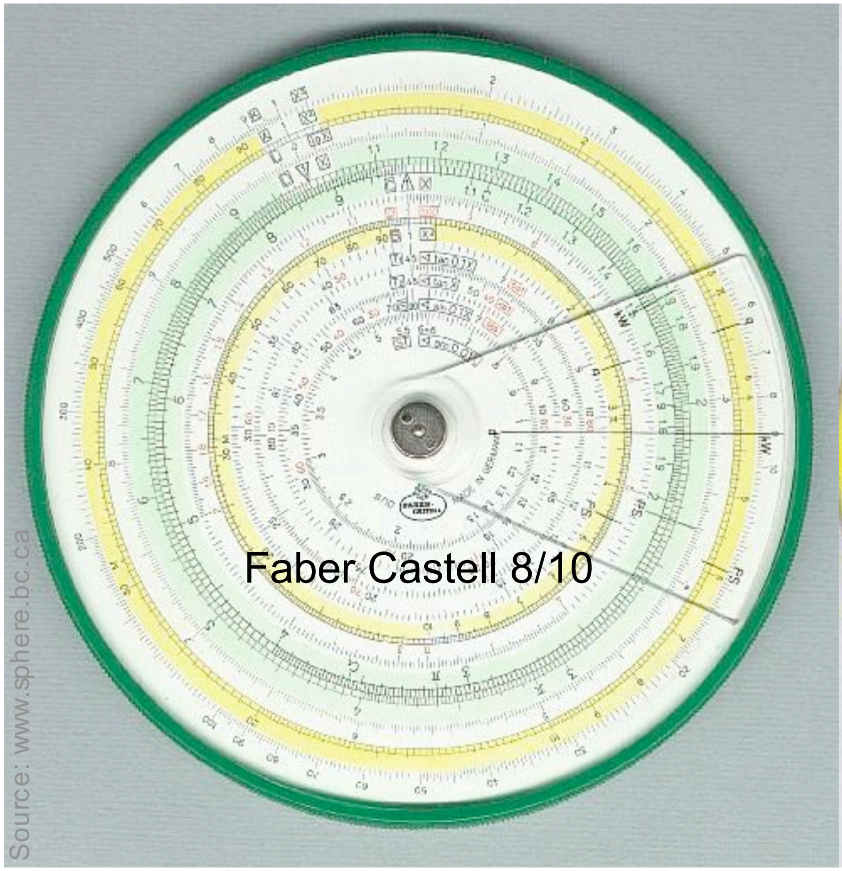
Slide Rule Topology

- Slide rules come in many
  - physical shapes and sizes
  - scale configurations, lengths, layout
- Precision
- Size
- Convenience

Linear

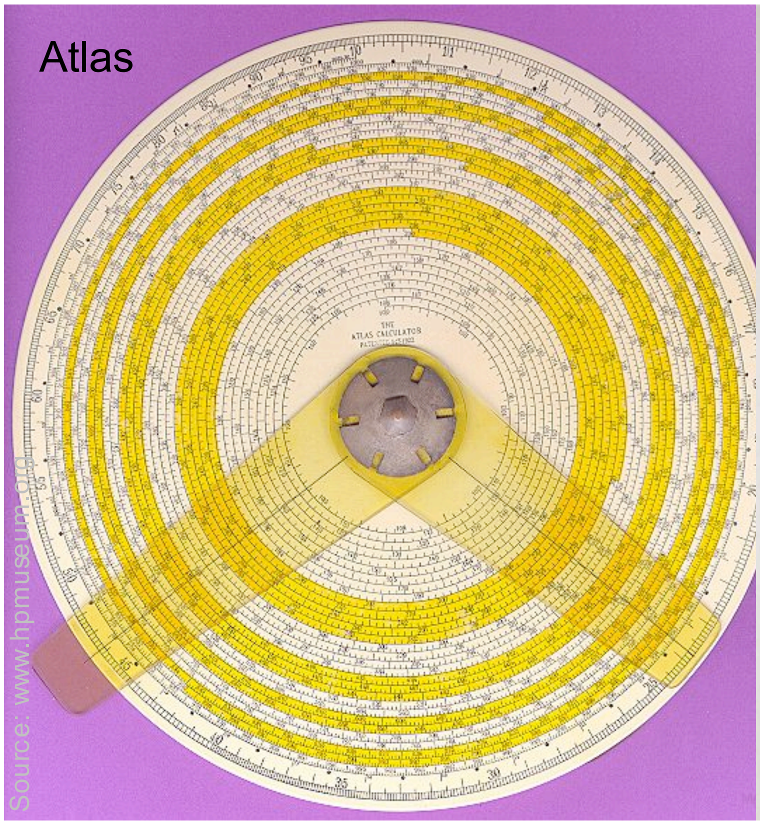

K&E Deci-Lon
Pickett N3
Post Versalog
Faber Castell 2/83N
Source: www.sphere.bc.ca

Circular


Faber Castell 8/10
Source: www.sphere.bc.ca


PICKETT
CIRCULAR SLIDE RULE
Pickett 110ES
Source: www.sphere.bc.ca

Spiral


Atlas
Source: www.hpmuseum.org


Sutton (1663)
SCIENCE & SOCIETY
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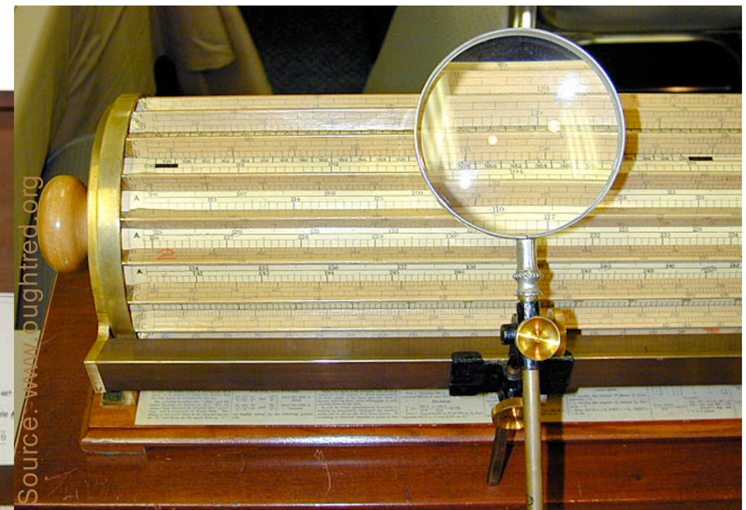
Source: www.scienceandsociety.co.uk

Cylindrical Spiral

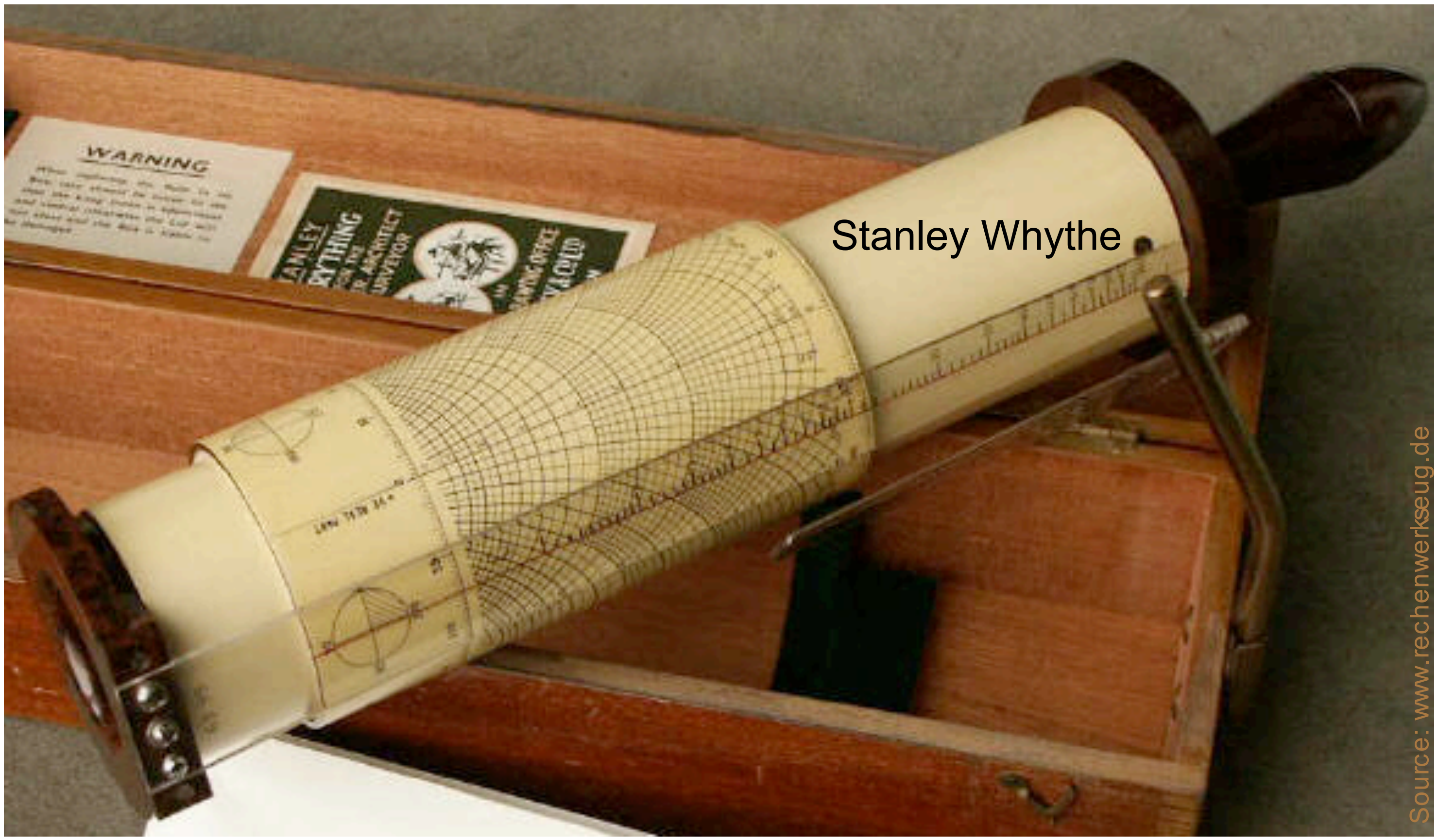
Cylindrical Grid

Thacher 4012

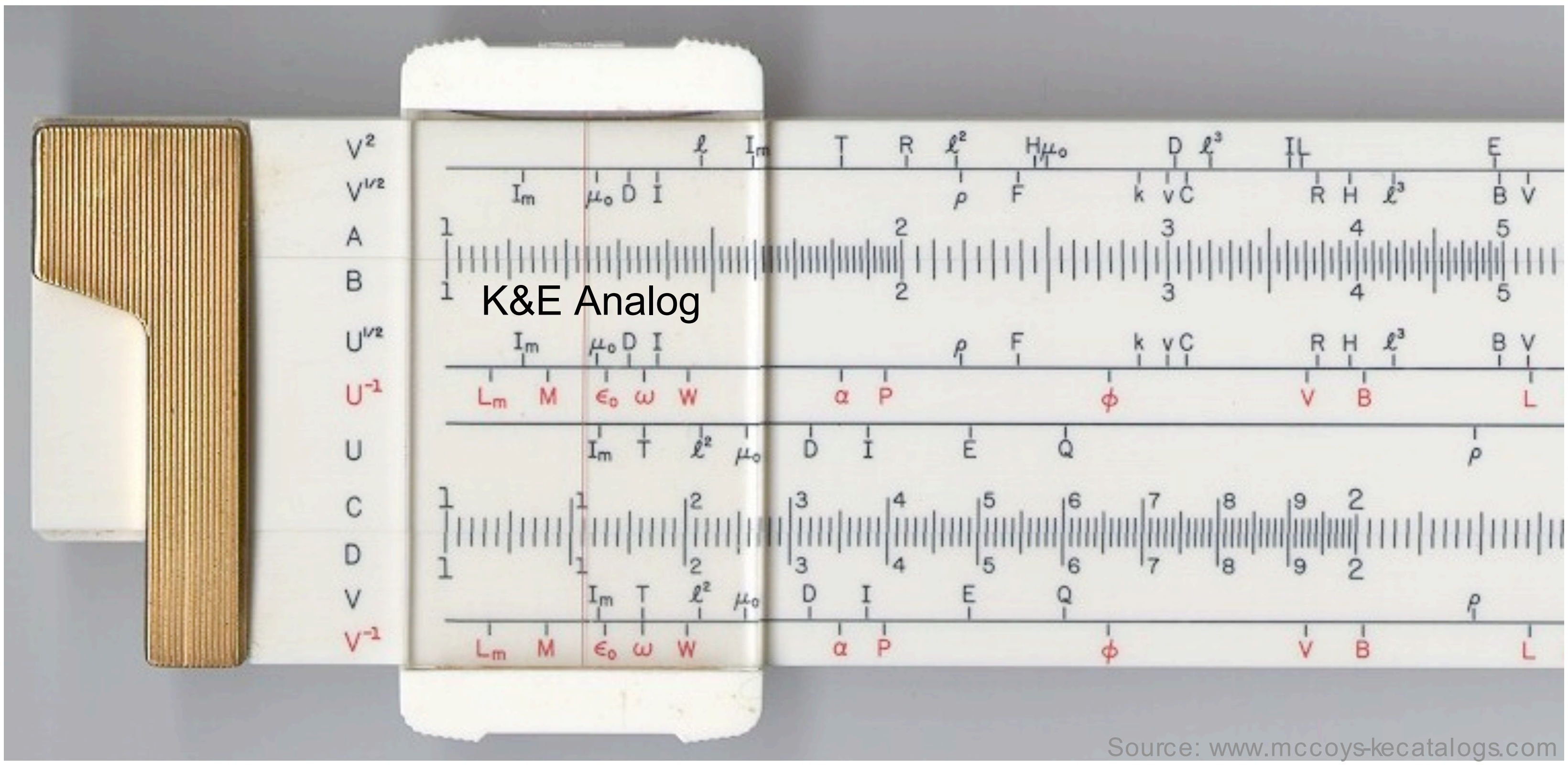


Thacher 4013

Complex Arithmetic



Dimensional Analysis



UCSD Freshman Seminar

Masters of the Slide Rule, Winter '03

What Students Learn

- How to use all scales
- Estimation
- Approximation
- Precision, accuracy
- Advanced topics
  - Scales from scratch
  - Benford’s Law

Larger Lessons

- Economy of calculating
  - slide rules
  - calculators
  - computers
- Social value
  - parents, grandparents
  - do so much with so little

Quotes

My skills of estimation are getting better … I like being engrossed in the calculations, instead of just punching them into my calculator. I make less mistakes, and find I know what I am talking about …

- Brian Robbins, W03

Quotes

I was looking at the A scale and I liked how it finds squares by just decreasing the size of the D scale by half … So then I found the cubed K scale, and of course, it is three times smaller than the D scale.

- Tracy Becker, W03

Quotes

I like being able to see mathematical operations in the visual way that a slide rule allows … This seminar has given me a better understanding of precision, relationship between logs and multiplication, and Benford's Law.

- Amy Cunningham, W03

Quotes

What amazes me the most about the slide rule is that it works … I can't help but marvel at its design and that someone actually was able to create such a device ... Its complexity is just mind boggling.

- Kendra Kadas, F03

Quotes

I was in physics class, and the professor explained how tan and sin are close for really small angles. The class didn't show much reaction, but my first thought was "hey, I learned that from my slide rule seminar."

- John Beckfield, F03

Quotes

The first couple of days with this slide rule have really been a learning experience for me … It took me some time to realize that you could multiply by any interval of 10 using the same number spectrum.

- Rajiv Rao, F04

Quotes

This slide rule seminar is the only thing saving me from a quarter full of literature writing, and other humanitarian monotony. After hours of "theory of literature," I realized I still had slide rule homework. Hurray!

- Lydia McNabb, F04

Quotes

The slide rule rules. The slide rule is truly an extension of a person, not something completely separate such as the calculator. I actually had to think before, during, and after getting the answer on the slide rule.

- Lynn Greiner, F04

Quotes

I'm actually quite amazed with the design of the slide rule. I find the folded scales especially ingenious … I definitely feel I understand what I'm doing - not quite the "black box" that calculators are.

- Ryan Lue, F04

Quotes

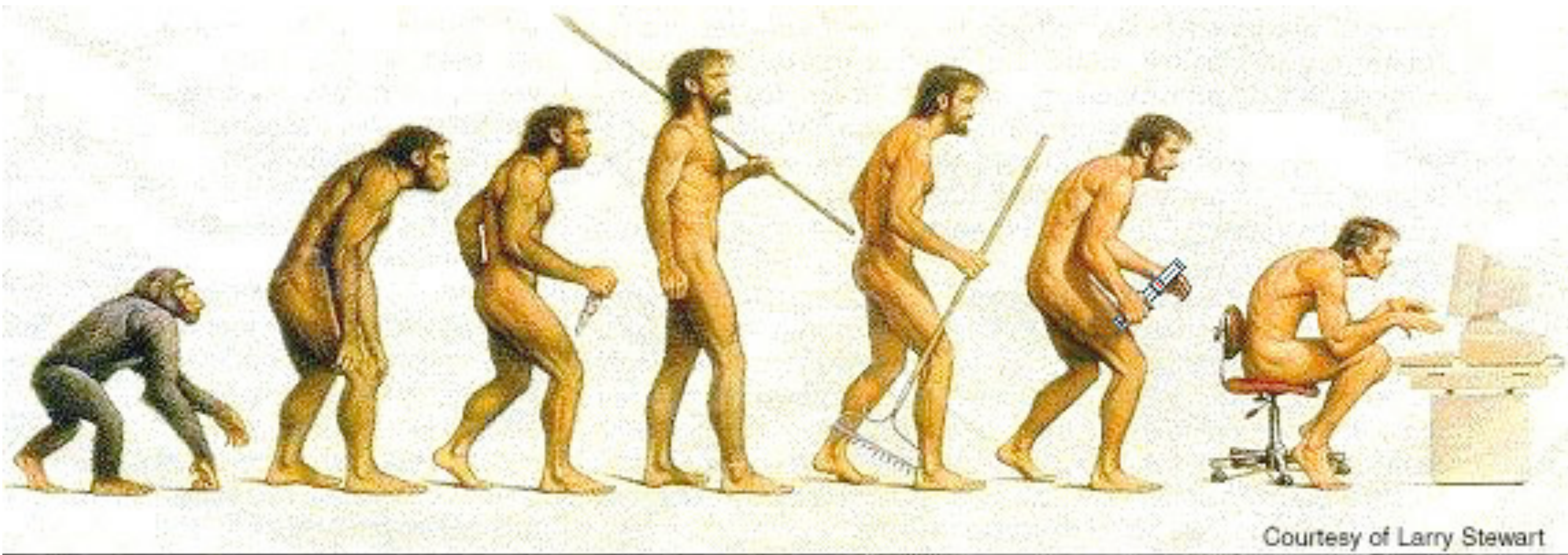
The more I use the slide rule, the greater the insight I have into how ingeniously the scales were put together. I hope I can re-teach my parents how to use it.

- Chris Brumbaugh, F04

Proof of Slide Rule Use in '76

Student shows teacher a slide rule calculation.
Weehawken High School, NJ, 1976

Are We Making Progress?



Somewhere, something went terribly wrong

FOR MORE INFO

Joe Pasquale
Dept. of Computer Science & Engineering
University of California, San Diego
9500 Gilman Drive
La Jolla, CA 92093-0404

pasquale@cs.ucsd.edu
http://www-cse.ucsd.edu/~pasquale

Supplemental

Optimal Length of Log Scale

- What integer total length L minimizes RMS error of integer tick mark values?
- Determine for each tick mark X
  - round (L * log(X))
- Compute Error
  - | true value - nearest integer value |
- RMS: Root Mean Square (of errors)

Survey of Best Values < 1000

Length	Error	Length	Error
63	10.86	505	9.52
176	9.99	**568**	**2.19**
239	7.89	744	10.22
329	5.90	807	9.93
392	10.24	897	4.16

Length of 568, 2.2% error

Location of major tick marks

1:	0	0.00	6:	442	441.99
2:	171	170.99	7:	480	480.02
3:	271	271.01	8:	513	512.96
4:	342	341.97	9:	542	542.01
5:	397	397.02	1:	568	568.00

Length of 329, 5.9% error

Location of major tick marks

1:	0	0.00	6:	256	256.01
2:	99	99.04	7:	278	278.04
3:	157	156.97	8:	297	297.12
4:	198	198.08	9:	314	313.95
5:	230	229.96	1:	329	329.00

Length of 392, 10.2% error

Location of major tick marks

1:	0	0.00	6:	305	305.04
2:	118	118.00	*7:*	*331*	*331.28*
3:	187	187.03	8:	354	354.01
4:	236	236.01	9:	374	374.06
5:	274	274.00	1:	392	392.00