K+E

ANALON™

ENGINEERING – SCIENCE

ANALYSIS SLIDE RULE

INSTRUCTION MANUAL

by

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ANALON

ENGINEERING—SCIENCE ANALYSIS SLIDE RULE

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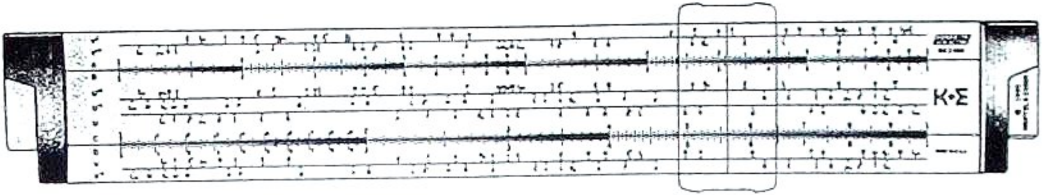

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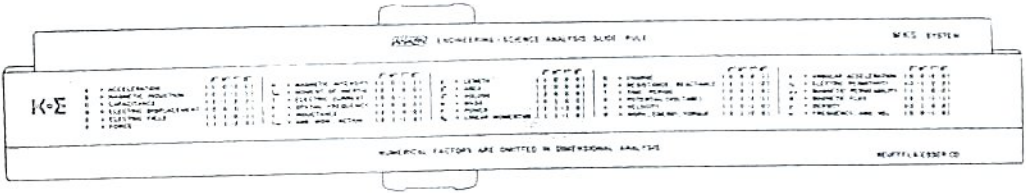

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INTRODUCTION

Chances are, since you have purchased the ANALON Slide Rule, you are interested in slide rules and know something about them.

You know, for instance, that all numerical slide rules are based on logarithms. You can add them to multiply, and subtract them to divide. The various scales—folded, inverted, to the base e, etc., are based on the same principles. The more types of scales, the more calculations you can make.

The basic problems that a numerical slide rule solves are multiplication, division, taking roots, and raising to powers. Until the ANALON Slide Rule was invented, there was no similar tool for dealing with physical quantities having both size and dimension. This new slide rule extends the basic principles of the slide rule to dimensions and can be a very useful device. With your ANALON Slide Rule you will be able to perform dimensional operations, just as with your numerical rule you perform numerical operations. Your ANALON Slide Rule also has several of the most useful numerical scales so you may perform many numerical operations directly on your dimensional rule. Your ANALON Slide Rule can be used in the following ways:

(1) For checking equations dimensionally

Is it true that $\dfrac{\text{resistance} \times \text{charge}}{\text{inductance}} = \text{current?}$

(2) As an aid in recalling formulas

For a conducting sphere of radius r, is the capacitance C given by

$$C = \frac{\epsilon_0 r}{4\pi} \quad \text{or by} \quad C = \frac{r}{4\pi\epsilon_0}\ ?$$

(3) **As an aid in deriving formulas**

If the formula for the period of a simple pendulum of length ℓ in a gravitational field of acceleration g has the form

$$T = A\ell^r g^s,$$

what are the values of r and s ?

(4) **For performing numerical calculations**

Multiplication, division, squares, square roots, and other numerical calculations may be performed. For example,

$$\frac{6.88 \times 176}{2\pi\sqrt{215 \times .0449}} = ?$$

The first three of the four uses fall into the category of *dimensional analysis*. You can obtain the answers to the above problems in a few seconds on the ANALON Slide Rule.

The ANALON Slide Rule is an aid to students, engineers, scientists, and teachers in understanding and employing dimensions and units in a simple yet rigorous manner.

Remember—a slide rule, like a list of formulas or a table of values, is not intended to substitute for thinking. It helps you by acting as your "bookkeeper" in performing tasks which would take much longer by hand. The ANALON Slide Rule will not eliminate the creative thinking process that leads you to a specific formula or derivation, but it will make the checking-out process faster and simpler.

Attain accuracy first. Let the speed come later. And keep in mind that numerical slide rules are accurate to about one percent on the more tightly compressed scales. A hairline's difference can mean a few thousandths error in some numerical calculations. The precise setting of scales is especially important in using the ANALON Slide Rule.

CHAPTER I

THE ANALON SLIDE RULE

1. What is the ANALON?

Your ANALON Slide Rule is equipped with scales on which physical dimensions such as *length*, *time*, *mass*, *charge*, *force*, $\sqrt{force}$, $(force)^2$, $(force)^{-1}$, etc. are located.

These dimensions can be multiplied, divided and operated upon in much the same way as ordinary numbers. For example, dividing *length* by *time* gives the result *velocity* in much the same way that dividing 24 by 6 gives the result 4. Similarly, multiplying *resistance* by $(current)^2$ gives the result *power* in much the same way that multiplying 12 by $(3)^2$ gives the result 108.

We must distinguish between **dimensions** and **units.**

The term dimensions refers to physical concepts such as *length*, *mass*, *time*, etc. Associated with every dimension are certain units of measure or simply units. For example, *length* may be measured in units of feet, centimeters, microns, etc.

A dimension is not affected by the size of the unit of measure employed; length remains length regardless of whether it is measured in units of feet, angstroms, or cubits.

Your ANALON Slide Rule multiplies and divides **dimensions,** not units. It is thus a *qualitative* slide rule rather than a *quantitative* slide rule. A length, if doubled, is still a length, and a mass, if halved, is still a mass. *Therefore, in dimensional calculations all numerical factors are ignored.* For example, in the formula

$$E = \tfrac{1}{2}mv^2,$$

the *dimensional* relationship is

$$Energy = mass \times (velocity)^2,$$

and the factor $\tfrac{1}{2}$ is ignored.

3. Reading the scales

The numerical scales are read as on a numerical slide rule. If you are not familiar with the theory and operation of the numerical slide rule, refer to the K&E DECI-LON® Manual No. 68 2069 or any of the other excellent texts available.

The dimensional scales might appear confusing at first, but they are extremely easy to read. Instead of the location of all possible numbers (up to four digits), these scales are concerned only with the location of a relatively small number of specific quantities.

The U and V scales

The physical quantities are shown directly on the U and V scales (see Fig. 2a). For example, the symbol F on the V scale represents the physical quantity FORCE. The vertical black line below the symbol F is its exact location. Unless you read that line, you are not reading FORCE.

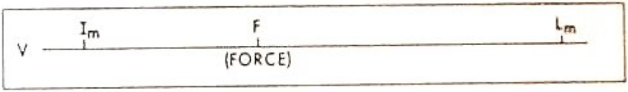

FIG. 2a

The U^{-1} and V^{-1} scales

The U^{-1} and V^{-1} scales are the inverse scales (see Fig. 2b). Following slide rule convention, the inverse scales are in red. The quantities located on these scales are the reciprocals of the symbols shown. For example, when F is located on either of these scales, it is equivalent to F^{-1} or $1/F$.

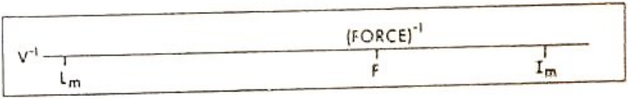

FIG. 2b

Operations with dimensions, such as multiplication, division, etc., are performed in the same manner as numerical calculations, with the dimensions taking the place of numbers.

2. Physical description

Your ANALON Slide Rule consists of three parts; the body, the slide, and the indicator. Readings are made with the use of the vertical hairline on the window of the indicator (see Fig. 1).

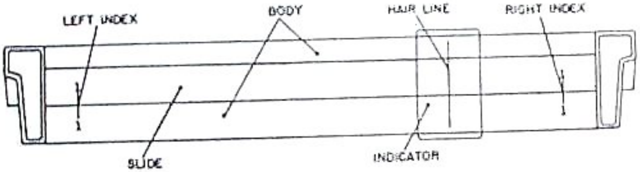

FIG. 1

The mark associated with the number 1 on a numerical scale is called the index of the scale. Each numerical scale (A, B, C, and D) has two indices, one at the left end and one at the right end.

The front face of the ANALON Slide Rule consists of seven dimensional scales (three on the slide and two on each rail) and the A, B, C, and D numerical scales.

The U and V scales contain dimensions which are so common as to have standard names and symbols. These are tabulated on the reverse face of the ANALON Slide Rule.

The other dimensional scales contain powers of the dimensions shown on the U and V scales. The various powers (-1, $\frac{1}{2}$, 2) have been placed on separate scales to make them easier to locate and to avoid congestion. The exponents (-1, $\frac{1}{2}$, 2) have been omitted to simplify the appearance. It is helpful to remember that in principle all dimensions (including powers) could have been crowded onto the U and V scales.

On the reverse face of the ANALON Slide Rule is a reference table for the 30 symbols used on the seven dimensional scales. This table also reduces each of these 30 dimensions to a combination of four basic dimensions; Length, Mass, Time, and Electric Charge.

The $U^{1/2}$ and $V^{1/2}$ scales

The $U^{1/2}$ and the $V^{1/2}$ scales are square root scales (see Fig. 2c). Quantities located on these scales are equivalent to the square roots of the symbols shown. For example, the symbol F on these scales represents $(\text{FORCE})^{1/2}$ or $\sqrt{\text{FORCE}}$.

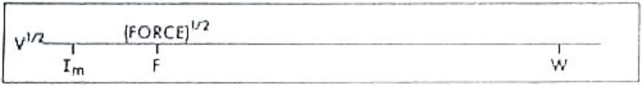

Fig. 2c

The V^2 scale

The V^2 scale is a square scale (see Fig. 2d). Quantities located on this scale correspond to the square of the symbols on it. For example, the symbol F on the V^2 scale represents $(\text{FORCE})^2$.

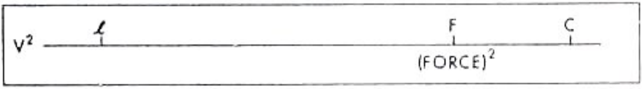

Fig. 2d

4. Multiplication and division

The scales of a *numerical* slide rule are sequences of numbers arranged according to a logarithmic law, whereas the *dimensional* scales on the Analon Slide Rule are arrays of symbols representing the physical quantities. The symbols are located according to the physical laws and definitions.

The dimensions of an equation obey the laws of algebra, thus the placement of the symbols on the rule is also by the logarithmic scheme. Each symbol has been assigned a numerical value. For example, the symbol v (Velocity) on the U scale is represented by the number 299 (we ignore the power of ten just as on numerical scales).

The physical quantities, e.g. Force, Density, $\sqrt{Mass}$, $(Velocity)^2$, etc., are treated as numbers and are multiplied and divided in the same way as ordinary numbers.

The U and V scales contain the physical quantities directly, and correspond numerically to the C and D scales. The inverse scales, U^{-1} and V^{-1}, contain the reciprocal values of the quantities listed on them, and the $U^{1/2}$ and $V^{1/2}$ and the V^2 scales contain the square roots and squares respectively of the quantities on them. These auxiliary scales (other than the U and V) are designed to aid and speed your use of the ANALON Rule.

The ANALON Slide Rule scales are set up in the MKS system of units. Conversion to other systems of units is possible. For conversion between systems, see Appendix B, page 27.

For practice, follow the simple examples given below:

Example 1. Dimensionally investigate the expression

$$\frac{Q^2}{4\pi\epsilon_0\ell^2}.$$

Method 1.

Set the left index of the C scale* opposite Q on the V scale, push hairline to Q on the U scale.†

Note that no quantity appears on the V scale under the hairline, but the number on the D scale, 257, must represent the quantity Q^2 (see Fig. 3a).

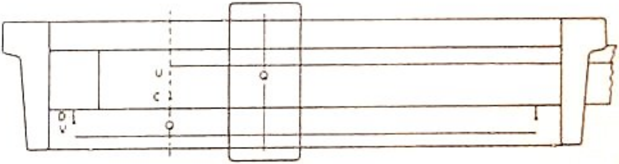

FIG. 3a

Draw ℓ on the U scale under the hairline.

Opposite the right index of C, it appears that Q^2/ℓ on the V scale has the dimension of α, but close inspection shows that it does not. Be careful! Remember what we said about accuracy.

* You will find it useful to use the hairline to help set the indices.

† We could just as well start by setting the hairline to Q on the V scale, and drawing Q on the U^{-1} scale under the hairline.

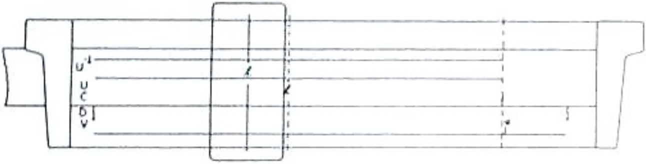

Fig. 3b

Push hairline to ℓ on the U^{-1} scale (see Fig. 3b),
(the V scale now shows Q^2/ℓ^2)
draw ϵ_0 on the U scale under the hairline,
opposite the index read F (Force) on the V scale under the hairline (see Fig. 3c).

The factor 4π is purely numeric, and does not enter into the dimensional calculations.

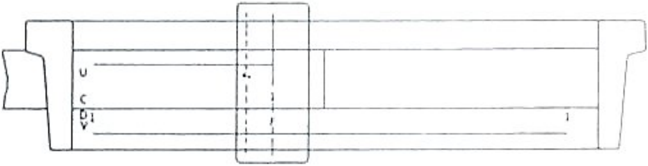

Fig. 3c

Method 2.

The amount of work in Method 1 may be considerably shortened through the use of the auxiliary scales.

Push hairline to Q on the V^2 scale,
draw ℓ^2 on the U scale under the hairline,
push hairline to ϵ_0 on the U^{-1} scale,
at the hairline read F on the V scale (see Fig. 4).

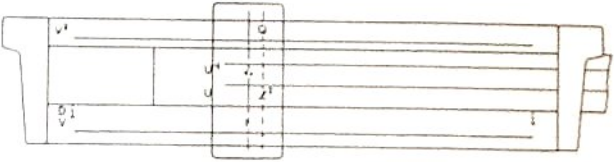

Fig. 4

The result shows that

$$\frac{Q^2}{4\pi\epsilon_0\ell^2}$$

is dimensionally equivalent to F (Force). This is as it should be since physically it represents the force of repulsion between two identical charges Q separated by a distance ℓ.

Example 2. Consider the calculation of

$$\sqrt{(\text{Inductance}) \times (\text{Capacitance})} = ?$$

Symbolically, this is

$$L^{1/2} \times C^{1/2} = ?$$

since the square root quantities appear directly on the Analon Rule.

Set the right index of the C scale opposite L on the $V^{1/2}$ scale,
push hairline to C on the $U^{1/2}$ scale,
the result T (or ω^{-1})* is read under the hairline on the V scale (see Fig. 5).

Fig. 5

* Remember, since numerical factors are ignored on the Analon, ω (radians/sec) and f (cycles/sec) are dimensionally equivalent.

5. Quantities not shown

When a specific quantity (or a combination of quantities) is of interest to you, and it does not appear on the rule, it may be easily added. Using the edge of the indicator as a guide, scratch the mark with the point of a compass (or other sharp instrument), and fill the indentation with ink. Record the new symbol in your manual, or on the reverse face of the ANALON Slide Rule in the same manner. If, for example, the dimension *pressure* (force divided by area) appears often in your work, it may be easily added. Since F/A falls at 1967 on the C & D scales, the edge of the indicator may be set to this value and used as a guide for scribing the mark.

CHAPTER II

DIMENSIONS, UNITS, AND DIMENSIONAL ANALYSIS

6. Dimensions and units

You do not need to understand the theory of logarithms to operate a numerical slide rule, but you must completely understand the mathematics to which you are applying the slide rule. Similarly, in order to use the ANALON Slide Rule, you must understand the fundamentals of dimensions and units.

A dimension is a tag or a label. It is the name we give to a physical quantity. Familiar examples are: force (F), velocity (v) and length (ℓ).

A unit is the amount of a physical quantity that is assigned the size or value 1, or unity. For the dimension "length", the unit of measurement might be the foot or the yard or the meter, etc. The size of units are dictated by convenience. A carpenter uses feet and inches, whereas an atomic physicist might employ the angstrom as the unit of length.

The relationships among the physical quantities—the physical laws—are expressed as equations. Such an equation, for example,

$$F = \frac{Gm_1m_2}{\ell^2}$$

expresses the gravitational force of attraction F between masses m_1 and m_2 separated by distance ℓ. The symbol G is a proportionality constant. The equation must be dimensionally consistent; that is, the combination of quantities on the right hand side must be equivalent to *force*. The constant G must be of such size and dimension that the equation balances numerically *and* dimensionally. The quantity G depends upon the units chosen. As an example, if F is in dynes, m in grams, and ℓ is in centimeters, then

$$G = \frac{F\ell^2}{m_1m_2} = 6.67 \times 10^{-8} \quad \text{dyne } cm^2/gm^2.$$

If a different system of units is chosen, G will have a different numerical value and different units, consistent with the new system. It should be clear that the dimensions of G will not in general change. It remains $\frac{\text{(Force) (Length)}^2}{\text{(Mass)}^2}$.

A similar example is Coulomb's law,

$$F = \frac{kq_1q_2}{\ell^2}.$$

This expresses the force of repulsion F between charges q_1 and q_2 separated by distance ℓ. By selecting k, various systems of units may be defined. If F is in newtons, q in coulombs, and ℓ in meters, then k is given by

$$k = \frac{1}{4\pi\epsilon} = 9 \times 10^9 \quad \text{newton (meter)}^2/\text{(coulomb)}^2.$$

If we select $k = 1$ and dimensionless, the units are those defined as electrostatic system of units (esu system), and we have absorbed the dimensions of the constant into the physical quantities of the esu system. For example, q in the esu system contains a power of the velocity of light in its dimension.

We have no control over the dimensions of quantities except as new quantities of interest are defined, but we have relative freedom over the choice of the system of units to be employed in a given situation.

There are several standard systems of units in use today. In all of the systems, the dimensions of all physical quantities can be expressed in terms of a few *basic* dimensions. For example, if length (L), mass (M), time (T) and electric charge (Q) are selected as basic, all other quantities* may be expressed in terms of them. Velocity will be $L\,T^{-1}$, force would be $M\,L\,T^{-2}$, etc. Which dimensions are chosen as basic is determined by the relative ability to measure the quantities of interest. A system often used in science and engineering is the MKSC system (Meter, Kilogram, Second, Coulomb). This is a system whose basic dimensions are length, mass, time, and charge. If current is easier to measure than electric charge, then the basic dimensions would be L, M, T, and I, where Q would be $I\,T$ dimensionally. In 1960 the General Conference on Weight and Measure

* If we want to include thermodynamic problems, we will need an additional basic dimension—Temperature.

adopted the MKSA system (Meter, Kilogram, Second, Ampere) as standard. This system is called the SI system—the *Système Internationale*.

7. Dimensional analysis

Physical equations are dimensionally homogeneous. This means that each term of an equation must be expressed in the same combination of basic dimensions. Clearly, a dimensional check of an equation will not show purely numerical factors, algebraic signs, etc., but it will disclose many possible errors. Consider the following example in which we wish to check the expression dimensionally:

$$v = \frac{xM}{m}\sqrt{\frac{g}{L}},$$

where v is the velocity of a bullet of mass m fired into a ballistic pendulum of mass M and length L, x being the forward travel of the pendulum and g the acceleration of gravity.

Replacing each quantity in terms of the basic dimensions M, L and T,

$$\left[\frac{L}{T}\right] = \frac{[L][M]}{[M]}\sqrt{\frac{[L\,T^{-2}]}{[L]}} = \left[\frac{L}{T}\right],$$

we see that the expression checks dimensionally.

If a physical situation may be formulated in terms of n quantities (Mass, Force, etc.), and the n quantities may be expressed in terms of m basic dimensions, then there are $(n - m)$ different dimensionless constants that may be formed. This concept is useful in deriving possible formulas.

If a problem involves only x (distance), v (velocity), and t (time), then $n = 3$ and $m = 2$ (L and T). There is only one dimensionless quantity which can be formed: $\frac{x}{vt}$ or $\frac{vt}{x}$. The Analon Slide Rule quickly shows that x and vt are dimensionally equivalent.

Consider the problem of deriving (or recalling) the formula for the period of oscillation of a simple pendulum. The period t is assumed to be a function of the length ℓ, the mass m and the acceleration of gravity g. We can express this combination in the form

$$k = \textit{dimensionless constant} = t^a \ell^b g^c m^d$$

where the exponents are to be found. Since we are looking for t in terms of the other quantities, we select the value of a as 1 (unity) and solve for t,

$$t = k\ell^{-b}g^{-c}m^{-d},$$

or in terms of basic dimensions,

$$[T] = k\,[L]^{-b}\,[L\,T^{-2}]^{-c}\,[M]^{-d}.$$

Equating the exponents of the basic dimensions on both sides of the equation, we get

$$\begin{array}{lcl} L & : & 0 = -b - c \\ M & : & 0 = -d \\ T & : & 1 = 2c \end{array}$$

Hence, $d = 0$, $c = \frac{1}{2}$, $b = -\frac{1}{2}$; and the final equation is

$$t = k\sqrt{\frac{\ell}{g}}\,.$$

This is known to be the correct form. An experiment would show that $k = 2\pi$. Note that if we had not selected the value of a, we would have had four unknowns and only three equations. We will do this problem with the ANALON Slide Rule in Chapter III.

Here is another simple example. We wish to determine a possible form for the kinetic energy W of a moving body of mass m and velocity v. Proceeding as before, we get

$$k = W^a m^b v^c,$$

and selecting a as 1, we obtain a solution,

$$W = km^{-b}v^{-c}.$$

In terms of basic dimensions obtained from the back of the ANALON,

$$[M\,T^{-2}\,L^2] = k\,[M]^{-b}\,[L]^{-c}\,[T]^{+c}.$$

Equating the exponents of the basic dimensions on both sides of the equation, we get

$$\begin{array}{lcl} L & : & 2 = -c \\ M & : & 1 = -b \\ T & : & -2 = c \end{array}$$

This gives $b = -1$, $c = -2$; and the final equation is

$$W = kmv^2.$$

You will recognize this when $k = \frac{1}{2}$.

Later you will see how the ANALON Slide Rule can help you solve the above problems with ease.

CHAPTER III

APPLICATIONS

8. Dimensional checking

Dimensional checking of equations is used to detect errors such as missing factors, incorrect powers of terms, etc. Examples 1 and 2 illustrate dimensional checking.

Example 1.

Check the expression

$$\Delta B = \frac{\mu_0 I \Delta \ell \sin \theta}{4\pi r^2}$$

giving a B field, ΔB, due to an elemental length of conductor, $\Delta \ell$, in terms of current I and the geometry of the problem. The left hand side is magnetic flux density. The $\sin \theta$ and the 4π are numerics (dimensionless) and $\Delta \ell / r^2$ is equivalent to one over length. On the Analon Slide Rule, $\frac{\mu_0 I}{\ell}$ is seen to be equivalent dimensionally to B, thus checking the equation as far as consistent dimensions are concerned.

Example 2.

Refer to the first example on page 1 in the Introduction.

Is it true that $\frac{\text{resistance} \times \text{charge}}{\text{inductance}}$ = current? Symbolically, is

$$\frac{RQ}{L} = I \ ?$$

To check this equation,

push the hairline to R on the V scale,
draw L on the U scale under the hairline,
push the hairline to Q on the U scale,
under the hairline read I on the V scale.

The answer is YES.

9. Recalling formulas

In trying to recall specific formulas, the physical quantities involved are often remembered, but the specific way in which they appear is not clear. The ANALON Rule is an aid to remembering the correct form of formulas. Examples 3 and 4 illustrate the procedure.

Example 3.

Is the repulsive force between two electric charges q_1 and q_2 separated by a distance r given by

$$F = \frac{q_1 q_2}{4\pi\epsilon_0 r} \quad \text{or} \quad F = \frac{q_1 q_2}{4\pi\epsilon_0 r^2} \;?$$

Set the left index to Q on the V^2 scale,
push the hairline to ϵ_0 on the U^{-1} scale,
draw ℓ^2 on the U scale under the hairline,
at the left index read Force, F, on the V scale.

Therefore the formula with r^2 is the correct one.

Example 4.

In trying to recall the formula for the capacitance C of a conducting sphere of radius r, you recall that C depends upon ϵ_0 and r, but you do not recall the exact form. Is it

$$C = \frac{\epsilon_0}{4\pi r}, \quad C = \frac{\epsilon_0 r}{4\pi}, \quad \text{or} \quad C = \frac{r}{\epsilon_0 4\pi} \;?$$

The ANALON Slide Rule shows that $\epsilon_0 \ell$ has the dimensions of capacitance, therefore $C = \frac{\epsilon_0 r}{4\pi}$.

10. Deriving formulas

It was shown in Chapter II that dimensional analysis is a powerful tool for deriving possible formulas. Since the ANALON Rule contains the dimensional information of the physical quantities, it may be employed to do the dimensional analysis. Examples 5 and 6 illustrate this procedure.

Example 5.

We wish to derive a possible formula for the period of oscillation of a simple pendulum (see Chapter II, Section 7). We know (or we assume) that t depends upon the length and the mass of the pendulum and the acceleration of gravity, of some form

$$t = k\ell^a g^b m^c.$$

We must determine values of a, b and c such that the right hand side of the equation has the dimensions of time. The ANALON Slide Rule shows that ℓ/g has the dimension of T^2 on the V^2 scale, and ℓg has the dimension of v^2, neither of which contain a mass term. The v^2 is not useful as we do not have an additional length term. Since mass cannot be included, we conclude that $c = 0$, and since ℓ/g is T^2, the result is

$$t = k\sqrt{\frac{\ell}{g}}.$$

Example 6.

What is the formula for the kinetic energy of a moving particle of mass m and velocity v? This one is very easy. Assuming a solution of the form

$$W = km^a v^b,$$

only two exponents are to be found. The ANALON Slide Rule shows that mv is momentum, p_m, and that multiplying p_m by v gives W, work or energy, and

$$W = kmv^2.$$

No other combination of m and v will dimensionally yield energy.

11. Performing numerical calculations

The A, B, C, and D scales of your ANALON Slide Rule may be used for multiplication, division, proportion, squares, and square roots. If you are not familiar with numerical calculations with a slide rule, refer to one of the many manuals on the subject, such as the K&E DECI-LON® Manual No. 68 2069.

12. Relating the numerical and dimensional scales

The numerical scales used in conjunction with the dimensional scales allow you to consider quantities and combination of quantities not marked on the ANALON Rule, and to work with combinations of quantities which do not represent a particular physical quantity. Remember that the physical quantities appear on the rule as symbols, but are in fact locations of specific numbers corresponding to the particular physical quantity.

A combination of quantities may fall between symbols on the rule. You recall this happened in Example 1 on page 16. The following example illustrates the numerical and dimensional scale interrelations.

Example 7.

The resistance of a uniform conducting wire of length L, cross section area A, and conductivity σ is given by

$$R = \frac{L}{\sigma A}.$$

If we solve this expression for σ, we get

$$\sigma = \frac{L}{RA}.$$

This has the dimensions of $(R\ell)^{-1}$. We see on the ANALON Slide Rule that $(R\ell)^{-1}$ falls near the quantity α on the V scale, at about 762. This number now represents σ, conductivity, and the symbol may be added to the rule, or the number may be recorded in your manual in Table I. If the number near 762 appears in future problems, you might expect it may be σ, and a detailed investigation of the problem will yield the answer.

You must be careful! If the number 755 occurs in a dimensional investigation, since it falls between α and σ, you must refer to the physics of the problem to determine the correct quantity.

A Word of Caution.

You must not let the ANALON Slide Rule do your thinking in a physical problem. A basic understanding of physics is necessary in the solution of a problem. Your ANALON Slide Rule is an aid. It saves you work to free your time, and it indicates possibilities, which you may follow through.

CHAPTER IV

PRACTICE PROBLEMS

13. Solving problems

The following group of problems are for practice in becoming familiar with the operation of the ANALON, and to further illustrate the type of problem for which the ANALON is applicable. The problems are relatively simple so that you may easily verify the solutions using conventional dimensional analysis techniques.

Work the problems using the ANALON Slide Rule alone, and then compare the ease of solution with that of the conventional methods. The solutions to the problems are listed at the end of the Chapter.

For additional practice, select relations from a physics or engineering book. You will be pleasantly surprised at the consistency of the ANALON Rule.

As you work these and other problems, keep the following points in mind:

A. When using the ANALON Slide Rule, as in regular dimensional analysis, an algebraic sign (+ or −) is ignored.

B. Quantitatively, 6 feet + 4 feet = 10 feet, but dimensionally length + length = length. The number of terms in an equation is unimportant dimensionally as long as they are consistent.

C. Numerical quantities such as 5, π, $\cos \frac{\pi}{6}$, log 7.5, etc. are not considered.

D. The symbols for the physical quantities are not all standard. In addition, in a given problem several symbols may be employed for the same type of physical quantity. For example, length ℓ, radius r, distance x, wavelength λ, etc. are all length. With a little practice you will easily find the correct symbol on the ANALON Rule.

E. In such functions as e^x, $\sin \theta$, $\cosh \phi$, etc., the quantities x, θ, ϕ must be dimensionless. For example, in $e^{\alpha t}$, α must have the dimension of $1/T$ such that αt is dimensionless.

F. Dimensionally, the interpretations of derivatives and integrals are very simple.

The n^{th} derivative of a quantity y with respect to x is dimensionally y/x^n. For example

$$\frac{d^2}{dt^2}i(t) \quad \text{is} \quad \frac{[I]}{[T^2]} \text{ dimensionally.}$$

The integral of y with respect to x is dimensionally yx. For example

$$\int i(t)dt \quad \text{is} \quad [IT] \text{ dimensionally.}$$

14. Exercises

Solve the following problems. Use your ANALON Rule first.

1. What is the physical meaning of the expression $\sqrt{\frac{F}{\mu}}$, where F is the tension in a stretched wire of mass per unit length μ?

2. Find the correct expression for the inductance of a solenoid of N turns, length ℓ, cross section area A, and permeability μ.

   (a) $L = \dfrac{N^2\mu A^2}{\ell}$, (c) $L = \dfrac{N^2\mu A}{\ell}$,

   (b) $L = \dfrac{N^2\mu\ell}{A}$, (d) $L = \dfrac{N^2\mu A}{\ell^2}$.

3. The expression for the period of oscillation of a compound pendulum is

$$T = 2\pi\sqrt{\frac{x}{mgh}},$$

   where m is the mass, g the acceleration of gravity, and h the distance from the center of gravity to the point of suspension. Determine the unknown quantity x.

4. What is the interpretation of

   (a) $\dfrac{1}{\sqrt{LC}}$, (b) $\dfrac{1}{\sqrt{\mu_0\epsilon_0}}$?

   Can you relate them?

5. What is the relationship between the energy W of a photon, and its frequency f?

6. Which of the following expressions are equivalent to frequency?

(a) t^{-1},

(b) R/L,

(c) C/R,

(d) $-\dfrac{R}{2L}+\sqrt{\dfrac{R^2}{4L^2}-\dfrac{1}{LC}}$,

(e) $\dfrac{C_1+C_2+C_3}{(R_1+R_2)C_1C_2C_3}$.

7. Find the incorrect term in the following expression involving translational and rotational kinetic energy, where I represents the moment of inertia.

$$W=\frac{mv^2}{2}+\frac{I^2\omega^2}{2m}.$$

8. The period of oscillation T, of a simple pendulum is a function of its length L (among other things). Two different periods T_1 & T_2 corresponding to the two length L_1 & L_2 are measured, and we would expect a relationship of the form

$$\frac{T_1}{T_2}=\left(\frac{L_1}{L_2}\right)^a.$$

Can you find the value of a employing the ANALON Rule?

9. A student finds in his class notes that power loss in an iron cored choke conducting an alternating current is given by the expression

$$P=\pi BHVf.$$

In his haste in recording the formula, he neglected to identify the quantities involved. He has forgotten whether V is voltage or volume. Which is it?

10. Find the value of n such that the combination

$$\frac{D\rho V^n}{\mu}$$

is a dimensionless group (a dimensionless combination of quantities related to a specific area). This group is called Reynolds number, employed in fluid dynamics.

D — Diameter of cross section of fluid channel
ρ — Fluid density
V — Average fluid velocity
μ — Fluid viscosity

Note that the symbol μ does not appear on the ANALON. It would appear at the value 2282 on the D scale, and has the dimensions of Mass per Length per Time or $\frac{[M]}{[LT]}$.

11. Find a possible expression for electrical power P in terms of voltage applied V, resistance R, inductance L, and capacitance C.

12. Perform a dimensional check on the differential expression

$$\frac{di}{dt} = \frac{V}{L} e^{-\frac{R}{L}t}.$$

13. Perform a dimensional check on the differential equation

$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{i}{C} = \frac{dv}{dt}.$$

14. Perform a dimensional check on the integral forms of Maxwell's equations (the basic equations of electromagnetic theory).

(a) Gauss' law (Electric): $\oint \vec{D} \cdot \vec{ds} = Q.$

(b) Gauss' law (Magnetic): $\oint \vec{B} \cdot \vec{ds} = O.$

(c) Ampere's law: $\oint \vec{H} \cdot \vec{dl} = i + \frac{d}{dt}\oint \vec{D} \cdot \vec{ds}.$

(d) Faraday's law of induction: $\oint \vec{E} \cdot \vec{dl} = -\frac{d}{dt}\phi.$

where E — Electric field intensity
B — Magnetic induction
H — Magnetic field intensity
s — surface (over a closed surface)
l — length (along a closed curve)
D — Electric displacement
ϕ — Magnetic flux
t — Time
q — Electric charge

15. Solutions

1. Velocity (of a transverse wave in the wire)

2. c

3. I_m (moment of inertia)

4. (a) f or ω (frequency)
   (b) v (velocity)
   $v = f\ell$, where ℓ must now be wavelength.

5. $W = L_m f$. Here, you will recognize this particular angular momentum as Plank's constant, h.

6. a, b, & d.

7. The expression $\dfrac{I^2\omega^2}{2m}$ should be $\dfrac{I\omega^2}{2}$.

8. No. Both terms are dimensionless ratios.

9. V is volume.

10. $n = 1$

11. $P = \dfrac{V^2RC}{L}$

12. The expression is dimensionally correct.

$$[I/T] = [V/L], \text{ and } [R/L]^{-1} = [T]$$

such that the exponent is dimensionless.

13. The expression is dimensionally correct.

$$[LI/T^2] = [RI/T] = [I/C] = [V/T].$$

14. (a) $[D\ell^2] = [Q]$. The expression is dimensionally correct.

    (b) Zero is dimensionally indeterminate, hence no dimensional check is possible.

    (c) $[H\ell] = [I] = [D\ell^2/T]$. The expression is dimensionally correct.

    (d) $[E\ell] = [\phi/T]$. The expression is dimensionally correct.

APPENDIX A

UNIT CONVERSIONS

Converting a physical quantity from one unit to another, such as converting power from watts to horsepower, or pressure from pounds per square inch to newtons per square meter, can be accomplished with the use of tables of unit equivalents such as Table II. The need for very extensive tables containing all possible equivalent unit conversions can be avoided by reducing the quantities of interest to a set of basic dimensions and converting the units of these basic dimensions. Any set of basic dimensions may be chosen, such as Length-Mass-Time-Charge, Length-Mass-Time-Current, Length-Force-Time-Charge, etc.; however, the choice will normally be dictated by the quantities to be converted.

As an example, it is required to convert a mass density of size ρ_0 in gm/cm^3 to its equivalent value in $lb/in.^3$. The quantity ρ_0 is in terms of basic dimensions Mass and Length, thus the needed relationships in the two systems are for Mass* (pounds & grams) and Length (inches & centimeters). From Table IIa,

$$1 \text{ kgm} = 2.205 \text{ lb (or } 1 \text{ gm} = 2.205 \times 10^{-3} \text{ lb);}$$

thus wherever the unit "grams" appears, it is to be replaced by its equivalent in pounds, or multiplied by the number of pounds in one gram (lb/gm). That is,

$$\text{gm(lb/gm)} = \text{lb, or gm}(2.205 \times 10^{-3}) = \text{lb,}$$

which is stated as "number of grams times 2.205×10^{-3} equals number of pounds."

* The pound (lb) is a measure of mass in the Foot/Pound-mass/Second (f lbm s) system, and a measure of force in the Foot/Pound-force/Second (f lbf s) system. In the latter, force is a basic dimension.

Similarly, since

$$1 \text{ meter} = 10^2\text{cm} = 3.28 \text{ ft.} = 39.4 \text{ in.},$$

$$1 \text{ cm} = .394 \text{ in.}, \quad \text{thus (in./cm)} = .394,$$

$$\text{and} \quad \text{cm(in./cm)} = \text{in.}, \quad \text{thus cm}^3\text{(in./cm)}^3 = \text{in.}^3,$$

$$\text{or} \quad \text{cm}^3(.394)^3 = \text{cm}^3\,(.0611) = \text{in.}^3.$$

The density conversion is now

$$\rho_0 \frac{\text{gm(lb/gm)}}{\text{cm}^3\text{(in./cm)}^3} = \rho_0 \frac{2.205 \times 10^{-3}}{.0611} \frac{\text{lb.}}{\text{in.}^3} = .0361\, \rho_0 \text{ lb/in.}^3.$$

APPENDIX B

DIMENSIONAL SYSTEM CONVERSIONS

Conversions among dimensional systems are generally performed through the use of tables such as Table IIb, but it is interesting to look at an example in some detail. The term "dimensional system conversion" might tend to be misleading, since systems are defined by themselves and are not dependent upon other systems. They are, however, related in terms of the defining equations of the systems.

Consider the force between two electric charges in the SI (MKS) and the esu systems.

SI System	esu System
$F = \dfrac{Q_1 Q_2}{4\pi\epsilon_r\epsilon_0 r^2}$	$F = \dfrac{Q_1 Q_2}{\epsilon_r r^2}$

where the quantities are:

F in newtons	F in dynes
Q in coulombs	Q in statcoulombs
r in meters	r in centimeters

ϵ_0 in farads/meter, or $\dfrac{\text{amp sec}}{\text{volt meter}}$

and ϵ_r is dimensionless in both systems.

To determine the relationship between electric charge in the SI system and electric charge in the esu system, consider a force of one newton at a separation of one meter in free space ($\epsilon_r = 1$). Recall that 1 newton = 10^5 dynes and 1 meter = 10^2 centimeters.

SI System	esu System
$1 \text{ newton} = \dfrac{Q^2}{4\pi\epsilon_0(1\text{m})^2}$	$10^5 \text{ dyne} = \dfrac{Q^2}{(10^2 \text{ cm})^2}$

or

$$Q = \sqrt{4\pi\epsilon_0}\ \text{coulomb} \qquad\qquad Q = \sqrt{10^9}\ \text{statcoulomb}$$

or

$$Q = \frac{\sqrt{10} \times 10^{-5}}{3}\ \text{coulomb} \qquad\qquad Q = \frac{10^5}{\sqrt{10}}\ \text{statcoulomb.}$$

Thus, $$Q_{(SI)}\,[2.998 \times 10^9] = Q_{(esu)}$$

or

1 coulomb = 2.998×10^9 statcoulomb (see Table IIb).

The reader is referred to any of the many text books on electromagnetic theory or dimensional systems for more complete discussions of the esu and emu systems.

Table I BASIC DIMENSIONAL EQUIVALENTS

Blank spaces at the bottom of this table are for the addition of other frequently used quantities.

Symbol	Quantity	Basic Dimensions			
		Length	Mass	Time	Charge
a	Acceleration	ℓ		T^{-2}	
B	Magnetic Induction (Magnetic Flux Density)		M	T^{-1}	Q^{-1}
C	Capacitance	ℓ^{-2}	M^{-1}	T^{2}	Q^{2}
D	Electric Displacement (Electric Flux Density)	ℓ^{-2}			Q
E	Electric Field	ℓ	M	T^{-2}	Q^{-1}
F	Force	ℓ	M	T^{-2}	
H	Magnetic Intensity	ℓ^{-1}		T^{-1}	Q
I_m	Moment of Inertia	ℓ^{2}	M		
I	Electric Current			T^{-1}	Q
k	Spatial Frequency	ℓ^{-1}			
L	Inductance	ℓ^{2}	M		Q^{-2}
L_m	Angular Momentum, Action	ℓ^{2}	M	T^{-1}	
ℓ	Length	ℓ			
ℓ^2	Area	ℓ^{2}			
ℓ^3	Volume	ℓ^{3}			
M	Mass		M		
P	Power	ℓ^{2}	M	T^{-3}	
P_m	Linear Momentum	ℓ	M	T^{-1}	

(Cont.)

Table I BASIC DIMENSIONAL EQUIVALENTS (Continued)

Symbol	Quantity	Basic Dimensions			
		Length	Mass	Time	Charge
Q	Charge				Q
R	Resistance, Reactance	ℓ^2	M	T^{-1}	Q^{-2}
T	Time, Period			T	
V	Potential (Voltage)	ℓ^2	M	T^{-2}	Q^{-1}
v	Velocity	ℓ		T^{-1}	
W	Work, Energy, Torque	ℓ^2	M	T^{-2}	
α	Angular Acceleration			T^{-2}	
ϵ_0	Electric Permitivity	ℓ^{-3}	M^{-1}	T^2	Q^2
μ_0	Magnetic Permeability	ℓ	M		Q^{-2}
ϕ	Magnetic Flux	ℓ^2	M	T^{-1}	Q^{-1}
ρ	Density	ℓ^{-3}	M		
ω	Frequency, Angular Velocity			T^{-1}	

Table IIa MECHANICAL UNIT RELATIONS

Quantity	MKS System	cgs System	f lbm s System	f lbf s System
Length	1 meter	10^2 cm	3.281 ft	3.281 ft
Mass	1 kilogram	10^3 gm	2.205 lbm	70.94 slug
Force	1 newton	10^5 dyne	7.233 poundal	0.2248 lbf
Work (Energy)	1 joule	10^7 erg	23.53 ft poundal	0.7376 ft lbf
Power	1 watt	10^7 erg/sec	23.53 ft poundal/sec	$1.341(10)^{-3}$ horse power
Density	1 kg/m^3	10^{-3} gm/cm^3	$62.43(10)^{-3}$ lbm/ft^3	2.009 slug/ft^3

Table IIb ELECTROMAGNETIC UNIT RELATIONS

Quantity	mksa (Absolute) System	cgs esu System	cgs emu System	Gaussian System
Permittivity of empty space (ϵ_0)	$8.855(10)^{-12}$ farad/m	1 [Dimensionless]	$1.1126(10)^{-21}$	1 [Dimensionless]
Permeability of empty space (μ_0)	$1.2566(10)^{-6}$ henry/m	$1.1126(10)^{-21}$	1 [Dimensionless]	1 [Dimensionless]
Charge (Q)	1 coulomb	$2.998(10)^9$ statcoulomb	0.1 abcoulomb	$2.998(10)^9$ statcoulomb
Potential (V)	1 volt	$3.336(10)^{-3}$ statvolt	$(10)^8$ abvolt	$3.336(10)^{-3}$ statvolt
Current (I)	1 ampere	$2.998(10)^9$ statampere	0.1 abampere	$2.998(10)^9$ statampere
Resistance (R)	1 ohm	$1.1126(10)^{-12}$ statohm	$(10)^9$ abohm	$1.1126(10)^{-12}$ statohm
Electric displacement (D)	1 coulomb/m^2	$3.767(10)^6$ statcoulomb/cm^2	$(10)^{-5}$ abcoulomb/cm^2	$3.767(10)^6$ statcoulomb/cm^2
Capacitance (C)	1 farad	$8.988(10)^{11}$ cm	$(10)^{-9}$ abfarad	$8.988(10)^{11}$ cm
Magnetic field strength (H)	1 ampere turn/m	$3.767(10)^8$ Statoersted	$1.257(10)^{-2}$ oersted	$1.257(10)^{-2}$ oersted
Magnetic flux density (B)	1 weber/m^2	$3.336(10)^{-7}$ statmaxwell/cm^2	$(10)^4$ maxwell/cm^2	$(10)^4$ gauss
Inductance (L)	1 henry	$1.1126(10)^{-12}$ stathenry	$(10)^9$ cm	$(10)^9$ cm