BRUNING

Instruction Manual for

BRUNING

SLIDE RULE NO. 68-540

(Old No. 2401)

(5 INCH RULE)

A·B·CI·C·D·K·S·L AND T SCALES

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BRUNING

Instruction Manual for

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SLIDE RULE

USES OF THE RULE

This rule affords one of the quickest and easiest ways of carrying out multiplications, divisions, and certain other types of computations (which are mentioned in the second paragraph below) when the results to be obtained are not required to be more than 99.5 percent accurate. This means that the slide rule should usually be reliable to three digits. Multiplications, divisions, etc., not easily performed mentally or by brief pencil computation can be quickly completed on the slide rule.

THE THREE PARTS OF THE RULE

This rule consists of three parts (Fig. 1), namely (1) a main body or frame which bears scales *A, D, K;* (2) a slide, which moves in grooves

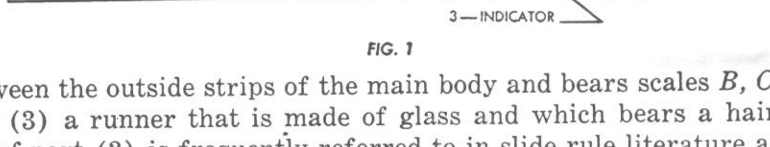

FIG. 1

between the outside strips of the main body and bears scales *B, C, CI,* and (3) a runner that is made of glass and which bears a hairline. All of part (3) is frequently referred to in slide-rule literature as the indicator. Realizing that it is the *hairline* which indicates position, we call this hairline itself the *indicator*. Therefore, we consider the slide rule as consisting of a main *body*, a *slide*, and an *indicator* (hairline)—the several scales on the main body and on the slide being divided, sub-divided, and sub-sub-divided by rulings of logarithmic scales; by this statement, we mean that each scale of the rule bears sets of marks which divide the scale into three kinds of divisions, called *primary, secondary,* and *tertiary divisions.* We explain such divisions in Section 1 below.

SUGGESTED METHOD OF LEARNING TO USE THE SLIDE RULE

Learn the following things (which we refer to as steps) in order:

1. How to read the *C* and *D* scales.
2. How to multiply, divide and solve proportions.
3. How to find the square or the square root of a number.
4. How to find the cube or the cube root of a number.
5. How to find the reciprocal of a number.
6. How to multiply or divide by use of the *CI* and *D* scales.
7. How to find the product of three factors at one setting of the slide by use of the *C, CI,* and *D* scales.
8. How to divide and multiply successively.
9. How to find sines of angles by use of the *S, A,* and *B* scales; and how to find associated cosecants, cosines, and secants.
10. How to find tangents of angles.
11. How to solve certain triangle problems with one setting of the slide.
12. How to find logarithms of given numbers, and numbers which have given logarithms.

We discuss Steps 1 to 12 in 12 sections which follow. Throughout this discussion, the reader should understand the terms *significant figures.* For the purposes of slide rule calculations, in any number the significant figures start with the first figure to the left that is not zero and end with the last figure to the right that is not zero. But since the 5″ slide rule cannot be read closer than to three figures we will be concerned only with the first three significant figures of any number in reading and setting the rule.

1. How to Read the *C* and *D* Scales.

The *C* and *D* scales are identical (see Figs. 2, 3, 4 below); *C* is on the slide and *D* is on the main body. Fig. 2 shows the primary marks on *C* and *D*. These are 1 [which is at the extreme left and

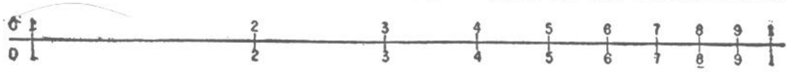

FIG. 2

is called the left index (l.i.)], 2, 3, etc., to 9, followed by another 1 [which is called the right index (r.i.)]. The r.i. is read as follows: 10 when the l.i. is read 1; 100 when the l.i. is read 10; 1000 when the l.i. is read 100; 1 when the l.i. is read .1; .1 when the l.i. is read .01; etc.

The first significant digit of a number is represented by one of the primary marks (Fig. 2). For example, the digit 5 in 52 would be represented by the mark that is labeled 5 in Fig. 2.

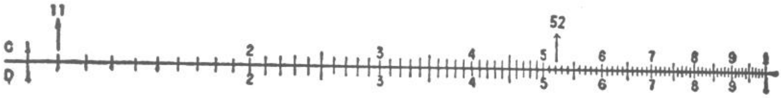

FIG. 3

Fig. 3 shows the primary and secondary marks on *C* and *D*. The second significant digit of a number is represented by either a secondary or by a primary mark. For an example of the former type, consider 11. To represent 11, we would let the l.i. on *D* represent 10. Then the first 1 to the right of the l.i. (between 1 and 2) would represent $10 + 1 = 11$. In Fig. 3, one arrow is drawn through 11 and another through 52. For an example in which the second significant digit of a number is represented by a primary mark, see Example 1 below.

FIG. 4

Fig. 4 shows the complete *C* and *D* scales of this slide rule. From the l.i. to the r.i., it contains 10 secondary divisions to each primary division, and tertiary divisions as follows: from the l.i. to 2, 5 tertiary divisions to each secondary division; from 2 to 5, 2 tertiary divisions to each secondary division; and from 5 to the r.i., no tertiary division. We next present several examples which indicate how to locate on *D* numbers that have three significant digits.

Example 1. Locate 102 on *D*.

Solution. Let the l.i. on *D* represent 100. Then the first tertiary mark to the right (Fig. 5) represents

$$100 + \frac{1}{50}(200-100) = 100 + 2 = 102$$

since there are 50 tertiary divisions between 1 and 2 (here 100 and 200).

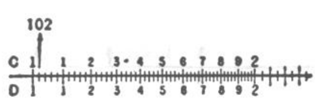

FIG. 5

Note. If we should let the l.i. on *D* represent 10, 1, or .1, then the position just determined in Example 1 to represent 102 would represent 10.2, 1.02, or .102, respectively.

Example 2. Locate 250, 251, 252, 253, 254, 255 on *D*.

Solution. Let the l.i. on *D* represent 100, so that the primary marks 2 and 3 (Fig. 6) represent 200 and 300, respectively. Then the fifth secondary mark to the right of 2, through which the

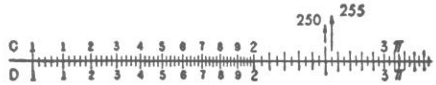

FIG. 6

arrow to the left is drawn, represents 250. The next mark to the right is a tertiary mark which represents

$$250 + \frac{1}{20}(300-200) = 250 + 5 = 255$$

since there are 20 tertiary divisions between 2 and 3 (here 200 and 300). The arrow to the right passes through this tertiary mark. To represent 251, 252, 253, 254, one estimates positions which are, respectively, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{4}{5}$ the way from the former mark (left arrow) to the latter (right arrow).

Note. If we should let the l.i. on *D* represent 10, then the positions located in Example 2 would represent 25.0, 25.1, 25.2, 25.3, 25.4, 25.5.

Example 3. Locate 640, 645, 650 on *D*.

Solution. Let the l.i. on *D* represent 100, so that the primary marks 6 and 7 (Fig. 7) represent 600 and 700, respectively. Then the fourth secondary mark to the right of 6, through which the arrow is drawn, represents 640. (There are no tertiary marks between 6 and 7). The fifth secondary mark to the right of 6,

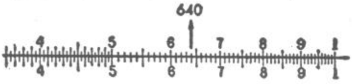

FIG. 7

the first one to the right of the arrow, represents 650. To represent 645, one estimates a position half way between the arrow and the first mark to its right.

Note. If we should let the l.i. on *D* represent 1, then the positions located in Example 3 would represent 6.40, 6.45, 6.50.

EXERCISE FOR THE READER

Verify (perhaps by aid of the indicator) that the arrows in Fig. 8 may be regarded as locating the numbers toward which they point.

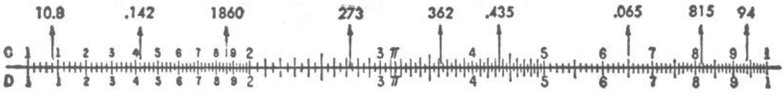

FIG. 8

2. How to Multiply, Divide, and Solve Proportions.

Example 1. Multiply 2 by 3.

Over 2 on *D* set the l.i. on *C* (Fig. 9);
under 3 on *C*, read the answer, 6, on *D*.

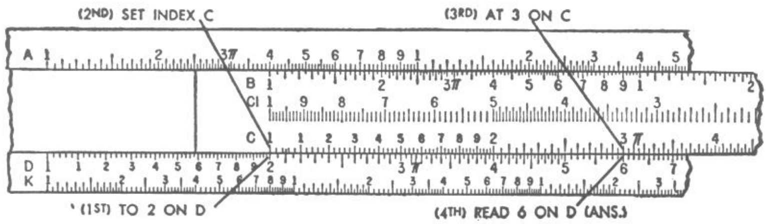

FIG. 9

Example 2. Multiply 17 by 69.

Set the r.i. on *C* over 69 on *D;*
under 17 on *C,* read the answer, 1170, on *D.*

Since the last digits of 17 and 69 are 7 and 9, respectively, and $7 \times 9 = 63$, the exact product would end in 3. Therefore, our answer is in error by 3 in the fourth place.

Note. In Example 1, we used the l.i. of *C* and in Example 2, we used the r.i. of *C.* If on using one index of *C,* the factor on the slide falls beyond the limits of *D,* then the other index of *C* is the *proper* one to use.

Rule for multiplication. To multiply two numbers together:

Set the proper index of *C* to one number on *D;*
under the other number on *C,* read the product on *D.*

Example 3. Divide 9 by 3.

Over 9 on *D* set 3 on *C* (Fig. 10);
under the l.i. of *C,* read the answer, 3, on *D.*

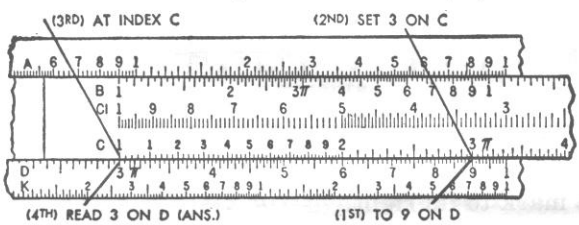

FIG. 10

Example 4. Divide 13.8 by 29.5.

Set 29.5 on *C* over 13.8 on *D;*
under the r.i. of *C,* read the answer, .468, on *D.*

Rule for division. Remembering that $\dfrac{\text{Dividend}}{\text{Divisor}} = \text{Quotient}$, we state:

set the divisor on *C* over the dividend on *D;*
under the index of *C,* read the quotient on *D.*

PROPORTION.

Example 5. If 5 yards of cloth cost \$2.30, how much would 30 yards cost?

We wish to find *X* in the proportion

$$\frac{2.30}{5} = \frac{X}{30}$$

Set 5 on *C* over 2.30 on *D;*
under 30 on *C*, read $X = 13.80$ on *D.*

Therefore the cost of 30 yards would be $13.80.

Note. In Example 5, the denominators 5 and 30 were both on *C,* and the numerators 2.30 and *X* were both on *D.* Since the *C* and *D* scales are identical, their roles could be interchanged, thus putting 5 and 30 both on *D,* and 2.30 and *X* both on *C.*

Note 2. In Example 5, we read on *D* beneath 30 on *C* to find the cost of 30 yards of cloth. We could have read on *D* beneath any number *N* on *C* to have found the cost of *N* yards of the same cloth.

EXERCISE FOR THE READER

Compute the following products and quotients by slide rule:

1. $13 \times 15 = 195$. (Set 1 on *C* over 15 on *D;* under 13 on *C,* read 195 on *D*).
2. $22 \times 18 = 396$. (Set 1 on *C* over 22 on *D;* under 18 on *C,* read 396 on *D*).
3. $1.3 \times 1.5 = 1.95$. (See Problem 1).
4. $\frac{138}{17} = 8.1$. (Set 17 on *C* over 138 on *D;* under r.i. on *C,* read 8.1 on *D*).
5. $\frac{13.8}{.17} = .81$ (See Problem 5).
6. If an airplane travels 2400 miles in 6.5 hours, how long would it take this plane to travel 1000 miles at the same rate? Hint. Find *X* in the proportion

$$\frac{6.5}{2400} = \frac{X}{1000}$$

Ans. $X = 2.71$ hours.

3. **How to Find the Square or the Square Root of a Number.**

The *A* and *B* scales are identical, and each one of these is a double scale. Each of the two *A* scales is built according to principles of logarithms, as is the *D* scale, but each *A* scale is just half as long as the *D.* Due to this fact, any number *N* on *A* is the square of the corresponding number *N′* on *D*:

(1) $$N = (N')^2; \; N' = \sqrt{N}.$$

We presently apply equations (1) in examples.

Example 1. Find the square of 29.5.

Solution. Set the indicator to 29.5 on *D;* under the indicator read the answer, 870 (rather than 87.0 or 8700), on *A.* (See Fig. 11.)

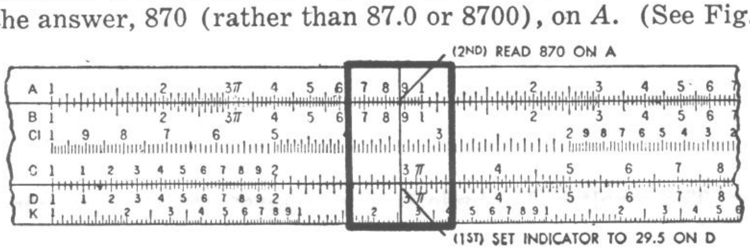

FIG. 11

Note. The result in Example 1 may be interpreted as implying that the square of 2.95 is 8.70; that of .295 is .0870; that of .0295 is .000870; that of 295 is 87000, to three significant digits, which are (8, 7, 0), the zero here being the first one to the right of 7 in 87000; etc.

Example 2. Find the length of the side of a square whose area is 45 sq. ft.

Solution. Since 45 has an even number of digits to the left of its decimal point (45.), we use the right hand A scale. Set the indicator to 45 on this scale; under the indicator, read the answer, 6.7 (in feet), on D. (The three figure result would be 6.70 feet in this case).

Example 2 illustrates how to find the square root of any number N with an *even number of digits to the left of its decimal point;* and this procedure holds also for numbers like .45, .0045, .000045 which can be obtained by shifting the decimal point in N an even number of places to the left. (See Problems 7, 8 in the next list of exercises for the reader).

If the square root of a number N with an *odd number of digits to the left of its decimal point* is desired, locate the number on the left hand A scale and its square root on D; and similar procedure applies for numbers like .06, .0006 which can be obtained from one of these numbers N by shifting its decimal point an even number of places to the left. (See Problem 9 in the next list of exercises).

EXERCISE FOR THE READER

By use of the slide rule, verify the following results, perhaps by referring to equations (1) above.

1. $3^2 = 9$.
2. $4^2 = 16$.
3. $15^2 = 225$.
4. $16^2 = 256$.
5. $\sqrt{25} = 5$.
6. $\sqrt{121} = 11$.
7. $\sqrt{60} = 7.75$.
8. $\sqrt{.60} = .775$.
9. $\sqrt{.060} = .245$.
10. $(3.3)^2 = 10.9$ to three significant figures.
11. $(4.02)^2 = 16.2$ to three significant figures.

4. How to Find the Cube or the Cube Root of a Number.

There are three K scales, each of which is constructed according to principles of logarithms and is just one-third as long as the D scale. Due to this fact, any number N on K is the cube of the corresponding number N' on D:

$$(1) \qquad N = (N')^3; \; N' = \sqrt[3]{N}.$$

We presently apply equations (1) in examples.

Example 1. Find the cube of 5.2.

Solution. Set the indicator to 5.2 on D; under the indicator, read the answer, 141 (rather than 14.1 or 1410), on K (See Fig. 12).

Note. The result in Example 1 may be interpreted as implying that the cube of .52 is .141; that of .052 is .000141; that of 52 is 141000, — each result being to three significant figures.

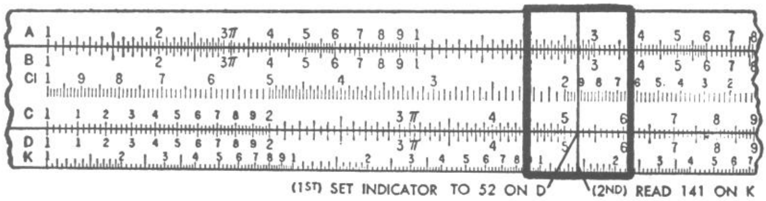

FIG. 12

Example 2. Find the length of the edge of a cube whose volume is 35 cubic feet.

Solution. Since 35 has two figures to the left of its decimal point (35.), we use the middle *K* scale. Set the indicator to 35 on this scale; under the indicator, read the answer, 3.27 (in feet), on *D*.

If we wish to find the cube root of any number *N* which has 2, 5, 8, . . ., $2 + 3n$ (*n* any positive integer) digits to the left of its decimal point, we locate *N* on the middle *K* scale. Similarly, if *N* has 1, 4, 7, . . ., $1 + 3n$ digits to the left of its decimal point, we locate *N* on the left hand *K* scale; and if *N* has 3, 6, 9, . . ., $3 + 3n$ digits to the left of its decimal point, we locate *N* on the right hand *K* scale.

In the last paragraph, *n* was restricted to be a positive integer. However, if the given number, *N*, be a decimal fraction, each statement of that paragraph still applies. Shift the decimal point to the right three places at a time until one or more significant figures are to the left of the transposed decimal point, then choose the correct *K* scale as noted in the preceding paragraph. By inspection and approximation determine the decimal point in the result. For example, to find the cube root of .000125, we shift the decimal point $2(3) = 6$ places to the right, getting 125, whose cube root is 5 (as one finds by using the right hand *K* scale and *D*). Then it is easily seen that the cube root of .125 is .5 and of .000125 is .05.

EXERCISE FOR THE READER

By use of the slide rule, verify the following results, each of which is a case of one of the equations (1) above:

1. $2^3 = 8$.
2. $4^3 = 64$.
3. $\sqrt[3]{27} = 3$.
4. $\sqrt[3]{512} = 8$.
5. $(3.5)^3 = 42.9$
6. $(4.5)^3 = 91$ (to the nearest integer).

5. How to Find the Reciprocal of a Given Number.

The *CI* scale is a *C* scale inverted, as the letters *C* and *I* suggest. Consequently, the *CI* scale is read from right to left. For any position of the indicator on *CI*, the number located is the reciprocal of the corresponding number on *C*, and vice versa. The reciprocal of any number is 1 divided by that number. We will illustrate the determination of the reciprocal of a given number with an example.

Example 1. Find the reciprocal of 4.

First solution. Set the indicator to 4 on *C* (Fig. 13) and under the indicator read the answer, .25, on *CI*.

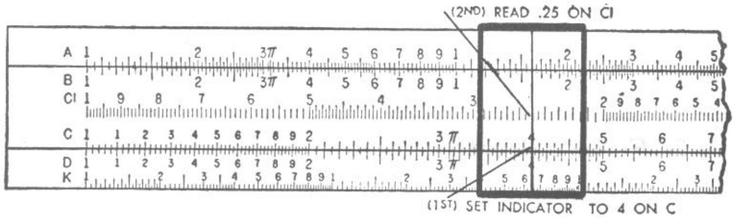

FIG. 13

Second solution. Set the indicator to 4 on *CI*, and read the answer, .25, on *C*.

From Example 1, it follows that the reciprocal of .4 is 2.5, of 40 is .025, of 400 is .0025, etc.

6. How to Multiply or Divide by Use of the *CI* and *D* Scales.

The next two examples are merely to prepare the reader for Section 7; *we still advise the reader to carry out simple multiplications and divisions by the method of Section 2.*

Example 1. By use of the *CI* and *D* scales, multiply 25 by 33.

Solution. Set the indicator to 33 on *D*; bring 25 on *CI* to the indicator; and under the r.i. on *C*, read the answer, 825, on *D* (Fig. 14).

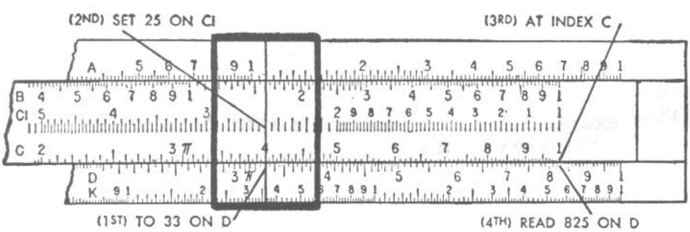

FIG. 14

We could have interchanged the roles of 33 and 25 in solving Example 1. Generally, multiplication with *CI* and *D* can be carried out as follows: to multiply two factors together, set the indicator to one factor on *D*; bring the other factor on *CI* to the indicator; and read the answer on *D* at that index of *C* which is on the scale.

Example 2. By use of the *CI* and *D* scales, divide 345 by 18.

Solution. Set the r.i. of *C* to 345 on *D;* move the indicator to 18 on *CI;* and read the answer, 19.2, under the indicator on *D*.

Generally, division with *CI* and *D* can be carried out as follows: to divide a given number *a* by a another number *b*, set the (appropriate) index of *C* to *a* on *D*; move the indicator to *b* on *CI;* and read the answer under the indicator on *D.*

7. How to Find the Product of Three Factors at One Setting of the Slide by Use of the *C*, *CI*, and *D* Scales.

We give one example which relates the desired procedure to that which was used in solving Example 1 of Section 6.

Example 1. By use of the *CI, D,* and *C* scales, compute the value of $25 \times 33 \times (.645)$.

Solution. Repetition of Example 1, Section 6, locates $25 \times 33 = 825$, under the r.i. of *C* on *D*. Our present problem is, therefore, to find the product of $825 \times (.645)$. The r.i. of *C* being on 825 on *D*, we read under .645 on *C*, the answer, 532 on *D* (Fig. 15), the result being obviously greater than 53.2 and less than 5320.

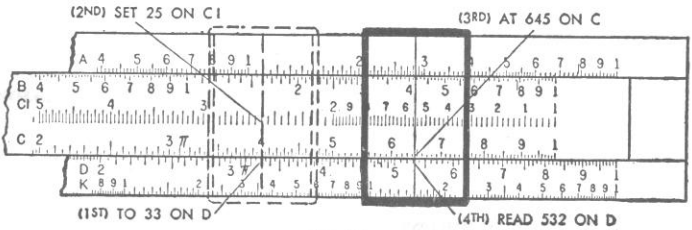

FIG. 15

EXERCISE FOR THE READER

By use of the *C* and *CI* scales, verify that:

1. the reciprocal of 2 is .5.
2. the reciprocal of 5 is .2.
3. the reciprocal of 12 is .0833.
4. the reciprocal of 35 is .0286.

By use of the *CI* and *D* scales, compute the following products and quotients:

5. $28 \times 32 = 896$.
6. $36 \times 4.4 = 158$.
7. $\frac{40}{17} = 2.35$.
8. $\frac{7300}{56} = 130$.

By use of the *CI, D,* and *C* scales, compute each of the following products with a single setting of the slide:

9. $17 \times 39 \times (.04) = 26.5$.
10. $48 \times (.21) \times 52 = 524$.
11. $23 \times 27 \times (.19) = 118.0$.
12. $135 \times (.0625) \times 3.45 = 29.1$.

8. How to Divide and Multiply Successively.

Example 1. By use of the slide rule, compute the value of $\frac{55 \times 43}{72}$.

Solution. (1) Set 72 on *C* over 55 on *D*;
(2) move indicator to 43 on C;
(3) under indicator, read the answer, 32.8 on *D*.

Discussion. After Step (1), the value of the quotient $\frac{55}{72}$, nearly .764, was under the r.i. of *C* on *D*. Step (2) multiplied this quotient by 43. It is interesting to note that in solving Example 1 it is not necessary to read the value of the quotient just mentioned.

Example 2. By use of the slide rule, compute the value of $\frac{645 \times 77 \times (.048)}{585 \times 68 \times (.051)}$.

Solution.
(1) Set 585 on *C* over 645 on *D*;
(2) move the indicator to 77 on *C*;
(3) bring 68 on *C* under the indicator;
(4) move the indicator to .048 on *C*;
(5) bring .051 on *C* under the indicator;
(6) under l.i. of *C*, read the answer, 1.18 on *D*.

Note. The area of a circle $= \frac{\pi}{4} \times (D)^2$. The value of $\frac{\pi}{4} = .7854$ is located by a special long mark near 8 on the right hand *A* and *B* scales. To find the area of a circle set this .7854 mark on *B* to the r.i. of *A*, set the indicator to the diameter on *D*, read the area under the indicator on *B*.

EXERCISE FOR THE READER

By use of the slide rule, verify that

1. $\frac{275 \times 35}{255} = 37.7.$

2. Area of circle with diameter of 7.65″ = 46.0 sq. in.

3. $\frac{385 \times 129 \times 86}{296 \times 142 \times 59} = 1.72$ (See Example 2; note that the slide must be moved its length to the left in multiplying by 86).

9. How to Find Sines of Angles by Use of the *S*, *A*, and *B* Scales; and How to Find Associated Cosecants, Cosines, and Secants.

If an angle *a* is as large as 34′ (34 minutes) and is less than 90°, there are two main ways of finding its (natural) sine. These we state and illustrate presently.

First method of finding sines of angles. Place the *S* and *T* scales outward, with the l.i. of *A* aligned with that of *S*. Then, to find the sine of any angle *a*, move the indicator to *a* on *S* and under the indicator read sin*a* on *A*.

Second method of finding sines of angles. With *S* and *T* on the reverse side of the slide, set the upper index line in the right hand slot on the reverse side of the rule to any angle on the *S* scale; then read the sine of this angle under the r.i. of *A* on *B*.

Caution. Before exemplifying the last two paragraphs above, we remark that all (natural) sines that are read on the left hand *A* or *B* scale have one zero between the decimal point and the first significant figure, while sines that are read on the right hand *A* or *B* scale have no zero between the decimal point and the first significant figure.

Example 1. Find sin30° by the first method, and by the second method.

Solution. Proceeding exactly as directed by the first method, we find sin30° opposite 5 on the second *A* scale. Therefore, sin30° = .5. Then applying the second method, we find that 5 on the second *B* scale is under the r.i. of *A* when the angle is at the upper index mark in the right slot on the reverse side of the rule (See Fig. 16) ; and so we conclude here also that sin30° = .5.

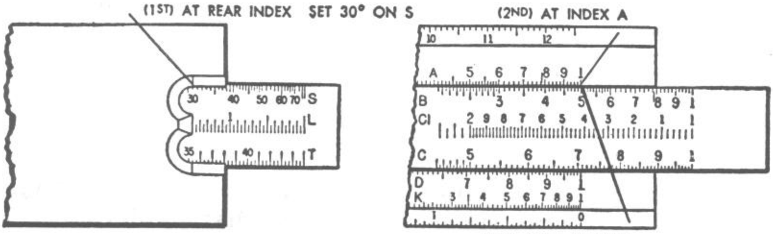

FIG. 16

Note. In applying the second method, there is no difficulty in finding *cosecants* of angles. They are read over the l.i. of *B* on *A*. For example, in the setting last used, we found sin30° = .5 under the r.i. of *A* on *B*. At the same time, over the l.i. of *B*, we find 2 on *A*; that is, csc30° = 2. *The second method is advantageous in that it gives cosecants and sines simultaneously.*

Cosines of angles can be found from sines by use of the known formula,

(1) $$\sin(90° - x) = \cos x.$$

Thus, from the case $x = 60°$ of (1), we have $\sin(90° - 60°) = \cos 60°$; i.e.,

(2) $$\sin 30° = \cos 60°.$$

Hence, from Example 1, we know that cos60° = .5. Further, *secants* can be found from cosines by the known equation

(3) $$\sec x = \frac{1}{\cos x}.$$

For example, with $x = 60°$, we know from (3) that sec60° is the reciprocal of .5; that is, sec60° = 2.

Example 2. Find sin2° 30′ and csc2° 30′.

Solution. With *S* and *T* on the reverse side of the rule, we set 2° 30′ on *S* to the upper index mark in the right slot; then under the r.i. of *A*, we read on the first *B* scale .0436 :sin2° 30′ = .0436. With the same setting, we read over the l.i. of *B* on *A* the value of csc2° 30′: csc2° 30′ = 22.9.

Sines of angles less than 34′.

The simplest method of finding sines of such angles is to reverse the slide, turning *S* and *T* outward, and proceed as in the following examples.

Example 3. Find sin12′ on the slide rule.

Solution.

Set the minute gauge mark on S to 12 on A;
over whichever index of S is on the rule,
read sin12′ = .0035 on A.

Note. Since it is known that sin1′ = .0003 nearly, in Example 3, two zeros should precede the first significant figure of the answer.

Example 4. Find sin36″ on the slide rule.

Solution.

Set the seconds gauge mark on S to 36 on A;
over the l.i. of S, read sin36″ = .000175 on A.

Note. Since it is known that sin1″ = .000005 nearly, in Example 4, three zeros should precede the first significant figure of the answer.

10. How to Find Tangents of Angles.

No angle less than 5° 43′ can be read on T. However, for slide rule purposes, the tangent of any angle less than 5° 43′ may be taken equal to the sine of the angle. Therefore, the tangent of such an angle can be found by methods previously given.

To read the natural tangent of any angle between 5° 43′ and 45°, place the S and T scales on the reverse side of the rule, set the angle on T to the lower index mark in the right hand slot, and then read over the r.i. of D on C the desired tangent. Further, a second method is to place the S and T scales outward, align the indices of T and D, and read directly on D the tangents of opposite angles on T, as illustrated in Example 1 below. Tangents read in either of these ways have no zero between the decimal point and the first significant figure.

Example 1. Find tan40°.

First solution. Place the S and T scales on the reverse side of the rule; set 40° on T to the lower index mark in the right hand slot; over the r.i. of D, read on C the answer, tan40° = .839.
Second solution. Place the S and T scales outward, with indices of D and T aligned; set the indicator to 40° on T; under the indicator, read on D the answer, tan40° = .839.

Remark. It is known that tan$x$ and cot$x$ are reciprocals of each other, when they are numbers different from zero. Consequently, in the setting used in the first solution of Example 1, one can *read under the l.i. of C on D the cotangent of 40°:* cot40° = 1.19. But cot$x$ = tan(90° − x), and so tan(90° − 40°) = tan50° = 1.19. The first method of solution of Example 1 can be used to find the tangent and the cotangent of any angle from 5° 43′ to 45°.

From these cotangents (tangents), one obtains, by the **formula**

(1) $$\tan(90^\circ - x) = \cot x,$$

the tangents (cotangents) of acute angles that are larger than 45°.

Example 2. Find $\tan 7^\circ\ 20'$ and $\cot 7^\circ\ 20'$.

Solution. Place the S and T scales on the reverse side of the rule; set $7^\circ\ 20'$ on T to the lower index mark in the right hand slot; over the r.i. of D, read on C the result $\tan 7^\circ\ 20' = .128$; under the l.i. of C, read on D the result $\cot 7^\circ\ 20' = 7.77$.

Note. Applying equation (1) in connection with the results of Example 2, we find

$$\tan 82^\circ\ 40' = 7.77, \quad \cot 82^\circ\ 40' = .128.$$

11. How to Solve Certain Triangle Problems with One Setting of the Slide.

If the angles of any triangle are A, B, C and the corresponding opposite sides are a, b, c, respectively, then the following equations are known to hold

(1) $$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}.$$

It is customary to call relations (1) **the law of sines. In particular**, if $C = 90^\circ$, so that $\sin C = 1$, then (1) reduces to

(2) $$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{1}{c}.$$

In the following example, we use the slide rule in connection with (2) to solve a right triangle with one setting of the slide.

Example 1. Given $A = 42^\circ$, $a = 37.5$; solve the right triangle ABC (in which $C = 90^\circ$).

Solution. We are to find B, b, c. $B = 90^\circ - A = 48^\circ$. The proportion to be solved is (See (2))

$$\frac{\sin 42^\circ}{37.5} = \frac{\sin 48^\circ}{b} = \frac{1}{c}.$$

We complete the solution as follows.

Set 42° on S under 37.5 on A (first or second A scale).
Over 48° on S, read $b = 41.6$ on A.
Over 90° on S, read $c = 56.0$ on A.

With $C = 90^\circ$ a single setting of the slide also suffices when A and c, B and c, or B and b are given.

Remark. In the solution of triangles, various proportions are employed; and if slide rule accuracy is sufficient, the slide rule solution should be used because of the speed with which it can be obtained.

12. How to Find Logarithms of Given Numbers and Numbers Which Have Given Logarithms.

First method. To find the mantissa of the logarithm of a given positive number N, set the l.i. of C to N on D; in the right hand slot, read on L beneath the slot index the required mantissa.

Example 1. Using the slide rule, find log35.

Solution. To 35 on D set the l.i. of C; at the slot index, read the required mantissa, namely .544. By the rule of characteristics of logarithms (which is stated below in this section), the characteristics of log35 is 1. Therefore,

$$\log 35 = 1.544.$$

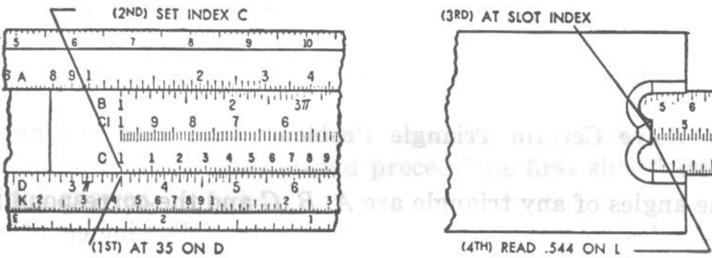

FIG. 17

The characteristic of the logarithm of a number which has an integral part is 1 less than the number of digits in the integral part. Thus the characteristics of the logarithms of 4, 40, 400 are 0, 1, 2, respectively.

The characteristic of the logarithm of a proper decimal is negative and 1 greater except for sign than the number of zeros between the decimal point and the first non-zero digit. Thus the characteristics of the logarithms of .4, .04, .004 are −1, −2, −3, respectively.

Second method. With L outward, set the zero of L to N on D. Set the indicator to the l.i. of D; under the indicator, read the mantissa of the logarithm of N on L.

If we are given the value of log$N$ and desire to find N, we may do so by reversing the procedure of either of the methods just described.

Example 2. Given $\log N = 1.432$; find N.

Since the characteristic of log$N$ is 1, there are two digits to the left of the decimal point in N. We complete the solution as follows, using the second method.

Set the indicator to the l.i. of D;
set .432 on L to the indicator;
under the zero of L, read the answer, 27.0, on D.

EXERCISE FOR THE READER

By slide rule, verify that:

1. sin11° = .191.
2. cos79° = .191.
3. sin7° = .122.
4. csc7° = 8.21.
5. tan13°30′ = .240.
6. cot13°30′ = 4.17.
7. sec20° = 1.06 (See Prob. 8).
8. tan3°20′ = .058.
9. log28 = 1.447.
10. log500 = 2.699.

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