The Log Log Vector

TRADE MARK

Slide Rule

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THE K & E LOG LOG VECTOR SLIDE RULE

TRADE MARK

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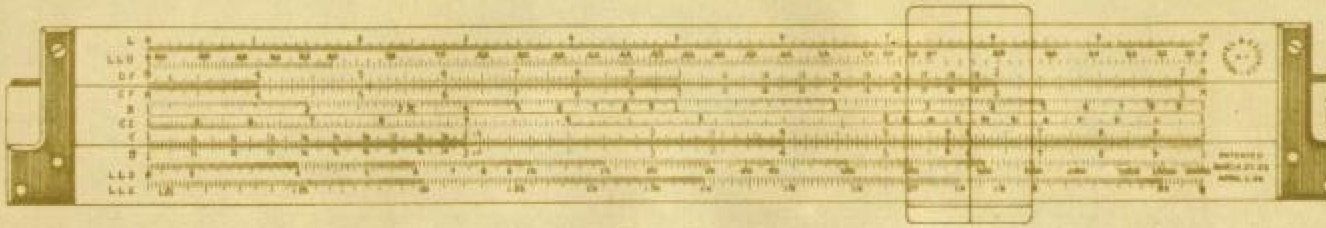
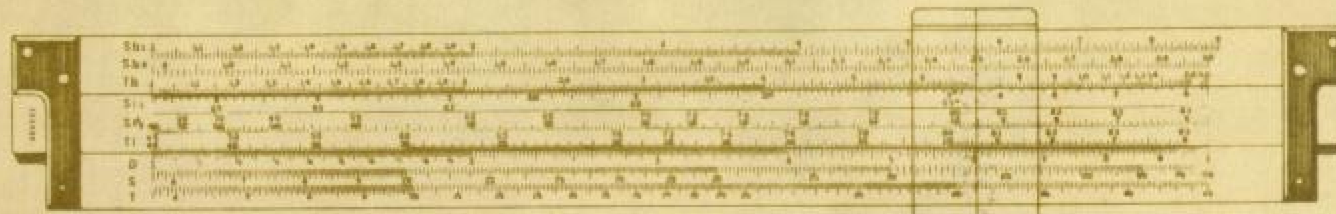


Front Face



Reverse Face

THE LOG LOG VECTOR SLIDE RULE

The Log Log Vector Slide Rule has in addition to the usual scales found on the K & E Log-Log rule, scales of circular and hyperbolic functions, which makes it well adapted to practically all vector calculations ordinarily met in Engineering problems.

The slide rule vector calculations are very simple when the relationship between the various scales and the scale D are properly understood.

THE CIRCULAR TRIGONOMETRIC SCALES

1. The scale marked S in conjunction with scale D gives the sine of any angle between 5.75° and 90°, and obviously the angles whose sines are between .1002 and 1. Thus over 10° on S read .1736 on D.

2. Scale marked SI_2 red in conjunction with CI scale on the opposite side is fully identical with scale S. It is marked from right to left. Hence in conjunction with scale D gives the reciprocals (cosecants) of angles between 90° and 5.75°.

3. Scale marked SI_1 red in conjunction with scale CI on the opposite side, gives the sines of angles between 0.58° and 5.75° (.01 to .1). When read from right to left, and in conjunction with scale D it gives 1/ sin 5.75° to 1/ sin 0.58° (10 to 100) on the D scale.

4. Scale marked SI_1 black in conjunction with scale CI gives the cosine of angles from 84.26° to 89.42° (.1 to .01) on CI. In conjunction with scale D it gives the secants 1/ cos 84.25° to 1/89.42° (10 to 100) on scale D.

5. Scale SI_2 black in conjunction with scale CI gives the cosine of any angle from 0° to 84.25° (1 to .1), and in conjunction with scale D the secants 1/ cos0° to 1/ cos 84.25° (1 to 10).

6. Scale TI red read from right to left and in conjunction with scale CI gives the tangents of angles between 5.75° and 45° (.1 to 1). In conjunction with scale D it gives the cotangents of any angle between the stated values (10 to 1).

7. Scale marked T in conjunction with scale D gives the tangents of angles between 5.75° and 45°.

8. Scale TI black in conjunction with scale D, gives the tangents of angles from 45° to 84.28° (1 to 10) and in conjunction with scale CI on the opposite side, the cotangents of angles within this range.

9. The sine or tangent of a small angle 1° or less, is practically equal to the radian value of the given angle.

The C-D scales on the reverse side have three constants, marked, R, ', and ", for giving the sine or tangent of small angles expressed in degrees, minutes or seconds, respectively, also the radian value of any size angle, when so expressed.

GENERAL RULES, FOR USING THE CONSTANTS

1. To given angle expressed in degrees, minutes, or seconds, on scales D, set the corresponding constant R, ', or ", respectively. On scale C, at index of slide read the value in radians, for any size angle, or if the angle is small, its sine or tangent may be read, or multiplied by any factor taken on scale C.

2. To the sine, tangent, or radian value, of an angle, on scale D, set an index of the slide. At the constant R, ', or ", on scale C read the corresponding angle, in degrees, minutes, or seconds, respectively, on scale D.

For locating the decimal point by inspection, it is convenient to remember that the

Radian, Sine or Tangent of	1°=	approximately	.018	or roughly	.02
" " "	1'=	"	.0003		
" " "	1"=	"	.000005		

Example, Convert 45° to radians,
To 45 on scale D set R on scale C. At index of slide read .7854 radians on scale D.

Example, Solve 540 sin 28',
To 28 on scale D set minute constant , on scale C. At 540 on scale C read 4.4 on scale D.

Example, Convert 1.5 radians to degrees,
Set left index of slide to 1.5 on scale D. At R on scale C read 86° on scale D.

PLANE VECTOR CALCULATIONS

Vector computations involve frequent changes from polar to rectangular coordinates and vice versa.

The solving of right triangles by ordinary methods, using log. and trig. tables is tedious and time consuming, hence a special Vector Slide Rule has been devised to abbreviate this work.

Vectors and their components are conveniently expressed algebraically in complex notation, thus $a+jb=Ae^{j\theta}=A\underline{/\theta}$. In this expression $a+jb$ represents the rectangular components of the vector $A\underline{/\theta}$, and $Ae^{j\theta}$ is an exponential expression of the vector in polar coordinates.

GENERAL RULES, FOR SOLVING RIGHT TRIANGLES

Given $a+jb$, find $Ae^{j\theta}$ (See Fig. 1)
Rule: To the smallest side <u>a</u> (or b) on scale D, set an index of the slide. Over the other side <u>b</u> (or a) on scale D read angle θ on scale TI. Move indicator to angle θ on scale SI, and read A on scale D.

If side <u>a</u> is the smaller, then angle θ is greater than 45°, and must be so read using the co-number in black, on scales TI and SI.

Given $A^{j\theta}$, find a+jb.

Rule: To A on scale D set angle θ on scale SI. At index "1" of the slide read the smaller side <u>b</u> (or a) on scale D. At angle θ on scale TI read the other side <u>a</u> (or b) on scale D.

Scale D here represents the numerical value of each of the three sides. However instead of denoting the sides by their numerical value they may be expressed in terms of any function, the scale of which is referred to scale D. Thus any side may be expressed as sinα, tanα, sinhα, tanhα, $\log_e x$ etc., and the triangle solved at a single setting of the slide.

Conversion of complex numbers of the form a+jb into exponentials of the form

$$Ae^{j\theta} = A\underline{/\theta} \qquad (1)$$

where

$$A = \sqrt{a^2+b^2} \qquad (2)$$

and

$$\theta = \tan^{-1}\frac{b}{a} \qquad (3)$$

Figure (1) shows how these quantities are related geometrically to one another.

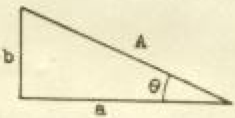


Fig. 1

From this figure we note that the angle θ may also be obtained by

$$\theta = \cot^{-1}\frac{a}{b} \qquad (4)$$

and the vector equivalent of the complex number by

$$A = \frac{b}{\sin\theta} \qquad (5)$$

or by

$$A = \frac{a}{\cos\theta} \qquad (6)$$

Expressions (4) and (5) are the operations embodied in the slide rule for the conversion of a+jb into A/θ for the case when a>b, i.e., when $\theta<45°$, and expressions (3) and (6) represent the operations for the case when a<b, i.e., when $\theta>45°$.

<u>CASE I. a>b $\theta<45°$</u>

1. Set right or left index as necessary, of scale TI opposite <u>b</u> on scale D.
2. Move indicator to <u>a</u> on scale D, and
3. Read θ opposite <u>a</u> on TI red.
4. Move indicator to θ on SI red, and read A opposite θ on D.

Examples:

Find the vector value of 4+j3.

1. Set left index of TI opposite 3 on D.
2. Move indicator to 4 on D, and read $\theta = 36.83^\circ$ opposite 4 on TI red.
3. Move indicator to 36.83° on SI_2 red, and read A = 5 opposite 36.83° on D.

Hence

$$4 + j3 = 5 \underline{/+36.83^\circ}$$

Example: $9.68 \pm j\,6.9$

Set left index of TI on 6.9 scale D.
Move indicator to 9.68 scale D, and read $\theta = 35.5^\circ$ on TI red.
Since 35.5° SI_2 red is beyond the right end of rule,
1. Reset right index of TI over 6.9.
2. Move indicator over 35.5° SI_2 red, and read A=11.9 on D.

Hence

$$9.68 \pm j6.9 = 11.9 \underline{/\pm 35.5^\circ}$$

Another method:

By using the folded scale DF, on the reverse side, instead of the D scale, resetting of the slide is obviated.
Thus set indicator to 6.9 on scale DF. Set left index of slide to indicator. Move indicator to 9.68 on DF and read $\theta = 35.5^\circ$ on TI, red. Set indicator to 35.5° red, on SI_2, and read 11.9 on DF.

Example: Find the vector value of 12+j4.5

1. Set right index of TI on 4.5 scale D.
2. Move indicator to 12 scale D, and read $\theta = 20.55^\circ$ on TI red.
3. Move indicator to 20.55° SI_2 red, and read A=12.8 on D.

Hence

$$12 + j4.5 = 12.8 \underline{/20.55^\circ}$$

Example: Find the vector value of 1.91+j.05

1. Set right index of TI opposite .05 on scale D.
2. Move indicator to 1.91 scale D, and read $\theta = 1.5^\circ$.
Since .05 is very small compared with 1.91, it follows that $1.91 + j.05 = 1.91 \underline{/1.5^\circ}$ for all practical purposes.

<u>CASE II. $a < b$; $\theta > 45^\circ$</u>

The operations are identical to those given for Case I, using scales TI black and SI black.

1. Set left or right index of TI, as necessary on <u>a</u> Scale D.
2. Move indicator to <u>b</u> scale D, and read θ on TI black.

3. Move indicator to θ on SI black and read A opposite θ on D.

Example: Find the vector value of 3+j4.

1. Set left index of TI on 3 scale D.
2. Move indicator to 4 scale D, and read θ=53.17° on TI black.
3. Move indicator to 53.17° SI_2 black, and read A=5 on D.

Hence

$$3 \pm j4 = 5\underline{/\pm 53.17^\circ}$$

Similarly:

For 4.5 ± j12

1. Set right index of TI opposite 4.5 scale D.
2. Move indicator to 12 scale D, and read θ=69.45° on TI black.
3. Move indicator to 69.45° SI_2 black, and read A=12.82 on D.

For .065+j1.49

1. Set right index of TI opposite .065 on Scale D.
2. Move indicator to 1.49 on scale D, read 87.5° on SI_1 black.

Since 1.49 is comparatively large with respect to .065, it follows that for all practical purposes

$$.065+j\,1.49 = 1.49\ \underline{/87.5^\circ}$$

ILLUSTRATIVE APPLICATIONS

1. A circuit consists of a resistance of 7.46 ohms, in series with an inductive reactance of 3.4 ohms. What is the vector impedance of the circuit?

$$Z = 7.46+j\,3.4 = 8.2\ \underline{/24.5^\circ} \text{ vector ohms}$$

2. Calculate the vector impedance of a circuit which consists of a resistance of .795 ohms in series with a capacitive reactance of 1.25 ohms.

$$Z = .795-j\,1.25 = 1.48\underline{/57.5^\circ} \text{ vector ohms}$$

3. A circuit consists of resistance in parallel with a capacitance. The measured current in the resistance is 10.5 amperes, and that in the capacitance is 5.7 amperes. What is the total current in the circuit? What is its time phase with respect to the difference of potential impressed upon the circuit? What is the power factor?

$$I = 10.5+j\,5.7 = 11.95\underline{/28.5^\circ} \text{ vector amperes}$$

The time phase is 28.5° leading.
The power factor is cos 28.5° = .878

4. A circuit consists of a conductance of .068 mhos in parallel with a practically pure inductive susceptance of .0872 mhos. What is the vector admittance of the circuit?

$$Y = .068+j.087 = .1105\underline{/52^{\circ}} \text{ vector mhos}$$

5. The attenuation constant and the wave length constant of No. 10 A.W.G. dry core cable at 800 cycles frequence are α=.0366 and β=.0559 hyperbolic and circular radians, respectively. What is the propagation constant of the line?

$$P = a+jb = .0366+j.0559$$
$$= .067\underline{/56.8^{\circ}}$$

<u>Conversion of $Ae^{j\theta} = A\underline{/\theta}$ into complex numbers of the form a+jb.</u>

The complex number equivalent to $A\underline{/\theta}$ is $A \cos \theta + jA \sin \theta$

Since the D scale gives the reciprocals of $\cos \theta$ for values of θ on the SI black scales, and the reciprocals of $\sin \theta$ for values of θ on the SI red scales, the slide rule operation for $A \cos \theta$ is $\frac{A}{1/\cos \theta}$ and for $A \sin \theta$ is $\frac{A}{1/\sin \theta}$

To find $A \cos \theta$ and $A \sin \theta$

Set θ SI black opposite A on scale D. Read $A \cos \theta$ on D at index of TI.

Set θ SI red opposite A scale D. Read $A \sin \theta$ on D at index of TI.

Examples:

Find the complex number equivalent to $1.49\underline{/87.5^{\circ}}$

1. Set 87.5°, SI_1 black opposite 1.49 scale D, and read $\cos \theta$ = .065 on D at right index of TI.
2. Set 87.5° SI_2 red opposite 1.49 scale D, and read 1.485 on D left index of TI.

Hence

$$1.49\underline{/87.5^{\circ}} = .065+j\ 1.485$$

Find the complex number equivalent to $11.9\underline{/-54.5^{\circ}}$

1. Set 54.5° SI_2 black opposite 11.9 on D, and read 6.91 = 11.9 cos 54.5° on D at right index of TI.
2. Set 54.5° SI_2 red opposite 11.9 on D, and read 9.69 = 11.9 sin 54.5° on D at right index of TI.

Hence

$$11.9\underline{/-54.5^{\circ}} = 6.9-j\ 9.69$$

Find the complex number equivalent of $12.8\underline{/20.5^{\circ}}$

1. Set 20.5° SI_2 black opposite 12.8 on D, and read 12.8 cos 20.5° = 11.98 on D at left index of TI.
2. Set 20.5° SI_2 red opposite 12.8 on D, and read 4.48 = 12.8 sin 20.5° on D at left index of TI.

ILLUSTRATIVE APPLICATIONS

1. The vector impedance of a circuit is $7.9\angle 25.5°$ ohms. Calculate its resistive and reactive components.

$$7.9\angle 25.5° = 7.12 + j\,3.4$$

Hence
$$r = 7.12 \text{ ohms}$$
$$x = 3.4 \text{ ohms}$$

2. The admittance of a parallel circuit is $.0955\angle -31.5°$ vector ohms. Calculate the conductance and susceptance of the circuit.

The conductance is

$$g = .0955 \cos(-31.5°) = .0813 \text{ mhos},$$

and the susceptance is

$$b = .0955 \sin(-31.5°) = -.0498 \text{ mhos}$$

The equivalent complex number is

$$g - jb = .0813 - j.0498$$

3. The vector propagation constant of a certain transmission line is

$$P = .065\angle 56.5°$$

Calculate the attenuation and phase constant.

The attenuation constant is

$$a = .065 \cos 56.5° = .03585 \text{ hyperbolic radians.}$$
$$\beta = .065 \sin 56.5° = .0542 \text{ radians}$$

The wave length constant is identical to the phase constant, but is usually expressed in degrees. Its value for the given line is $.0542 \times 57.3 = 3.11°$ (see mark "R" on C and D scales for the conversion of radians to degrees).

HYPERBOLIC FUNCTIONS

The Hyperbolic Scales

1. Since $\sinh x = x$, and $\tanh x = x$, for all practical purposes, when $x<0.1$, scale D gives the sinh x and tanh x directly for any value of $x<0.1$.

2. Scale Sh_1 in conjunction with scale D, gives the hyperbolic sines of hyperbolic angles x of any value between $.1<x<.882$; Thus for $x = .515$ set indicator on .515 scale Sh; and read $.5375 = \sinh .515$ on scale D.

Similarly:
$$\sinh .645 = .69$$
$$\sinh .1945 = .1958$$
$$\sinh .273 = .276$$

It also gives the hyperbolic angle x when the hyperbolic sine (sinh x) is known, between values of .1 and .882. Thus for sinh x = .758 on D, read X = .7 on Sh_1 scale. For sinh x = .249 on D, read x = .247 on Sh_1.

3. Scale Sh_2 in conjunction with scale D, gives the hyperbolic sines of hyperbolic angles x of values between $.882<x<3$, and the values of x when sinh x is known and is within .882 and 10. Thus for x = 1.245 on Sh_2, read 1.592 = sinh 1.245 on D.

Similarly:
sinh 1.466 = 2.05
sinh 1.875 = 3.18
sinh 2.24 = 4.64
sinh 2.95 = 9.53

For:
sinh x = 5.1 on D read x = 2.332 on Sh_2
sinh x = 4.35 on D read x = 2.176 on Sh_2
sinh x = 1.84 on D read x = 1.37 on Sh_2

4. Scale Th gives in conjunction with scale D, the hyperbolic tangents of hyperbolic angles x between the values $1<x<3$, and approximately for all values of $x>3.5$, since tanh x for $x>3.5$ is 1. Thus for x = .175 on Th, read .1733 on D.

Similarly:
tanh .224 = .2202
tanh .435 = .409
tanh .94 = .735
tanh 1.45 = .895
tanh 2. = .965
tanh 3. = .995

For:
tanh x = .795 on D, read x = 1.082 on Th.
tanh x = .52 on D, read x = .576 on Th.
tanh x = .137 on D, read x = .1378 on Th.

The Hyperbolic Cosine, Cosh x

The hyperbolic cosine of any real number x, may be obtained by any of the following formulas:

$$\cosh x = \frac{\sinh x}{\tanh x} \qquad (1)$$

$$= \sqrt{1+\sinh^2 x} \qquad (2)$$

or

$$= \frac{\sinh x}{\sin\left[\tan^{-1}(\sinh x)\right]} \qquad (3)$$

Formula (1) is best suited for slide rule calculation, and the value of cosh x, can be obtained at a single setting of the slide.

RULE: Set an index of the slide to x on scale Th.
Move indicator to x on scale Sh_1 or Sh_2 and read cosh x on scale C, on the reverse side of the rule.

Examples: Find cosh .662.

Set left index of slide to .662 on scale Th.
Move indicator to .662 on scale Sh_1 and read 1.224 = cosh x, on scale C.

Find cosh 1.57.

Set right index of slide to 1.57 on scale Th.
Move indicator to 1.57 on scale Sh_2, and read 2.506 = cosh x, on scale C.

Formula (3) is also suited for slide rule calculation, and in certain cases may be useful. This formula may obviously be written.

$$\cosh x = \frac{\sinh x}{\sin \theta}, \text{ where } \theta = \tan^{-1} (\sinh x)$$

When $x < .1$, $\sinh x = \tanh x$, whence $\cosh x = 1$.
When $.1 < x < .882$, $\sinh x < 1$, hence $\tan^{-1} (\sinh x) < 45°$.
It follows therefore that for values of x within this range:

1. Set right index of TI opposite x on Sh_1 and read $\theta = \tan^{-1} (\sinh x)$ on scale T, opposite index of D.
2. Move indicator to θ on SI_2 red, and, read cosh x on D, under hair line.

Example: Find cosh .292.

1. Set right index of TI opposite .292 Sh_1 and read $\theta = 16.5°$ opposite index on scale D.
2. Move indicator to 16.5° on scale SI_2 red, and read 1.043 = cosh x, on D under hair line.

When $.882 < x < 3$, $\sinh x > 1$, and $\tan^{-1} (\sinh x) > 45°$, it follows, therefore, that for values of x within this range:

1. Opposite x on Sh_2 scale read $\theta = \tan^{-1} (\sinh x)$ on TI black.
2. Set left index of TI opposite x, scale Sh_2.
3. Move indicator hair line over θ, scale SI_2 red, and read cosh x on D, under hair line.

Example: Find cosh 2.34.

1. Opposite 2.34 scale Sh_2, read $\theta = 79°$ on TI black.
2. Set left index of TI opposite 2.34, scale Sh_2.
3. Move indicator hair line over 79°, scale SI_2 red, and read 5.24 = cosh 2.34 on D, under hair line.

COMPLEX HYPERBOLIC FUNCTIONS

Vector Equivalent of the Hyperbolic Sine of Complex Numbers

The complex number equivalent of the hyperbolic function

$$\sinh (x + j\theta) \text{ is}$$

$$\sinh x \cos \theta + j \cosh x \sin \theta \qquad (1)$$

The vector value of this expression may, therefore, be written

$$\sinh (x+j\theta) = \frac{\sinh x \cos \theta}{\cos [\tan^{-1}(\tan \theta / \tanh x)]} \; \underline{/\tan^{-1}(\tan\theta/\tanh x)}$$

$$= A \; \underline{/\beta} \qquad (2)$$

where

$$\beta = \tan^{-1} \frac{\tan\theta}{\tanh x} \qquad (3)$$

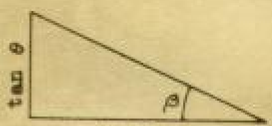
and

$$A = \frac{\sinh x \cos\theta}{\cos\beta} \qquad (4)$$

Expressions (3) and (4) represent the operations to be carried out to evaluate the vector value of sinh $(x + j\theta)$. This may be done by actual substitution of values (obtained thru slide rule) in the equations, and carrying out the calculations involved by slide rule operation, or by the rules explained and illustrated below.

Since the value of θ is frequently expressed in radians when used in conjunction with complex hyperbolic function, it is first necessary to convert it to degrees, by the use of the mark "R" on the C-D scales.

Expression (3) indicates that we may think of tan θ and tanh x in their relationship to the vector angle β, as the sides of a right triangle as shown in the figure.



It follows, therefore, that we may write

$$\tanh x + j \tan\theta = a + jb,$$

and the slide rule method for determining the vector angle β becomes identical with that described in section "B" for plane vector calculation. The value of x is on scale Th and tanh x = a is on scale D opposite x. The value of θ is on scale T when $5.75° < \theta < 45°$, and on scale TI black when $\theta > 45°$, while corresponding values of tan θ = b are on scale D.

To obtain the vector angle β

1. Determine first whether tan θ or tanh x is smaller. For $\theta > 45°$, tanh x is always smaller than tan θ. The angle β in such a case is larger than 45°, and is read on TI black. For values of $\theta < 45°$, set indicator hair line over θ on scale T. If x on scale Th is on the right of the hair line, then tanh x$>$tan θ; the angle β will be smaller than 45° and should be read on scale TI red. If, on the other hand, x on scale Th is to the left of the hair line, then tanh x$<$tan θ. In such a case, the angle $\beta > 45°$ and should be read on TI black.

2. Set left or right index (as necessary) of scale TI opposite θ scale T, if tan$\theta <$ tanh x, or opposite x, scale Th, if tanh x$<$tan θ;

3. Move indicator to x if tanh x is larger, and read β on TI red; or over θ, when tan θ is larger, and read β on TI black.

Example:

Determine the vector angle of sinh (.256+j 10.5°)

1. Set indicator over 10.5° scale T. It shows that tanh .256>tan 10.5°
2. Set left index of TI opposite 10.5° scale T;
3. Move indicator to .256 scale Th, and read β = 36.5° on TI red.

Similarly for sinh(1.28+j 30.5°)

1. Set indicator over 30.5°, scale T; it shows that tanh 1.28>tan 30.5°. Hence:
2. Set left index of TI opposite 30.5° scale T;
3. Move indicator to 1.28 scale Th, and read β = 34.5° on TI red.

For sinh (.505+j 20°)

1. Set indicator over 20° scale T; it shows that tanh .505>tan 20°. Hence:
2. Set left index of TI opposite 20° scale T;
3. Move indicator to .505 scale Th, and read β = 38°.

Find the vector angle of sinh (.43+j 30.5°)

1. Set indicator over 30.5° scale T; it shows that tanh .43<tan 30.5°. Hence:
2. Set left index of TI opposite .43 scale Th.
3. Move indicator over 30.5° scale T, and read β = 55.5° on TI black.

Similarly, for sinh (.17+j 35°)

1. Set indicator on 35° scale T; it shows that tanh .17<tan 35°. Hence:
2. Set left index of TI on .17 scale Th;
3. Move indicator to 35° scale T, and read β = 76.5° on TI black.

For sinh (.224+j 27.5°)
tanh .224<tan 27.5, β = 67.08°

Find Vector angle of sinh (.68+j 62°)
Note that θ = 62° (larger than 45°). Hence tanh .68<tan 62°.
1. Set indicator over 62° TI black;
2. Set right index of TI (SI) opposite .68 on Th scale, and read β = 72.55° on TI black.

Similarly, For sinh (.243+j 53.5°)

1. Set indicator hair line over 53.5° TI black;
2. Set right index of SI_1 opposite .242 scale Th, and read β = 80° on TI black.

If the angle θ of the complex hyperbolic function sinh (x+j θ) is less than 5.75°, we have to use scale SI_1 red instead of scale T. Since scale SI_1 red in conjunction with scale D gives cotangent values, we may use the following formula in calculating the vector angle β.

$$\beta = \cot^{-1} \frac{\tanh x}{\tan \theta} = \cot^{-1} (\tanh x \cot \theta)$$

In accordance with this formula, when $\theta < 5.75°$,
1. Set left index of TI opposite x on Th scale
2. Move indicator over θ, scale SI_1 red,
3. Reset index of TI on 1 scale D, and read β on scale TI red, under hairline.

Example: Find the vector angle of sinh (.216+j 3.5°)

1. Set left index of TI opposite .216 scale Th;
2. Move indicator to 3.5° scale SI_1 red;
3. Reset index of scale TI opposite index of D, and read β = 16° on TI red.

Similarly, For Sinh (.339+j 4.5°) β = 13.5°

The numerical value of the function sinh (x+j θ), is obtained in accordance with expression (4).
1. Set θ, scale SI_1 black opposite x scale Sh_1 or Sh_2 as necessary.
2. Move indicator to β (obtained as outlined above) scale SI_1 black, and read A under hair line on scale D.

Example: Find the vector value of sinh (.256+j 10.5°)

1. Find β
Set left index of TI on 10.5° scale T, move indicator over .256 Scale Th, and read β = 36.5°, which should be written down.
2. Set 10.5° scale SI_2 black opposite .256, scale Sh_1;
Move indicator to 36.5° scale SI_2 black, and read A = .316 on D scale under hair line.

Hence

$$\text{Sinh } (.256+j\ 10.5°) = .316\underline{/36.5°}$$

Similarly: For Sinh (1.28+j 10.5°)

1. Find β
Set left index of TI on 30.5° scale T;
Move indicator over 1.28 scale Th, and read β = 34.5° on TI red.
2. Set 30.5° SI_2 black opposite 1.28 scale Sh_2.
Move indicator to 34.5° SI_2 black, and read A = 1.735 on D under hair line.

Hence

$$\text{Sinh } (1.28+j\ 30.5°) = 1.735\underline{/34.5°}$$

Similarly: For Sinh (.17+j 35°)

1. Find β = 76.5°

2. Find $A = \frac{\sinh .17 \cos 35°}{\cos 76.5°} = .6$

Hence

$$\text{Sinh } (.17+j\ 35°) = .6\underline{/76.5°}$$

ILLUSTRATIVE APPLICATION

A 25 mile No. 10 A.W.G. line, whose characteristic impedance at 800 cycles frequency is

$Z_0 = 723\underline{/-11.1^\circ}$ vector ohms, and whose propagation constant is $P = \underline{.0292/71.8^\circ}$ is short circuited at the receiving end. Calculate the short circuit current under the assumption that the difference of potential impressed at the sending end is 100 volts.

The expression for the desired current is

$$I = \frac{V_s}{Z_0 \sinh pS}$$

For the values given above, this expression becomes

$$I = \frac{100\underline{/0}}{723\underline{/-11.1^\circ} \times \sinh (o.0292 \times 25\underline{/71.8^\circ})}$$

Calculations:

$$.0292 \times 25\underline{/71.8^\circ} = .228+j\ .694$$
$$.694 \text{ (radians)} = 39.6^\circ$$

Hence

$$\text{Sinh } (.0292 \times 25\underline{/71.8^\circ}) = \sinh (.228+j\ 39.6^\circ)$$
$$= .677\underline{/74.83^\circ}$$

Substituting in the expression for I, we get

$$I = \frac{100\underline{/0^\circ}}{723\underline{/-11.1^\circ} \times .677\underline{/74.83^\circ}}$$

$$= \frac{100\underline{/0^\circ}}{489\underline{/63.73^\circ}} = .2045\underline{/-63.73^\circ} \text{ vector amperes}$$

with reference in time phase to the impressed voltage.

<u>Vector Equivalent of the Hyperbolic Cosine of Complex Numbers</u>

The complex number equivalent of the hyperbolic function

$$\text{Cosh } (x+j\,\theta) \text{ is}$$

$$\text{Cosh } x \cos\theta + j \sinh \sin\theta \qquad (5)$$

The vector value of this expression may, therefore, be written

$$\text{Cosh } (x+j\,\theta) = \frac{\sinh x \sin\theta}{\sin[\tan^{-1}(\tanh x \tan\theta)]} \underline{/\tan^{-1}(\tanh x \tan\theta)} \qquad (6)$$

$$= B\underline{/\alpha}$$

where

$$\alpha = \tan^{-1} (\tanh x \tan\theta) \qquad (7)$$

and

$$B = \frac{\sinh x \sin\theta}{\sin\alpha} \qquad (8)$$

Expression (7) represents the operation to be carried out in evaluating the vector angle α, and (8) the operation to be carried out to determine the value of B.

To obtain the value of α, when $\theta < 5.75°$

1. Set θ, scale SI_1 red opposite x on scale Th, and read α on SI_1 red opposite right index of D.

Example: Find the vector angle cosh (.347+j 4.5°)

1. Set 4.5° scale SI_1 red opposite .347 scale Th, and read α = 1.5° on SI_1 red opposite right index of D.

Similarly: For cosh (.491+j 5.5°)

1. Set 5.5° scale SI_1 red opposite .491 scale Th, and read α = 2.5° SI_1 red opposite right index of D.

Similarly: For cosh (.278+j 3.7°) ——— α = 1°.

<u>To obtain the value of α, when $5.75° < \theta < 45°$.</u>

1. Set θ scale TI red opposite x scale Th, and read α on SI_1 red opposite left index of D or on TI red at the right index of TI red as necessary.

Examples:

Find the vector angle of cosh (.282+j 9°)

Set θ scale TI red opposite .282 scale Th, and read α = 2.5° on SI_1 red opposite left index of D.

Similarly: For cosh (.178+j 16.5°)

1. Set 16.5° scale TI red opposite .178 scale Th, and read α = 3° on SI_1 red cpposite left index of D.

Find the vector angle of cosh (.41+j 21°)

1. Set 21° scale TI red opposite .41 scale Th, and read α = 8.5° on TI red opposite right index of D, or on T at left index of TI.

Similarly: For cosh (.635+j 35.4°) ——— α = 21.75°
For cosh (1.16+j 42°) α = 36.48°

<u>To obtain the value of α when $\theta > 45°$.</u>

1. Set left index of TI opposite x on scale Th.
2. Move indicator over θ on TI black, and read α on scale T under hair line.

Example: Find the vector angle of cosh (.1855+j 56.5°)

1. Set left index of TI opposite .1855 scale Th,
2. Move indicator over 56.5° scale TI black, and read α = 15.5° on scale T under hair line.

Similarly: For cosh (.231+j 68.6°) ——— α = 30°

If after setting left index of TI opposite x on Th, we find that θ is beyond the right index of D, the angle β is larger than 45°. In such a case:

1. Set right index of TI opposite x on scale Th;
2. Move indicator over θ on TI black;
3. Reset index of TI opposite index of D, and read ∢ on TI black under hair line.

Example: Find the vector angle of cosh (.282+j 78.5°)

1. Set right index of TI opposite .282 on scale Th.
2. Move indicator over 78.5° scale TI black.
3. Reset index of TI opposite index of D, and read ∢ 53.5° on TI black, under hair line.

Similarly: For cosh (.915+j 80.5°) —— ∢ = 77°
For cosh (1.05+j 72°) ∢ = 67.5°

<u>To determine the numerical value B of the function cosh (x+jθ)</u>

1. Determine first the vector angle ∢, as outlined above.
2. In accordance with expression (8) set θ scale SI red opposite x on Sh scale;
3. Move indicator over θ scale SI red and read B on scale D under hair line.

Examples:

Find the vector value of cosh (.282+j 78.5°)

1. Obtain the vector angle ∢:-
Set right index of TI opposite .282 scale Th;
Move indicator over 78.5° scale TI black, and read ∢ = 53.5° on TI black under hair line after resetting index of TI opposite index of D.
2. Set 78.5° scale SI_2 red opposite .282 scale Sh_1
Move indicator over 53.5° scale SI_2 red, and read B = .348 on D under hair line.

Hence

cosh (.282+j 78.5°) = .348/53.5°

Similarly: For cosh (1.05+j 72°)

1. Find ∢ = 67.45°
2. Set 72° scale SI_2 red opposite 1.05 Sh_2
Move indicator over 67.45° scale SI_2 red and read B = 1.29 on scale D under hair line.

Hence

Cosh (1.05+j 72°) = 1.29/67.45°

Find the vector value of cosh (.347+j 4.5°)

1. Obtain first the value of ∢:-
Set 4.5° scale SI_1 red opposite .347 Th.
Read ∢ = 1.5° on SI_1 red opposite right index of D.
2. Set 4.5° scale SI_1 red opposite .347 scale Sh_1.
Move indicator over 1.5° SI_1 red ——
Note that 1.5° is beyond scale index. Hence reset right index of TI at position of left index, move indicator over 1.5° scale SI_1 red, and read B = .1061.

Hence

$$\text{Cosh } (.347+j\ 4.5^\circ) = .1061\underline{/1.5^\circ}$$

Find the vector value of cosh (.25+j 36°)

1. Obtain first the vector angle ∢:-
Set 36° scale TI red opposite .25 scale Th, and read 10.09° on scale TI red opposite right index of D, or on scale T opposite left index of TI.
2. Set 36° scale SI_2 red opposite .25 scale Sh_1
Move indicator over 10.09 scale SI_2 red, and read B = .847 on scale D under hair line.

Hence

$$\text{Cosh } (.25+j\ 36^\circ) = .847\underline{/10.09^\circ}$$

Similarly: Cosh (.41+j 21°)

1. Obtain ∢ = 8.5°
2. Set 21° scale SI_2 red opposite .41 scale Sh_1
Move indicator over 8.5° scale SI_2 red ———
Note that it is beyond right index of D, hence set right index of TI at position of left index (1513 on D)
Move indicator over 8.5° on SI_2 and read 1.021 on D under hair line.

Hence

$$\text{Cosh } (.41+j\ 21^\circ) = 1.021\underline{/8.5^\circ}$$

ILLUSTRATIVE APPLICATIONS

Calculate the open circuit receiving-end voltage of a 50 mile No. 10 A.W.G. line, whose characteristic impedance at 800 cycles frequency is 723/-11.1° vector ohms, and whose propagation constant is 0.0292/71.8°, under the assumption that the impressed difference of potential at the sending end is 100 volts.

The expression by means of which this may be calculated is

$$V = \frac{V\ s}{\cosh P\ S}$$

where

$$Vs = 100\underline{/0^\circ}$$

$$P = 0.0292\underline{/71.8^\circ}$$

$$S = 50$$

substituting in the above expression, we get

$$V = \frac{100\underline{/0^\circ}}{\cosh\ (.0292 \times 50\underline{/71.8^\circ})}$$

$$\text{Cosh } (.0292 \times 50\underline{/71.8^\circ}) = \cosh\ (1.46\underline{/71.8^\circ})$$

$$= \cosh\ (.456+j\ 1.385)$$

$$= \cosh\ (.456+j\ 79.4^\circ)$$

$$= .507\underline{/66.3^\circ}$$

Hence

$$V = \frac{100\angle 0^\circ}{.507\angle 66.3^\circ} = 197.5\angle{-66.3^\circ} \text{ vector volts lagging}$$

the sending end voltage in time phase by 66.93°.

<u>Evaluation of the Hyperbolic Tangent of a Complex Function</u>

Since

$$\text{Tanh } (x+j\,\theta) = \frac{\sinh (x+j\,\theta)}{\cosh (x+j\,\theta)} = \frac{A\angle\beta}{B\angle\alpha} = C\angle\delta$$

it follows that the value of the tanh of such a function must be obtained by evaluating the sinh and cosh as outlined in the preceding two sections and taking the ratio as indicated.

angle δ associated with the vector value of the function is $\delta = \beta - \alpha$, where β is the angle associated with the vector value of the sine function, and the angle α with the cosine function, and whose values are obtained as outlined in the preceding two sections.

ILLUSTRATIVE APPLICA.ION

Calculate the sending end impedance of an open circuited 50 mile No. 10 A.W.G. line, whose characteristic impedance is $Z_o = 723\angle 11.1^\circ$ and whose propagation constant is $P = .0292\angle 71.8^\circ$.

The value of the sending end impedance is given by the expression

$$Z_s = \frac{Zo}{\text{tanh PS}}$$

$$\text{Tanh PS} = \text{tanh } (.0292 \times 50\angle 71.8^\circ)$$

$$.0292 \times 50\angle 71.8^\circ = .456+j1.385 \text{ radians}$$

$$= .456+j\ 79.4^\circ$$

$$\text{tanh } (.456+j\ 79.4^\circ) = \frac{\sinh (.456+j\ 79.4^\circ)}{\cosh (.456+j\ 79.4^\circ)}$$

$$\sinh (.456+j\ 79.4^\circ) = 1.087\angle 85.42^\circ$$

$$\cosh (.456+j\ 79.4^\circ) = .508\angle 66.3^\circ$$

Hence

$$\text{tanh } (.456+j\ 79.4^\circ) = \frac{1.087\angle 85.42^\circ}{.508\angle 66.3^\circ}$$

$$= 2.14\angle 19.12^\circ$$

Substituting in the expression for the sending end impedance we get

$$Z_s = \frac{723\angle{-11.1^\circ}}{2.14\angle 19.12^\circ} = 337\angle 30.22^\circ$$

2. Calculate the impedance at the input terminals of the above line when terminated with an impedance of $250\angle 30^\circ$ vector ohms.

The expression by means of which this impedance may be calculated is

$$Z_s = \frac{Z_r Z_o \cosh PS + Z_o^2 \sinh PS}{Z_o \cosh PS + Z_r \sin PS}$$

$$\cosh PS = .508\angle 66.3^\circ$$

$$Z_o Z_r \cosh PS = 723\angle -11.1^\circ \times 250\angle 30^\circ \times .508\angle 66.3^\circ$$

$$= 91800\angle 85.2^\circ$$

$$Z_o^2 \sinh PS = (723\angle -11.1^\circ)^2 \times 1.087\angle 85.42^\circ$$

$$= 523000\angle -22.2^\circ \times 1.087\angle 85.42^\circ$$

$$= 568000\angle 63.22^\circ$$

$$Z_o \cosh PS = 723\angle -11.1^\circ \times .508\angle 66.3^\circ$$

$$= 368\angle 55.2^\circ$$

$$Z_r \sinh PS = 250\angle 30^\circ \times 1.087\angle 85.42^\circ$$

$$= 271.7\angle 115.42^\circ$$

Substituting in the expression for Z_s we get

$$Z_s = \frac{91800\angle 85.2^\circ + 568000\angle 63.22^\circ}{368\angle 55.2^\circ + 271.7\angle 115.42^\circ}$$

$$91800\angle 85.2^\circ = 7670 + j\,91000$$
$$568000\angle 63.22^\circ = 256000 + j\,507000$$
$$\text{sum} = 263670 + j\,598000 = 654000\angle 66.03^\circ$$

$$271.7\angle 115.42^\circ = 271.7\angle 90^\circ | 25.42^\circ$$
$$= -271.7 \sin 25.42^\circ \; j\,271.7 \cos 25.42^\circ$$
$$= -116.7 + j\,245$$
$$368\angle 54.83^\circ = 210.3 + j\,302.5$$
$$\text{sum} = 93.6 + j\,547.5 = 555\angle 80.35^\circ$$

Substituting these values in the equation for Z_s we get

$$Z_s = \frac{654000\angle 66.03^\circ}{555\angle 80.35^\circ}$$

$$= 1179\angle -24.05^\circ \text{ vector ohms}$$

What would be the line current at the sending end if the impressed difference of potential is assumed 100 volts?

$$I_s = \frac{V_s}{Z_s} = \frac{100\angle 0^\circ}{1179\angle -24.05^\circ}$$

$$= 0.0848\angle 24.05^\circ \text{ vector amperes}$$

leading the voltage by 24.05°.

What would be the receiving end voltage?

The receiving end voltage may be calculated by the expression

$$V_r = \frac{V_s Z_r}{Z_r \cosh PS + Z_0 \sinh PS}$$

$$= \frac{100\angle 0^\circ \times 250\angle 30^\circ}{250\angle 30^\circ \times .508\angle 66.3^\circ + 723\angle -11.1^\circ \times 1.087\angle 85.42^\circ}$$

$$127\angle 96.3^\circ = -127 \sin 6.3^\circ + j\, 127 \cos 6.3^\circ$$

$$= -13.95 + j\, 126.8$$

$$784\angle 74.32^\circ = \frac{212.00 + j\, 755.}{199.05 + j\, 881.} = 903\angle 77.25^\circ$$

Substituting in the equation for V_r we get

$$V_r = \frac{25000\angle 30^\circ}{903\angle 77.25^\circ} = 27.7\angle -47.25^\circ \text{ vector volts}$$

What would be the receiving end current?

$$I_r = \frac{V_r}{Z_r} = \frac{27.7\angle -47.25^\circ}{250\angle 30^\circ}$$

$$= .1107\angle -77.25^\circ \text{ vector amperes}$$

lagging the sending end voltage by 77.25°.

Decibel Computations

In Radio or Communication engineering, power, current and voltage ratios are often expressed in decibels (db).

For power ratios:

$$db = 10 \log_{10} \frac{P_1}{P_2}$$

The quickest solution of this formula is obtained with the aid of the log log scales as follows:

To 10 (the base of the common logarithms) on LL3 set 10 (the factor) of the C scale; then opposite any power ratio on the log log scales, the corresponding decibel value can be read on the C scale.

Example 5. How many db correspond to a power ratio $\frac{P_1}{P_2} = 4.36$?

Solution: With slide set as explained.
Opposite 4.36 on LL3 scale
read 6.4 on C scale. (The decimal point is easily placed by remembering that the index, set to 10 on LL3, is 10).

Example 6. $\frac{P_1}{P_2} = 20{,}000$.

Solution: Slide set as explained.
Opposite 20,000 on LL3 scale
read 43. on C scale.