Stadia or Tachymetrical Slide Rules,
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Stadia or Tachymetrical Slide Rules

Introduction
Slide rules with scales for specific professions have been designed since Oughtred's time. Surveying was one of those professions.
From the 19th century, a number of slide rule manufacturers have produced special versions of slide rules for one particular method of surveying: stadia surveying or tachymetry.

Tachymetry
Figure 1 shows the set-up of a tachymetric measurement. Tachymetry (or tacheometry) means "fast measurement", also called "stadia surveying" in countries like England and the United States ("stadia" were the old Greek length-units of 185 meter, or 600 Greek feet).

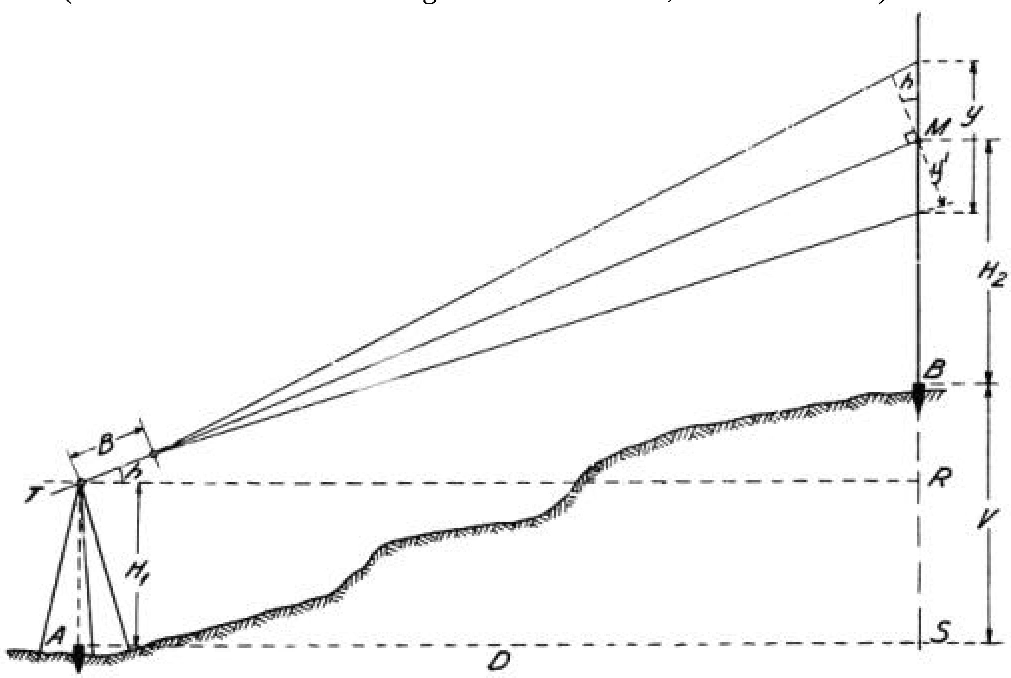

Fig. 1: Principles of Tachymetry from Dutch textbooks by Schermerhorn and van Steenis

Note that the word tachymetry may cause confusion with the tachymeter-scales of sportsman watches for calculating between time, distance and speed.
While in the past, distances were measured by the "surveyor's chain", this can be done easier and faster using a telescope equipped with stadia hairlines in combination with a stadia rod. Especially in rough and broken grounds, the gain in speed and accuracy by eliminating the chain can be substantial. The stadia mechanism consists of two or three hairlines, horizontally placed in the viewing field of the telescope, which is often integrated into a theodolite. The scale length on a vertical stadia rod, as observed between the hairlines, is used to calculate the

distance to the rod. The principle of the stadia hairlines was already known about the end of the 17th century, but James Watt was one of the first who applied this method in 1790, during his field surveying for the construction of the Crinan canal in Scotland.

The calculations in stadia surveying involve corrections in case the measuring direction TM in figure 1 is not horizontal (which can happen even in level Holland). In that case the following approximations can be derived from the figure, see [1]:

Horizontal **D**istance:	$D = A \cdot y \cdot \cos^2(h)$	(1)
Vertical Distance[1] :	$V = A \cdot y \cdot \sin(h) \cdot \cos(h)$	(2)
V can be expressed in D:	$V = D \cdot \tan(h)$	(3)

with:

A = Multiplication factor of the stadia mechanism, in most cases set to 100, but for large area surveys in the USA set to 200

y = Scale length on the vertical stadia rod, as observed between the hairlines

h = The angle of elevation of the telescope

The occurrence of a second "cosine" function in these formulae is caused by the fact that the observed distance is not measured correctly, because the rod stands vertically (see *y*), and not perpendicular to TM (see y^1).

These corrections from measured distance to horizontal and vertical distance (D and V respectively) were also called "reductions". They could be calculated in different ways.

The highest precision was achieved with stadia reduction tables, but those were not very convenient when working outside. They were mostly used back in the office to produce terrain maps from a large number of field measurements.

Nomograms have also been used for reduction calculations.

Slide rules were less precise, but easier to use and more resistent to the weather, especially the versions in stainless steel or other non-magnetic metals.

The continuing development of self-reducing tachymeters, like the Redta from Zeiss and the RDH from Wild, and the more modern optical distance meters, eventually made the manual reductions in stadia surveying redundant.

Slide rules for Stadia Surveying

Most slide rules in the 20th century, and even long before, did have sine and tangent scales, see figure 2 from the "Slide Rule Catalogue" [2].

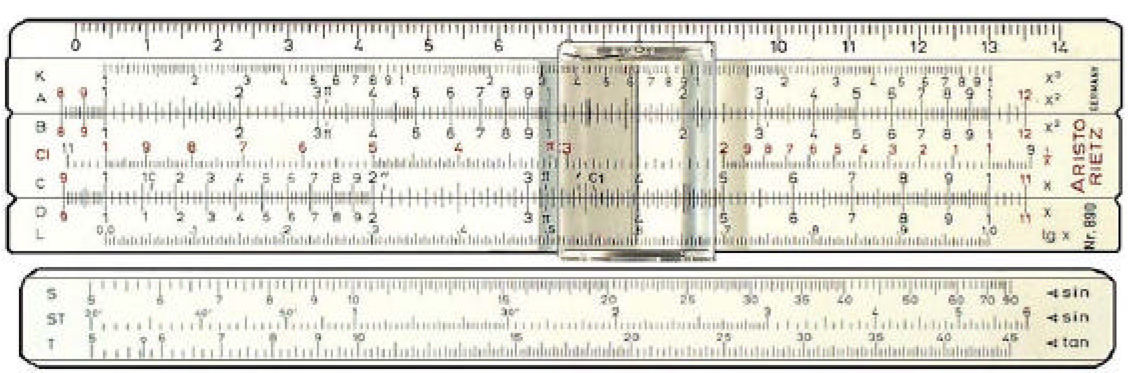

Fig. 2: Sine and tangent scales on the back of a simple slide rule (type Rietz)

[1] Formula (2) actually has to be corrected for height of telescope and of rod centre by adding $H_1 - H_2$

In the usual notation for scales in the SR Catalogue, where the = sign indicates the edge between body and slide, the Rietz scales can be described as

front: K A = B CI C = D L, back: = S ST T =

The "backward" position of the sine scale on generic slide rules made trig calculations cumbersome, while the product of sine and cosine, required in formula (1), could not be done without writing down an intermediate result.

So it is not surprising that a need existed for specialised slide rules to calculate the sin.cos- and the $\cos^2$-functies in stadia surveying. Stadia rules can often be recognised by the scale namings $sin.cos$ and cos^2. In stead of sin.cos, an equivalent formula was sometimes used: ½ sin (2a), especially in the USA.

Table 1 gives an overview of some stadia slide rules which have been produced, with information like originating country, and special form or dimensions. For 19th century rules, the year of introduction is given or estimated. This information has been distilled from various collections, manuals and brochures.

Many stadia rules had both a sexagesimal (360°) and a centesimal (400 gon, also written as 400° of 400^g) version. Anglo-Saxon brands were often restricted to sexagesimal versions.

Geodetical slide rules without stadia scales have been omitted from table 1.

Undoubtedly there are many more brands and types of stadia slide rules.

COUNTRY	BRAND	TYPE
China	Sjanghai	Flying Fish nr. 1005
Denmark	DIWA	Topographical 231 (360°), 241 (400^g)
England	Stanley	Fuller Bakewell Type 3 (cylinder with spiraling scales, 1879)
France	H. Morin [4]	No type indication (400°, German Silver, 40 cm, ˜ 1860)
	Graphoplex	Géomètre Topographe 630 (400^g)
	Tavernier Gravet	"Règle Moinot" nr. 20
Germany	Aristo	Tachymeter 23 (System C. Werner, Vienna, since 1898) Universal-Tachymeter 24 Schweizer Topograph 44 Universal 48 GEODÄT 958, 0958 Stadia Computer 680 (disc Ø 15 cm)
	Faber - Castell	Tachymeter 1/38, 111/38, 4/38 (50 cm), 67/38 (12.5 cm)
	Nestler	9a (Spain, German Silver, 40 cm) Universal 28 Geometer 280 (360°), 281 (400^g), 285 & 286 (50 cm)
	Reiss	System Seiffert 1145
Japan	Sun - Hemmi	Stadia 2690 (bamboo) Civil 269 (bamboo)
	Concise	Stadia Computer, still for sale in 2005! (disc Ø 9.6 cm)

Netherlands	Matthijssen [5]	1) Nr. 170 2) No type indication
Switzerland	Kern (Aarau) [3]	No type indication (German Silver, 22 cm, before 1880)
	Loga	1) Topo 6400 A‰ (military disc in 6400 mils, Ø 12.6 cm) 2) 30 Tt (400°, Ø 12.6 cm)
USA	Gurley Dempster [6], Dietzgen	Cox's Stadia Computer, (Ø 5" disc in a 6" square, copyright 1899 by W. & L.E., Troy, N.Y., USA) Dempster RotaRule type A & AA (1928 – 1948, Ø 12.9 cm, stadia scales Gh and Gv)
	Keuffel & Esser	Colby's Stadia 1749 (1895), 1749-3, 4125 Stadia 4100 (1900), N-4100, 4101, N-4101 (50 cm) Surveyor's Duplex 4102, N-4102 Webb's Stadia Rule 4105 (cylindrical form) Kissam Stadia 4143 (681486, plastic successor to N-4100)

Table 1: Overview of some stadia slide rules

Descriptions of selected types

NESTLER Geometer No. 280 (mid 20th century)

This is an example of a basic tachymetrical slide rule. To a general-purpose Nestler slide rule, two scales were added to the slide, the $\cos^2$ scale and the sin.cos scale. These scales were combined on one line to allow reading D and V value in one setting, and an extra scale was added underneath for small angles of the sin.cos scale (between about 40' and 6°).
These tachymetrical scales were used in combination with the precise D- scale which was copied for that purpose to the usual position of the A-scale. This type therefore had no square scales A and B.

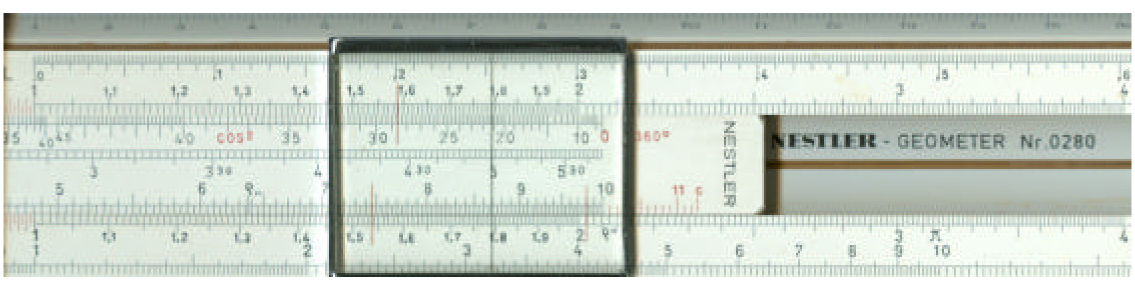

Fig. 3: Stadia slide rule from Nestler

ARISTO GEODÄT No. 0958 (mid 20th century)

This duplex slide rule provides space for many more scales on front and back, thereby increasing the number of functions substantially.
Next to the standard goniometrical S, T and ST scales, additional scales have been provided like a second tangent scale for larger angles, and a P-scale $\surd(1-x^2)$ for sine-cosine conversions. Also the square scales are present.

The $\cos^2$ scale and the sin.cos scale are organised in the same way as the Nestler above, but here in combination with the lower D-scale.
Separate scales for "1-cos" are used for stadia measurements with a horizontal rod.
Other geodetical functions on this Aristo consisted of a separate scale "1/tan(a/2)" for calculating a hypotenuse minus opposite line in a orthogonal triangle, and the possibility to correct for earth curvature and refraction (ER for "Erdkrümmung" & "Refraktion").
Figure 4 shows on a 0958 (400^g) the setting of slide and cursor for measured value A.y = 132 at elevation h = 7^g = 6° 18' : in that case the result is D = 130.4 and V = 14.4.
The figure also shows that the cursor hairline is not even needed when the values used (132 and 7) coincide on exact scale divisions.

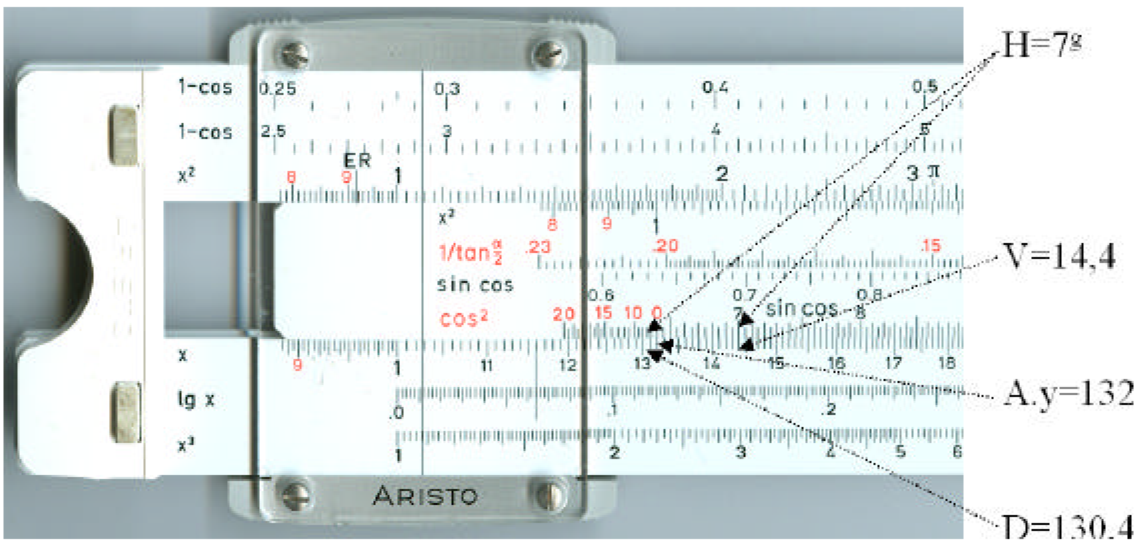

Fig. 4: Stadia slide rule from Aristo

KERN (AARAU), see [3]
The manufacturer of geodetic instruments Kern (before 1885 known as J. Kern) in Switzerland, produced from before 1880 until mid 1900's a compact tachymetrical slide rule from German Silver, with a clever incorporation of a reduction scale in the cursor. The horizontal reduction of a measured distance on the upper logscale (value under the 0° mark on the cursor) could be read on the same upper scale, under the angle of elevation h on the cursor. The vertical reduction could also be read on the upper logscale via the elevation angle on the SIN.COS scale (on the slide). Also this rule has a separate scale with corrections for earth curvature and refraction.
Figure 5 gives for the Kern rule the setting of slide and cursor for a measured value of A.y = 400 with an angle of h=10°; then D = 388 and V = 68.4 (directly over the * indicator of the sin.cos scale).

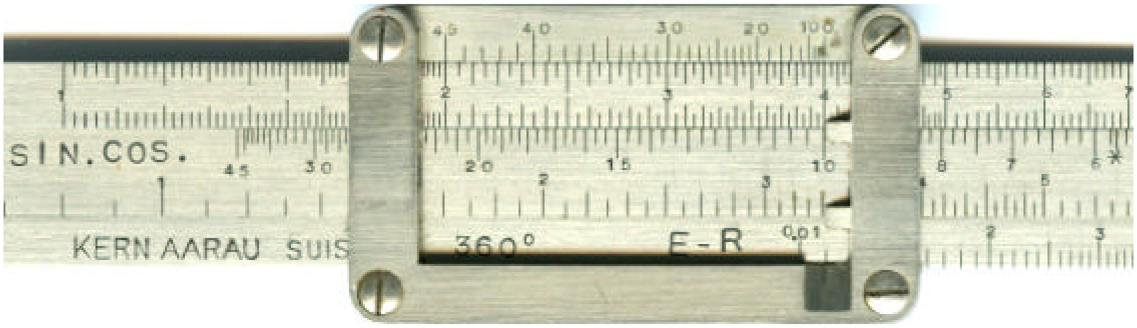

Fig. 5: Stadia slide rule from Kern

GURLEY's STADIA DISC (USA)

This slide chart was designed only for reductions in stadia surveying. Even a normal multiplication or division is not possible. The chart was made of cardboard or celluloid; Perrygraf has also produced this model as Cat. No. 3100 in plastic.

Gurley, a large American manufacturer of surveying instruments since 1845, probably used this chart to go with stadia equipment; maybe it was also presented as a customer gift? The Directions for Use on the disc are self-explaining.

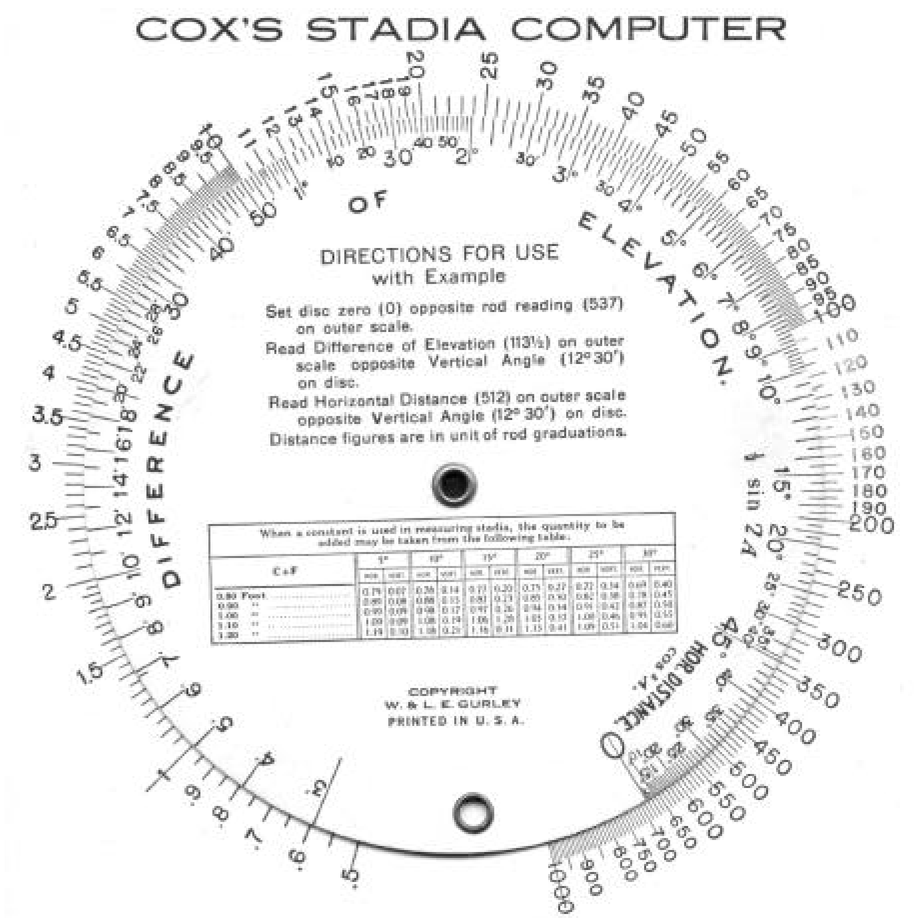

Fig. 6: Cox's Stadia Computer

MORIN

Mid 19th century the Paris firm H. Morin, then at Rue Boursault nr. 3, produced a special tachymetrical slide rule in German Silver.

This type of slide rule has also been supplied as an attribute to Richer theodolites, by "Maison Richer. Guyard & Canary (Paris)", see [5].

This slide rule is not so easy recognisable as the abovementioned ones because the scales do not mention the functions sin.cos and $\cos^2$: instead the tachymetrical scales bear the inscriptions Tang (and the complementary cotangent) and Sin^{-2} (the "–" sign before the "2" is not to be taken literally). The designer Moinot, see [4], used the zenith-angle "z", measured from the zenith, vertically above the observer downward, in stead of the elevation angle h from formula (1) to (3). So "z" is the complementary angle z = 90° - h.

An advantage is that measurements along negative slopes do not have a negative sign (which could get lost during calculations): the zenith-angle is always positive, from 0° - 180°.

The formulae (1) and (3) are converted into:

Horizontale **D**istance:	$D = A \cdot y \cdot \cos^2(h)$	$= A \cdot y \cdot \sin^2(z)$	(4)
Vertical distance :	$V = D \cdot \tan(h)$	$= D \cdot \cot(z)$	(5)

This slide rule has other interesting aspects. The design is without cursor, so all interacting scales have to be adjacent on body and slide. In the usual scale notation of the SR Catalogue [2], the Morin scales are described as follows:

Generic side of the slide up:	Nomb = Sinus	Nomb	= Nomb
Tachymetrical side of the slide up:	Nomb = Tang	P.E. / Sin^{-2} / Sin^{-2}	= Nomb

"Nomb" means Nombre, a 2-decade logscale, so the Morin rule with the generic side of the slide up has the scale configuration: A = S B = A, as compared to figure 2. This means that multiplication and division is possible between number and sine, and between numbers.

When the slide is reversed, the reduced distance can be calculated according to formula (4) on the two lower scales. Keeping the slide in the same position, we can read the vertical distance V on the tang/cotang scale according to formula (5). This particular rule uses the 400 gon system, see figure 7.

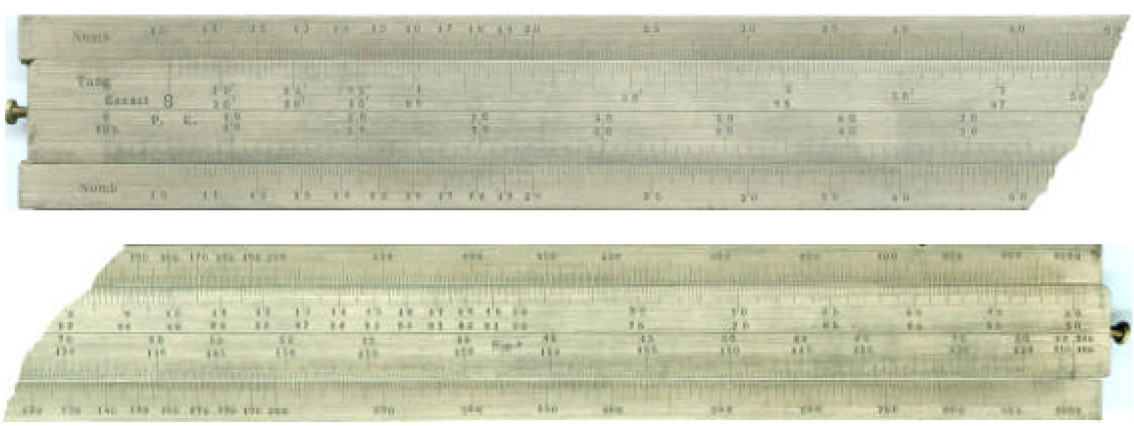

Fig. 7: Stadia slide rule from H. Morin

Another notable characteristic of the Morin rule is the caption *Caract 8* and *9* on the sine and tangent lines. This means "Caractère", the characteristic of the logarithm, which is the part before the decimal point. The part after the decimal point is called "mantissa".

Slide rule users calculate with the mantissa, and they determine the characteristic by estimation and expectation. For the sine and tangent scales however there is a fixed relationship with the characteristic: in a log-tangent table we find for tan(45°) = 1 the logarithm 10.00000, so the characteristic is 10 [2]. In the two decades on the logline left of tan(45°)=1, the *Caract 9* on the Morin rule indicates the range 0.1 to 1, and *Caract 8* indicates the range 0.01 tot 0.1.
The last scale on the Morin rule to be mentioned, is the P.E. scale ("parties égales" or equal parts), representing the L-scale in figure 2 (the logarithmic function itself).

Conclusion

Although the tachymetrical or stadia slide rule could not provide the high precision of the tachymeter tables, a surprising variety and number of this type of slide rules has been designed and produced since the mid-19th's century. In the beginning these calculating rules were supplied as attribute to geodetical instruments (Morin, Richer, Dennert & Pape, Kern, Gurley and Keuffel & Esser), but in the last century the tachymetrical slide rule became indispensable in the product catalogue of major slide rule manufacturers.

Acknowledgments

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Literatuur

[1] Schermerhorn, Prof. Ir. W., Steenis, Ir. H.J., "*Leerboek der landmeetkunde*", Amsterdam, 2e edition, 1946. In Dutch: tachymetry is described in Chapter XIX, the formulae are in XIX.5
[2] Herwijnen, Ir. H. van, "*Slide Rule Catalogue*", Voorschoten, 1990 - 2003
[3] Aeschlimann, Dr. H., "Topographic slide rules by Kern Aarau", *4th International Meeting of Slide Rule Collectors*, Huttwil, Zwitserland, 1998
[4] Moinot, I., "*Levés de Plans à la Stadia*", J. Bounet, Perigueux, 1874
[5] "*Collectie Geodesie*", Technical University Delft, www.museum.tudelft.nl/collecties/index.html , Department of Geodesy, containing a number of tachymetrical slide rules which can be found by inventary number, or by name:
- Richer/Moinot, same design as the Morin rule described above (Inv. Nr. 2000150 and 2000151)
- Dennert & Pape (Inv. Nr. 2000152)
- P. Matthijssen, a Dutch designer and producer of specialised slide rules (Inv. Nr. 2000147 is prototype, Nr. 170, Inv. Nr. 2000148 is a more complex productiontype, without type nr.)
- Slide rule of wood (Inv. Nr. 2000264) like the Kern rule described above, probably from Dennert & Pape. An identical slide rule, by D&P/K&E, is part of the MIT Museum's Keuffel & Esser Cabinet, described and depicted in the Journal of the Oughtred Society, Vol. 14, No. 1, Spring 2005, p. 11

[6] Shepherd, R., "The Dempster RotaRule", *Journal of the Oughtred Society*, Vol. 7, No. 1, Spring 1998, p. 4 – 6

[2] since Gunter's first goniometrical table of logarithms in 1620, this value has always been increased by 10, at that time with the only goal to prevent non-integer values smaller than one