THE 50 CENTIMETRE SLIDE-RULE

AS APPLIED TO

TACHEOMETRY AND CURVE-RANGING

WITH TABLES

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THE USE OF THE 50 CENTIMETRE SLIDE-RULE IN TACHEOMETRY AND CURVE RANGING[1]

In the writer's experience, he has not met with surveyors who make use of this form of slide-rule for all the purposes for which it is adapted. The 25 centimetre rule is in common use, limited as it is by its size to operations which do not render the surveyor independent of books.

The 50 centimetre slide-rule is of an accuracy which nearly meets all cases, and with the addition of the table of cosine squares, specially prepared by the writer for use with the slide-rule, it meets all requirements in tacheometry, both for reduction of observations in the field and for the office work. In curve ranging, with the addition of the table of cosines it is also sufficient without any other tables. These tables of cosine squares and cosines are so condensed that they can be pasted in the fly-leaf of the field-book and thus do not add to the surveyor's impedimenta, neither do they require any turning over of pages to find the needed figure.

Many years ago, a metallic slide-rule was devised by Mr Kern of Aarau, by means of which the multiplication by cosine square and by sine and cosine were given direct, but this slide-rule was only 20 cm. in length and consequently could only be read by means of a magnifying glass. The scope

[1] Since the publication of the writer's *Preliminary Survey and Estimates*, Longmans Textbooks of Science Series (First Edition 1890. Fourth Edition, 1905), containing what he believes to have been the first practical description of tacheometry published in England, many improvements have taken place both in instruments and methods. To the readers of the above-mentioned book the present notes are offered as supplementary to, and in some respects superseding the description therein given.

of the instrument was not such as to dispense with more precise calculation by logarithms.

In order to illustrate the completeness of the work which can be done by the 50 cm. slide-rule, in conjunction with the simple tables appended, a short description will be given of the methods adopted by the writer both in tacheometry and curve-ranging.

The reader is assumed to be a surveyor or student of surveying and therefore to be familiar with the ground-work of the subject. Metric measurement is used throughout the pamphlet although all the formulae and description are equally adapted to any other unit of measurement.

THE REDUCTION OF TACHEOMETRIC FIELD-WORK BY MEANS OF THE SLIDE-RULE.

Tacheometry, in which distances are measured optically by a telescope, without a measuring chain, is generally admitted to be most suitable in difficult country, and where great accuracy is not required.

These are, however, emphatically *not* the limits of tacheometry. Tacheometry is of universal application and in all kinds of country. The surveyor can in this manner do everything quicker and better than with the chain; only his instruments must be adapted to the class of work which he has to do. The first question to be decided is the *degree* of accuracy required, in other words, the *scale* upon which the survey is to be mapped. To aim at a minuteness of field-work which cannot afterwards be plotted, is a waste of time. The commonest mistake of beginners is to fill up their field-book with a mass of detail which cannot possibly be reproduced on paper.

The second question is the *degree* of accuracy with which the observations in the field must be reduced by calculation to the form suitable for plotting. Beginners are frequently found working out by logarithms to the nearest millimetre, distances which they have not yet learned to measure correctly with the instrument. The principal source of error in tacheometry is the optical measurement itself, and the surveyor should first practise this until he is quite sure of his work, not only on test lines but, wherever practicable, by checking each primary base with the steel tape. Errors often arise by attempting measurements of too great a length for the size of instrument. The tacheometer should be as powerful as possible without being unwieldy. The most powerful telescope, which can be carried on an ordinary tripod; the complete instrument being portable without needing to be dismounted when changing stations; is one with a 1·8 inch aperture and a 13 inch focal length. The eyepiece may give 42 times magnification. An anallatic lens is very convenient but by no means essential.

The Stadia-rod should be held vertically, as thus the rodman is more reliable than when he has to hold it at right angles to the line of vision. The vertical angle should be measured by directing the optical axis to the same height upon the rod as that of the optical axis above the peg of the instrumental station. The stadia, or distance upon the rod subtended by the stadia lines in the telescope may for convenience be read by directing the lower stadia line on the nearest even decimetre line of the rod, as the error therefrom is in almost all cases unmeasurable, and much time is saved as compared with reading all three lines of the telescope. The subtense lines should be fixed to read 1:100. A double reading of 1 : 50 and also 1 : 100 is unnecessary and objectionable. The stadia-rod should be maintained in a vertical position by means of a wire and thin cylindrical lead plummet hanging on the back of the rod. The wire passes through the eye of a staple with a little swinging freedom, and the rodman must see that the wire hangs freely without touching the eye of the staple. This method is preferable to a circular bubble.

The slide-rule should be of 50 cm. length with celluloid face. The objection sometimes raised that this rule is too sensitive to climatic changes is unfounded. The writer has used the same slide-rule in the Tropics and in Europe without any trouble arising from the change of climate, wet seasons and dry.

The trigonometrical functions on the reverse of the slide are necessarily made to read with the upper or smaller scale of the rule, and the graduation does not permit of the cosines (complements of the sines) of small arcs being read correctly. By means of the table appended hereto, however, the cosines or cosine squares of the table are used with the lower and more open scale of the rule and slide, and thus an equal accuracy is obtained to that of the other operations.

It is never desirable to reduce all the observations in the field, but that of the principal or instrumental stations is very advisable. The bulk of the reduction and of the plotting should be carried on in the office at the same time as the field-work. For this purpose, duplicate field-books should be kept. The surveyor, at the end of his day's work, should have no more to do than to examine the plot of the preceding day's work.

The stadia-rod being held vertically, the reduction of the field observations is as follows:

(1) The horizontal distance is determined from the stadia observation of distance and vertical angle; by the equation,

$$(1) \quad \text{Hor. Dist.} = M \times \cos^2 V$$

in which M = the stadia measurement and V the vertical angle.

(2) The difference of level between the two points, or pegs, is determined by the equation,

$$(2) \quad \text{Diff. Lev.} = \text{Hor. Dist.} \times \tan V$$

from which the reduced elevation above sea-level or some other fiducial base is obtained, the elevation of the instrumental starting station being known or assumed.

(3) It is also preferable to plot the work by the method of rectangular coordinates of Latitude and Departure, a method which, whilst admittedly more accurate, is not frequently adopted on account of the labour of using traverse tables. By the slide-rule however, it is as rapid as, and free from the objections of protractor work.

$$(3) \quad \text{Lat.} = \text{Hor. Dist.} \times \cos \text{Az. (North or South)}$$

and $$(4) \quad \text{Dep.} = \text{Hor. Dist.} \times \sin \text{Az. (East or West)}$$

in which Az. = the horizontal angle or azimuth of the base line, measured from the North or South point towards East or West respectively.

By this method the surveyor enters in his field-book not only the horizontal and vertical coordinates of each base line but also the reduced Latitude and Departure of each instrumental station from the origin of these latter coordinates; namely from the starting point of the survey. If for instance he has made an elongated traverse and when midway on his return, wishes to run a check line to some previous station about midway on his outgoing line; thus roughly bisecting his traverse with the check line; having the reduced coordinates of the two stations, he can, even when the one station is not visible from the other, set his instrument by calculation upon the bearing and run his check line to join the required previous station.

The reduction of the principal stations by logarithms is very seldom needed. The following example is given, worked out independently by the 50 cm. slide-rule and by logarithms.

Example.

The measurement, M, of the distance by stadia on the inclination of the line of sight was 173·4 metres.

The vertical angle, V, was 5° 45′.

The horizontal angle, or bearing, was 335° 10′, of which the Azimuth, Az. is N. 24° 50′ W.

	By slide-rule metres.	By logarithm metres.
Hor. Dist. $= M . \cos^2 V$	171·6	171·66
Latitude = Hor. Dist. × cos Az.	155·9 N.	155·79 N.
Departure = Hor. Dist. × sin Az.	72·0 W.	72·09 W.
Diff. Lev. = Hor. Dist. × tan V	17·28	17·286.

The distance of 173·4 metres (nearly 570 feet) is quite a long sight in ordinary practice. The accuracy obtainable is in proportion to the distance. It will be seen therefore from the preceding example how large the scale must be upon which the plan is plotted before the additional accuracy obtainable from logarithms becomes a scaleable quantity.

The greatest difference in the preceding Example, on a scale of 1 : 500, is less than half a millimetre on paper. For most purposes a scale of 1 : 2500 is sufficiently large, and upon this scale the difference could barely be detected by a magnifying glass. For Form of Field-book see page 17.

Labour-saving tacheometers.

Numerous instruments have been devised, some of them remarkably ingenious, for dispensing with the trigonometrical functions and with traverse tables. No instrument has however been or is likely to be devised from which the reduced results can be read off without further calculation. In the writers opinion, the operation of reduction, when performed by the slide-rule from observations made with an instrument of ordinary graduation, cannot be excelled for rapidity and accuracy by any special instrument yet devised. A specially graduated tacheometer has moreover the disadvantage that it cannot be put into any surveyor's hands until he has well proved his weapon.

CURVE-RANGING FOR LIGHT RAILWAYS, TRAMWAYS, ETC., BY PRISMATIC COMPASS, WITH OR WITHOUT BOX SEXTANT AND THE 50 CM. SLIDE-RULE.

With a prismatic compass which can be read to a half-degree and estimated to a quarter-degree, curves can be set out with sufficient accuracy for the above purpose without any tables, and with greater rapidity and more accuracy than is usually attained when ranging by means of offsets to equal chords.

The compass bearings should always be read both ways, by back-sight and fore-sight, and the mean taken of the two readings in order as far as possible to eliminate magnetic deviation.

With the Box Sextant, the deflection angle can be read to one minute. The writer has however not found the Box Sextant necessary.

The Deflection angle, which is the only angle needing to be measured, can be determined within about 2′ by tape and slide-rule alone, in the following manner:

With a fine string lay out any two equal lengths on the prolongation from the point of intersection of the leading-in tangent and the leading-out tangent. From the two extremities measure the distance subtended. All three measurements with a steel tape. The deflection angle is then determined by the following equation by the slide-rule: (1) $\sin\frac{\Delta}{2} = \frac{y}{2x}$.

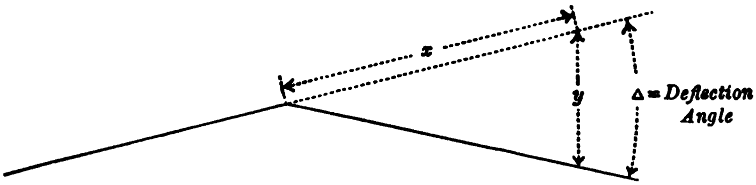

Fig. 1.

The following formulae are equally applicable to precise curve-ranging with the theodolite. When, however, the work is of a nature to require the more precise instruments, it is also advisable to fix the points in the curve at even distances along the continuous chainage of the centre line. This entails the setting out of the first and last points of the curve at odd distances from the termini, which takes more time; is not in any case essential; and in work of the description here dealt with, superfluous.

The method here described, is to divide the whole curve length into chords of equal length. The only angle measured on the ground (if the intersection point is accessible) is the deflection angle or difference of bearing between the leading-in and leading-out tangents.

When the intersection point is inaccessible, the operation is almost as simple, but for this a separate description will be given.

The slide-rule used, for similar reasons to those already given in the notes on tacheometry, is the 50 centimetre rule, preferably with a celluloid face. With this slide-rule, all the operations can be performed without any tables whatever.

As the use of the cosine is also involved in the formulae, unless a rather complicated equation be substituted for the one where the cosine occurs, a table of cosines, so arranged as to suit the capacity of the slide-rule has been added, by which means, using with it the lower scales of slide and rule, additional accuracy is obtained.

The General Diagram, Fig. 2, illustrates the method, and the explanation of the symbols is as follows:

Bearing of leading-in tangent .... B_I

Bearing of leading-out tangent .... B_O

Deflection angle, being the difference of bearing between the two tangents .... Δ

Bearing of the apex of the curve from the point of intersection .... B_A

Radius .... R

Chord of complete curve C

Chord of semi-curve (semi-chord) .... C_S

Sub-chord into which the curve is divided n times c

Number of sub-chords in the curve .... n

Length of curve, measured in segments, the sub-chord being chosen sufficiently short to make the difference between the theoretical length of curve and the length measured in segments a negligible quantity L

Sub-tangent .... T

Portions of sub-tangent from the terminal points of the curve measured to intersections of radii passing through the ends of the sub-chords .... T_1; T_2; T_3 etc.

Apex distance, from intersection point of sub-tangents to apex of curve .... A

Distances from the points T_1; T_2; etc. to the terminal points of the sub-chords, measured along the respective radii .... A_1; A_2; A_3 etc.

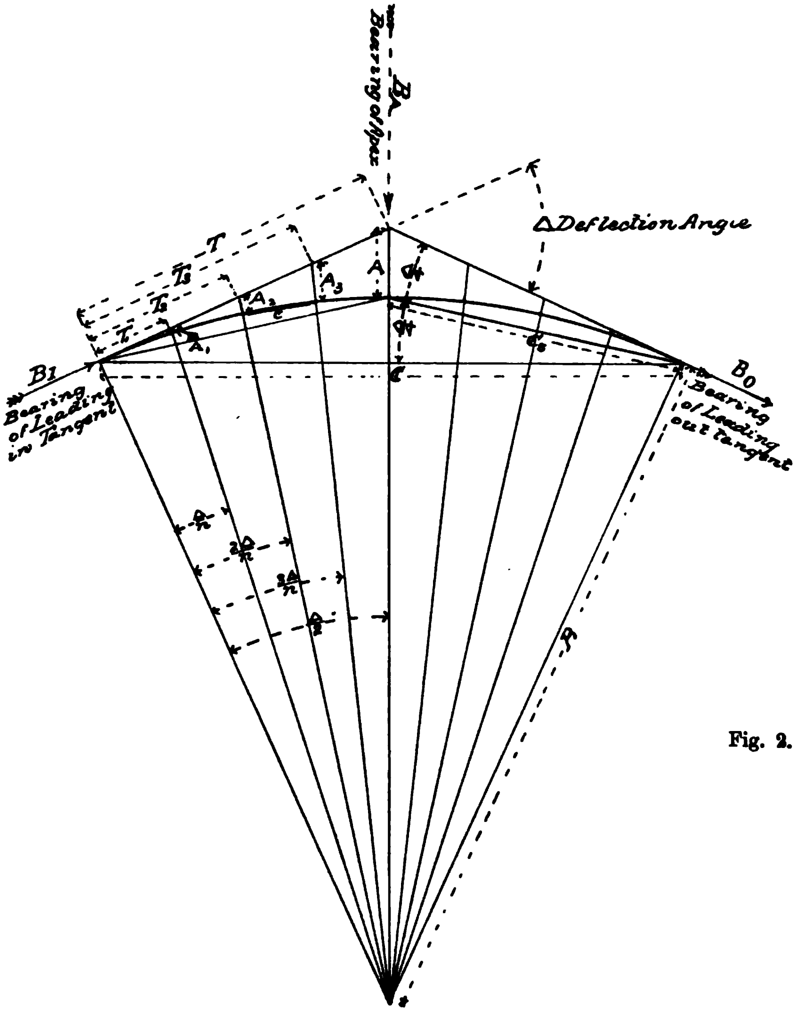

Fig. 2.

Formulae.

Deflection angle of curve—in right-handed curves $\Delta = B_O - B_I$.
left „ „ $\Delta = B_I - B_O$.

Bearing of apex. Right-handed curves $B_A = B_I + \left(90^\circ + \frac{\Delta}{2}\right)$.

Left „ „ $B_A = B_I - \left(90^\circ + \frac{\Delta}{2}\right)$.

When the result of the above equations is a minus quantity add to the lesser angle 360°

$$C = 2R \times \sin\frac{\Delta}{2}; \quad C_S = 2R \times \sin\frac{\Delta}{4}; \quad c = 2R \times \sin\frac{\Delta}{2n}.$$

$$L = n \times c.$$

$$T = R \times \tan\frac{\Delta}{2}, \qquad A = R \div \cos\frac{\Delta}{2} - R,$$

$$T_1 = R \times \tan\frac{\Delta}{n}, \qquad A_1 = R \div \cos\frac{\Delta}{n} - R.$$

For T_2; T_3 etc., and A_2; A_3 etc., multiply in the preceding two equations Δ by the number of T or A respectively. Thus:

$$T_5 = R \times \tan\frac{5\Delta}{n} \text{ etc.} \qquad A_5 = R \div \cos\frac{5\Delta}{n} - R \text{ etc.}$$

TABLES OF COSINES AND COSINE SQUARE FROM 0° TO 45°, ARRANGED FOR FACILITATING CALCULATION WITH THE SLIDE-RULE.

	Cosine	Cosine Square		Cosine	Cosine Square		Cosine	Cosine Square		Cosine	Cosine Square		Cosine	Cosine Square
1° 30′	0·9996	0·9993	16° 45′	0·9575	0·9169	23° 55′	0·9141	0·8356	31° 5′	0·8564	0·7334	38° 15′	0·7853	0·6167
2° 0′	0·9994	0·9988	50	0·9571	0·9161	24° 0′	0·9135	0·8346	10	0·8557	0·7322	20	0·7844	0·6153
30	0·9990	0·9981	55	0·9567	0·9153	5	0·9129	0·8335	15	0·8549	0·7309	25	0·7835	0·6139
3° 0′	0·9986	0·9973	17° 0′	0·9563	0·9145	10	0·9123	0·8324	20	0·8541	0·7296	30	0·7826	0·6125
30	0·9981	0·9963	5	0·9559	0·9137	15	0·9117	0·8313	25	0·8534	0·7283	35	0·7817	0·6110
4° 0′	0·9976	0·9951	10	0·9554	0·9129	20	0·9112	0·8302	30	0·8526	0·7270	40	0·7808	0·6096
20	0·9971	0·9943	15	0·9550	0·9121	25	0·9106	0·8291	35	0·8519	0·7257	45	0·7799	0·6082
40	0·9967	0·9934	20	0·9546	0·9112	30	0·9100	0·8280	40	0·8511	0·7244	50	0·7790	0·6068
5° 0′	0·9962	0·9924	25	0·9541	0·9104	35	0·9093	0·8269	45	0·8503	0·7231	55	0·7780	0·6054
20	0·9957	0·9914	30	0·9537	0·9096	40	0·9087	0·8258	50	0·8496	0·7218	39° 0′	0·7771	0·6040
40	0·9951	0·9902	35	0·9533	0·9087	45	0·9081	0·8247	55	0·8488	0·7205	5	0·7762	0·6025
6° 0′	0·9945	0·9891	40	0·9528	0·9079	50	0·9075	0·8236	32° 0′	0·8480	0·7192	10	0·7753	0·6011
15	0·9940	0·9881	45	0·9524	0·9071	55	0·9069	0·8225	5	0·8473	0·7179	15	0·7744	0·5997
30	0·9936	0·9872	50	0·9519	0·9062	25° 0′	0·9063	0·8214	10	0·8465	0·7166	20	0·7735	0·5982
45	0·9931	0·9862	55	0·9515	0·9054	5	0·9057	0·8203	15	0·8457	0·7153	25	0·7725	0·5968
7° 0′	0·9925	0·9851	18° 0′	0·9510	0·9045	10	0·9051	0·8192	20	0·8449	0·71[illegible]9	30	0·7716	0·5954
12	0·9921	0·9843	5	0·9506	0·9036	15	0·9044	0·8180	25	0·8442	0·7126	35	0·7707	0·5940
24	0·9917	0·9834	10	0·9501	0·9028	20	0·9038	0·8169	30	0·8434	0·7113	40	0·7698	0·5925
36	0·9912	0·9825	15	0·9497	0·9019	25	0·9032	0·8158	35	0·8426	0·7100	45	0·7688	0·5911
48	0·9907	0·9816	20	0·9492	0·9011	30	0·9026	0·8147	40	0·8418	0·7087	50	0·7679	0·5897
8° 0′	0·9203	0·9806	25	0·9488	0·9002	35	0·9019	0·8135	45	0·8410	0·7073	55	0·7670	0·5882
12	0·9898	0·9796	30	0·9483	0·8993	40	0·9013	0·8124	50	0·8402	0·7060	40° 0′	0·7660	0·5868
24	0·9893	0·9787	35	0·9479	0·8984	45	0·9007	0·8113	55	0·8394	0·7047	5	0·7651	0·5854
36	0·9887	0·9776	40	0·9474	0·8976	50	0·9001	0·8101	33° 0′	0·8387	0·7034	10	0·7642	0·5839
48	0·9882	0·9766	45	0·9469	0·8967	55	0·8994	0·8090	5	0·8379	0·7020	15	0·7632	0·5825
9° 0′	0·9877	0·9755	50	0·9464	0·8958	26° 0′	0·8988	0·8078	10	0·8371	0·7007	20	0·7623	0·5811
10	0·9872	0·9746	55	0·9460	0·8949	5	0·8981	0·8067	15	0·8363	0·6994	25	0·7613	0·5796
20	0·9867	0·9737	19° 0′	0·9455	0·8940	10	0·8975	0·8055	20	0·8355	0·6980	30	0·7604	0·5782
30	0·9863	0·9728	5	0·9450	0·8931	15	0·8969	0·8044	25	0·8347	0·6967	35	0·7595	0·5768
40	0·9858	0·9718	10	0·9446	0·8922	20	0·8962	0·8032	30	0·8339	0·6954	40	0·7585	0·5753
50	0·9853	0·9708	15	0·9441	0·8913	25	0·8956	0·8021	35	0·8331	0·6940	45	0·7576	0·5739
10° 0′	0·9848	0·9698	20	0·9436	0·8904	30	0·8949	0·8009	40	0·8323	0·6927	50	0·7566	0·5724
10	0·9843	0·9688	25	0·9431	0·8895	35	0·8943	0·7997	45	0·8315	0·6913	55	0·7557	0·5710
20	0·9838	0·9678	30	0·9426	0·8886	40	0·8936	0·7986	50	0·8307	0·6900	41° 0′	0·7547	0·5696
30	0·9832	0·9668	35	0·9421	0·8877	45	0·8930	0·7974	55	0·8298	0·6886	5	0·7537	0·5681
40	0·9827	0·9657	40	0·9417	0·8867	50	0·8923	0·7962	34° 0′	0·8290	0·6873	10	0·7528	0·5667
50	0·9822	0·9647	45	0·9412	0·8858	55	0·8917	0·7951	5	0·8282	0·6859	15	0·7518	0·5653
11° 0′	0·9816	0·9636	50	0·9407	0·8849	27° 0′	0·8910	0·7939	10	0·8274	0·6846	20	0·7509	0·5638
10	0·9810	0·9625	55	0·9402	0·8840	5	0·8903	0·7927	15	0·8266	0·6832	25	0·7499	0·5624
20	0·9805	0·9614	20° 0′	0·9397	0·8830	10	0·8897	0·7915	20	0·8258	0·6819	30	0·7489	0·5609
30	0·9799	0·9602	5	0·9392	0·8820	15	0·8891	0·7903	25	0·8249	0·6805	35	0·7480	0·5595
40	0·9793	0·9591	10	0·9387	0·8811	20	0·8883	0·7892	30	0·8241	0·6792	40	0·7470	0·5580
50	0·9787	0·9579	15	0·9382	0·8802	25	0·8877	0·7880	35	0·8233	0·6778	45	0·7460	0·5566
12° 0′	0·9781	0·9568	20	0·9377	0·8793	30	0·8870	0·7868	40	0·8225	0·6765	50	0·7451	0·5551
10	0·9775	0·9556	25	0·9372	0·8783	35	0·8863	0·7856	45	0·8216	0·6751	55	0·7441	0·5537
20	0·9769	0·9544	30	0·9367	0·8774	40	0·8856	0·7844	50	0·8208	0·6737	42° 0′	0·7431	0·5523
30	0·9763	0·9532	35	0·9362	0·8764	45	0·8850	0·7832	55	0·8200	0·6724	5	0·7422	0·5508
40	0·9756	0·9519	40	0·9356	0·8754	50	0·8843	0·7820	35° 0′	0·8191	0·6710	10	0·7412	0·5494
50	0·9750	0·9507	45	0·9351	0·8745	55	0·8836	0·7808	5	0·8183	0·6696	15	0·7402	0·5479
13° 0′	0·9744	0·9494	50	0·9346	0·8735	28° 0′	0·8829	0·7796	10	0·8175	0·6683	20	0·7392	0·5465
7½	0·9739	0·9484	55	0·9341	0·8725	5	0·8822	0·7784	15	0·8166	0·6669	25	0·7382	0·5450
15	0·9734	0·9475	21° 0′	0·9336	0·8716	10	0·8816	0·7772	20	0·8158	0·6655	30	0·7373	0·5436
22½	0·9729	0·9465	5	0·9330	0·8706	15	0·8809	0·7760	25	0·8149	0·6641	35	0·7363	0·5421
30	0·9724	0·9455	10	0·9325	0·8696	20	0·8802	0·7748	30	0·8141	0·6628	40	0·7353	0·5407
37½	0·9719	0·9445	15	0·9320	0·8686	25	0·8795	0·7735	35	0·8133	0·6614	45	0·7343	0·5392
45	0·9713	0·9435	20	0·9315	0·8677	30	0·8788	0·7723	40	0·8124	0·6600	50	0·7333	0·5378
52½	0·9708	0·9425	25	0·9309	0·8667	35	0·8781	0·7711	45	0·8116	0·6586	55	0·7323	0·5363
14° 0′	0·9703	0·9415	30	0·9304	0·8657	40	0·8774	0·7699	50	0·8107	0·6573	43° 0′	0·7313	0·5349
7½	0·9698	0·9404	35	0·9299	0·8647	45	0·8767	0·7686	55	0·8099	0·6559	5	0·7304	0·5334
15	0·9692	0·9394	40	0·9293	0·8637	50	0·8760	0·7674	36° 0′	0·8090	0·6545	10	0·7294	0·5320
22½	0·9687	0·9384	45	0·9288	0·8627	55	0·8753	0·7662	5	0·8082	0·6531	15	0·7284	0·5305
30	0·9681	0·9373	50	0·9283	0·8617	29° 0′	0·8746	0·7650	10	0·8073	0·6517	20	0·7274	0·5291
37½	0·9676	0·9362	55	0·9277	0·8607	5	0·8739	0·7637	15	0·8064	0·6503	25	0·7264	0·5276
45	0·9670	0·9352	22° 0′	0·9272	0·8597	10	0·8732	0·7625	20	0·8056	0·6489	30	0·7254	0·5262
52½	0·9665	0·9340	5	0·9266	0·8587	15	0·8725	0·7612	25	0·8047	0·6475	35	0·7244	0·5247
15° 0′	0·9659	0·9330	10	0·9261	0·8576	20	0·8718	0·7600	30	0·8038	0·6462	40	0·7234	0·5232
5	0·9655	0·9323	15	0·9255	0·8566	25	0·8711	0·7588	35	0·8030	0·6448	45	0·7224	0·5218
10	0·9652	0·9315	20	0·9250	0·8556	30	0·8703	0·7575	40	0·8021	0·6434	50	0·7214	0·5203
15	0·9648	0·9308	25	0·9244	0·8546	35	0·8696	0·7563	45	0·8012	0·6420	55	0·7203	0·5189
20	0·9644	0·9301	30	0·9239	0·8535	40	0·8689	0·7550	50	0·8004	0·6406	44° 0′	0·7193	0·5174
25	0·9640	0·9293	35	0·9233	0·8525	45	0·8682	0·7538	55	0·7995	0·6392	5	0·7183	0·5160
30	0·9636	0·9286	40	0·9227	0·8515	50	0·8675	0·7525	37° 0′	0·7986	0·6378	10	0·7173	0·5145
35	0·9632	0·9278	45	0·9222	0·8505	55	0·8667	0·7513	5	0·7977	0·6364	15	0·7163	0·5131
40	0·9628	0·9271	50	0·9216	0·8494	30° 0′	0·8660	0·7500	10	0·7969	0·6350	20	0·7153	0·5116
45	0·9624	0·9263	55	0·9211	0·8484	5	0·8653	0·7487	15	0·7960	0·6336	25	0·7143	0·5102
50	0·9620	0·9256	23° 0′	0·9205	0·8473	10	0·8646	0·7475	20	0·7951	0·6322	30	0·7132	0·5087
55	0·9617	0·9248	5	0·9199	0·8463	15	0·8638	0·7462	25	0·7942	0·6308	35	0·7122	0·5073
16° 0′	0·9613	0·9240	10	0·9194	0·8452	20	0·8631	0·7449	30	0·7933	0·6294	40	0·7112	0·5058
5	0·9608	0·9232	15	0·9188	0·8442	25	0·8624	0·7437	35	0·7925	0·6280	45	0·7102	0·5044
10	0·9604	0·9225	20	0·9182	0·8431	30	0·8616	0·7424	40	0·7916	0·6266	50	0·7092	0·5029
15	0·9600	0·9217	25	0·9176	0·8421	35	0·8609	0·7411	45	0·7907	0·6252	55	0·7081	0·5014
20	0·9596	0·9209	30	0·9171	0·8410	40	0·8601	0·7399	50	0·7898	0·6238	45° 0′	0·7071	0·5000
25	0·9592	0·9201	35	0·9165	0·8399	45	0·8594	0·7386	55	0·7889	0·6224	5	0·7061	0·4985
30	0·9588	0·9193	40	0·9159	0·8389	50	0·8587	0·7373	38° 0′	0·7880	0·6210	10	0·7051	0·4971
35	0·9584	0·9185	45	0·9153	0·8378	55	0·8579	0·7360	5	0·7871	0·6195	15	0·7040	0·4956
40	0·9580	0·9177	50	0·9147	0·8367	31° 0′	0·8572	0·7347	10	0·7862	0·6181	20	0·7030	0·4942

Formulae.

Deflection of curve; Δ.

For right-handed curves (2) $\Delta = B_O - B_I$.
For left-handed curves (3) $\Delta = B_I - B_O$.

Bearing of apex; B_A.

For right-handed curves (4) $B_A = B_I + \left(90^\circ + \frac{\Delta}{2}\right)$.

For left-handed curves (5) $B_A = B_I - \left(90^\circ + \frac{\Delta}{2}\right)$.

In Equations (2) to (5); when the result becomes a minus quantity add to the lesser angle 360°.

(6) $C = 2R \times \sin\frac{\Delta}{2}$; (7) $C_S = 2R \times \sin\frac{\Delta}{4}$; (8) $c = 2R \times \sin\frac{\Delta}{2n}$.

(9) $L = n \times c$.

(10) $T = R \times \tan\frac{\Delta}{2}$; (12) $A = R \div \cos\frac{\Delta}{2} - R$.

(11) $T_1 = R \times \tan\frac{\Delta}{n}$; (13) $A_1 = R \div \cos\frac{\Delta}{n} - R$.

For T_2; T_3 etc. and A_2; A_3 etc., in Equations (11) and (13) multiply Δ by the number of T and A respectively, thus:—

$$T_2 = R \times \tan\frac{2\Delta}{n} \text{ etc.}; \qquad T_2 = R \div \cos\frac{2\Delta}{n} - R \text{ etc.}$$

After the deflection angle is measured, the sub-divisions of Δ, required by the number of sub-chords n are first entered in the field-book, after which the required functions of the curve can be read off from the slide-rule as rapidly as they can be measured on the ground by the assistants. In most work of this kind, no more than 8 sub-chords are necessary. The example to be presently given represents more field-work than the average. The length of sub-chord chosen, should not be greater than will allow the grading ganger to set out the grading between the pegs with his boning sticks, and the track layer to curve the rails between the pegs with the crowbar or jimcrow. For ordinary railways, sub-chords of about 20 metres are usual. For light railways, about 10 metres. For lines with very sharp curves such as "Decauvilles" it may be necessary to put in sub-chords of about 5 metres.

The radius of curve is ruled by the character of the line having regard principally to the amount of earthwork involved which varies in direct proportion to the length of radius, when the ground is much accidentated. The minimum radius is that which the locomotive will go round safely at the maximum permissible speed. Very light lines such as "Decauvilles" may have radii as short as 15 metres; light railways about 25 metres, ordinary railways from 100 metres and upwards.

The general rule is never to put in *unnecessarily short radii.*

Example of eight segment curve.

At a distance of 7413 metres along the line of railway, a deviation to the right will be made. The bearings of the leading-in and leading-out tangents are respectively 17° 30′ N.E. and 56° 26′ N.E. Deflection curve; $\Delta = B_O - B_I = 38° 56'$.

$$\frac{\Delta}{2n} = \frac{\Delta}{16} = 2° 26'; \quad \frac{\Delta}{n} = \frac{\Delta}{8} = 4° 52'; \quad \frac{2\Delta}{n} = \frac{\Delta}{4} = 9° 44';$$

$$\frac{3\Delta}{n} = \frac{3\Delta}{8} = 14° 36'; \quad \frac{\Delta}{2} = 19° 28'.$$

Bearing of apex—$B_A = B_I + \left(90° + \frac{\Delta}{2}\right) = 126° 58'$(or 53° 2′ S.E.).

The radius chosen is 127 metres $= R$.

$$C = 2R \times \frac{\sin \Delta}{2} = 84{\cdot}65; \quad C_S = 2R \times \sin\frac{\Delta}{4} = 43{\cdot}00.$$

$$c = 2R \times \sin\frac{\Delta}{2n} = 10{\cdot}80; \quad L = 8 \times 10{\cdot}80 = 86{\cdot}40.$$

$$T = R \times \tan\frac{\Delta}{2} = 44{\cdot}90, \quad A = R \div \cos\frac{\Delta}{2} - R = 7{\cdot}7.$$

$$T_1 = R \times \tan \frac{\Delta}{n} = 10{\cdot}80, \qquad A_1 = R \div \cos \frac{\Delta}{n} - R = 0{\cdot}47.$$

$$T_2 = R \times \tan \frac{2\Delta}{n} = 21{\cdot}80, \qquad A_2 = R \div \cos \frac{2\Delta}{n} - R = 1{\cdot}85.$$

$$T_3 = R \times \tan \frac{3\Delta}{n} = 33{\cdot}10, \qquad A_3 = R \div \cos \frac{3\Delta}{n} - R = 4{\cdot}25.$$

Commencement of curve at $7413 - T = 7368{\cdot}10$ m.

Apex of curve at $7368{\cdot}10 + \frac{L}{2} = 7411{\cdot}30$ m.

Termination of curve at $7368{\cdot}10 + L = 7454{\cdot}50$ m.

Procedure of actual setting out.

1. From the point of intersection set out on the bearing 126° 58′ the distance $A = 7{\cdot}70$ to apex of curve and set in peg.

2. From the point of intersection set out along the two subtangents lengths $= T$ and drive pegs at the two curve termini.

3. Proceed to the point of commencement and from this point measure off in the direction of the point of intersection T_1; T_2 and T_3 setting up at these points temporary pickets.

From T_1 and from point of commencement measure off simultaneously A_1 and $c, = 0{\cdot}47$ and $10{\cdot}80$, the intersection of these two gives point 1 in the curve.

From T_2 and point 1 measure similarly 1·85 and 10·80, the intersection gives point 2, similarly fix point 3.

Check.

The formula for ranging curves by equal offsets is:—

1st offset from tangent $= \frac{c^2}{2R} = 0{\cdot}46$ metre.

2nd and following offsets from chord $= \frac{c^2}{R} = 0{\cdot}92$ „

Check round the points by this method.

If the whole of the curve is accessible but for some reason, the point of intersection is not so, as shown in Fig. 3.

Take the bearings B_I and B_O of the two tangents; place two pickets in each tangent to fix their direction.

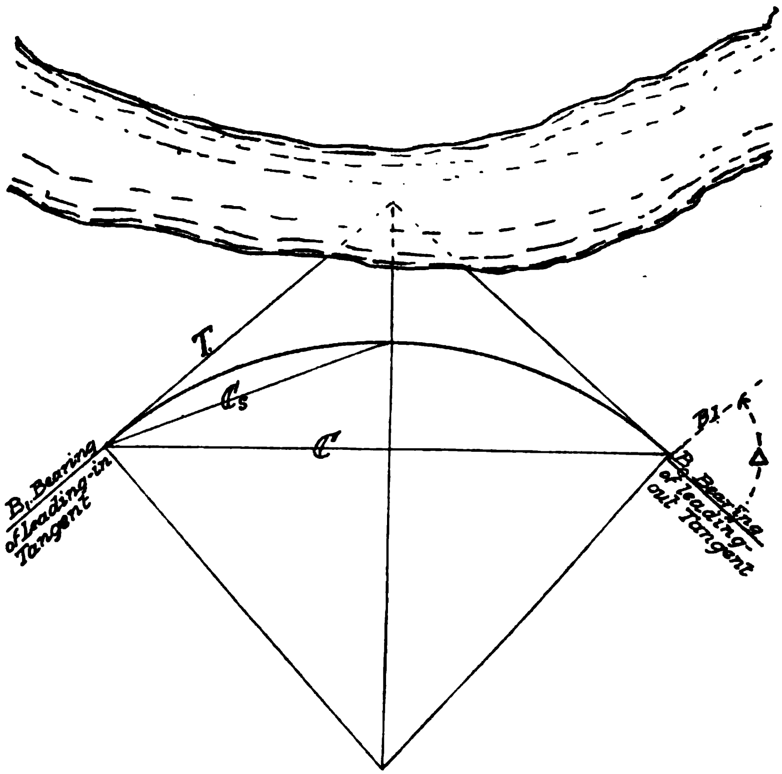

Fig. 3.

Calculate the deflection angle from Equation 2 or 3. Assume a point in the leading-in tangent for the commencement of curve.

Set out the bearing of the chord C by the Equation (14) $B_C = B_I \pm \frac{\Delta}{2}$ according as the curve is right or left.

Measure the length of the chord to its intersection with the leading-out tangent.

From Equation (6) transposed; the radius is found $R = \frac{C}{2} \div \sin\frac{\Delta}{2}$.

Set out the bearing of the semi-chord C_S by the Equation (15) $BC_S = B_I \pm \frac{\Delta}{4}$ according as the curve is right or left.

From Equation (7), the length of the semi-chord C_S is found $= 2R \times \sin\frac{\Delta}{4}$.

At the extremity of this line fix the apex point.

Proceed as previously described to fix points 1, 2, 3, etc.

If when fixing the bearing of, or running the semi-chord C_S it is found that the apex point will come in or too near the river, the point of commencement of the curve can be shifted backwards or forwards without requiring to remeasure the chord.

Calling the distance which the point of commencement is shifted backwards or forwards, x, then the increase or decrease of length of the chord C and of the radius R are given by the equations:

$$(16) \quad Cx = 2x \cos\frac{\Delta}{2},$$

and

$$(17) \quad Rx = x \div \tan\frac{\Delta}{2}.$$

The increased or decreased lengths of R and C can then be determined from the preceding equations as $C_1 = C \pm Cx$, and $R_1 = R \pm Rx$ according as the point of commencement is shifted backwards or forwards respectively. The increased or decreased length of C_S is determined by Equation 7 using R_1 instead of R. The bearings of C and C_S remain unchanged.

In working with the slide-rule, there is the great advantage over tables, that radii and sub-chords of any irregular length can be chosen, such as best suit each particular case, and the curves calculated with no more trouble than with even figures.

If for instance, at a certain point in a curve, it is found desirable to compound the curve, or to turn a reverse curve, and the deflection angle Δ, has been measured between a tangent to the old curve and the leading-out tangent of the proposed new curve, the sub-tangent of which is a fixed quantity; the radius of the new curve, which will have a common point of termination and commencement with the old curve, can be found by the transposition of Equation (10) into:

$$R = T \div \tan\frac{\Delta}{2}.$$

The new curve can then be set out as previously described.

If the point of intersection is inaccessible, the radius can be determined as previously described by running the chord to intersection with the leading-out tangent of the new curve.

FORM OF TACHEOMETRIC FIELD-BOOK

From Instrumental Station						To Stadia Station																			
Station	Height above base	Coordinates of Position				Station	Height above base	Coordinates of Position				Height of Inst.	Stadia	Constant	Distance on Slope	Vertical Angle	Hor. Dist.	Diff. Level	Bearing	Azimuth	Latitude		Departure		
		N.	S.	E.	W.			N.	S.	E.	W.										N.	S.	E.	W.	
A	100·0	0	0	0	0	A_1	117·28	155·9	—	—	72·0	1·53	172·9	0·5	173·4	+5° 45′	171·6	+17·28	335° 10′	N. 24° 50′ W.	155·9	—	—	72·0	

Process. 1. Enter names of Instrumental Station with Height above base and Coordinates of Position.

2. Enter Height of Instrument above peg.
3. Enter names of Stadia Station, whether principal or intermediate.
4. Read Vertical Angle, with central hair at same height on rod as Height of Instrument above peg, and enter plus or minus.
5. Read Stadia Distance.
6. If no anallatic lens is provided, add Constant to Stadia and enter it as Distance on Slope.
7. Enter Bearing.

   The above entries are all that are made when no field calculations are required.

8. Calculate and enter Hor. Distance and Diff. of Level.
9. Enter Height above base of Stadia Station.
10. Reduce Bearing to Equivalent Azimuth.
11. Calculate and enter Latitude and Departure.
12. Enter Coordinates of Position of Stadia Station.

   The Plotting does not usually demand the entry of the Coordinates of Position of any other than the principal stations. The intermediate stations are plotted from the principal stations by the Latitude and Departure, more quickly and with sufficient accuracy.

TABLES OF COSINES AND COSINE SQUARE FROM 0° TO 45°, ARRANGED FOR FACILITATING CALCULATION WITH THE SLIDE-RULE.

	Cosine	Cosine Square		Cosine	Cosine Square		Cosine	Cosine Square		Cosine	Cosine Square		Cosine	Cosine Square
1° 30′	0·9996	0·9993	16° 45′	0·9575	0·9169	23° 55′	0·9141	0·8356	31° 5′	0·8564	0·7334	38° 15′	0·7853	0·6167
2° 0′	0·9994	0·9988	50	0·9571	0·9161	24° 0′	0·9135	0·8346	10	0·8557	0·7322	20	0·7844	0·6153
30	0·9990	0·9981	55	0·9567	0·9153	5	0·9129	0·8335	15	0·8549	0·7309	25	0·7835	0·6139
3° 0′	0·9966	0·9973	17° 0′	0·9563	0·9145	10	0·9123	0·8324	20	0·8541	0·7296	30	0·7826	0·6125
30	0·9981	0·9963	5	0·9559	0·9137	15	0·9117	0·8313	25	0·8534	0·7283	35	0·7817	0·6110
4° 0′	0·9976	0·9951	10	0·9554	0·9129	20	0·9112	0·8302	30	0·8526	0·7270	40	0·7808	0·6096
20	0·9971	0·9943	15	0·9550	0·9121	25	0·9106	0·8291	35	0·8519	0·7257	45	0·7799	0·6082
40	0·9967	0·9934	20	0·9546	0·9112	30	0·9100	0·8280	40	0·8511	0·7244	50	0·7790	0·6068
5° 0′	0·9962	0·9924	25	0·9541	0·9104	35	0·9093	0·8269	45	0·8503	0·7231	55	0·7780	0·6054
20	0·9957	0·9914	30	0·9537	0·9096	40	0·9087	0·8258	50	0·8496	0·7218	39° 0′	0·7771	0·6040
40	0·9951	0·9902	35	0·9533	0·9087	45	0·9081	0·8247	55	0·8488	0·7205	5	0·7762	0·6025
6° 0′	0·9945	0·9891	40	0·9528	0·9079	50	0·9075	0·8236	32° 0′	0·8480	0·7192	10	0·7753	0·6011
15	0·9940	0·9881	45	0·9524	0·9071	55	0·9069	0·8225	5	0·8473	0·7179	15	0·7744	0·5997
30	0·9936	0·9872	50	0·9519	0·9062	25° 0′	0·9063	0·8214	10	0·8465	0·7166	20	0·7735	0·5982
45	0·9931	0·9862	55	0·9515	0·9054	5	0·9057	0·8203	15	0·8457	0·7153	25	0·7725	0·5968
7° 0′	0·9925	0·9851	18° 0′	0·9510	0·9045	10	0·9051	0·8192	20	0·8449	0·7139	30	0·7716	0·5954
12	0·9921	0·9843	5	0·9506	0·9036	15	0·9044	0·8180	25	0·8442	0·7126	35	0·7707	0·5940
24	0·9917	0·9834	10	0·9501	0·9028	20	0·9038	0·8169	30	0·8434	0·7113	40	0·7698	0·5925
36	0·9912	0·9825	15	0·9497	0·9019	25	0·9032	0·8158	35	0·8426	0·7100	45	0·7688	0·5911
48	0·9907	0·9816	20	0·9492	0·9011	30	0·9026	0·8147	40	0·8418	0·7087	50	0·7679	0·5897
8° 0′	0·9903	0·9806	25	0·9488	0·9002	35	0·9019	0·8135	45	0·8410	0·7073	55	0·7670	0·5882
12	0·9898	0·9796	30	0·9483	0·8993	40	0·9013	0·8124	50	0·8402	0·7060	40° 0′	0·7660	0·5868
24	0·9893	0·9787	35	0·9479	0·8984	45	0·9007	0·8113	55	0·8394	0·7047	5	0·7651	0·5854
36	0·9887	0·9776	40	0·9474	0·8976	50	0·9001	0·8101	33° 0′	0·8387	0·7034	10	0·7642	0·5839
48	0·9882	0·9766	45	0·9469	0·8967	55	0·8994	0·8090	5	0·8379	0·7020	15	0·7632	0·5825
9° 0′	0·9877	0·9755	50	0·9464	0·8958	26° 0′	0·8988	0·8078	10	0·8371	0·7007	20	0·7623	0·5811
10	0·9872	0·9746	55	0·9460	0·8949	5	0·8981	0·8067	15	0·8363	0·6994	25	0·7613	0·5796
20	0·9867	0·9737	19° 0′	0·9455	0·8940	10	0·8975	0·8055	20	0·8355	0·6980	30	0·7604	0·5782
30	0·9863	0·9728	5	0·9450	0·8931	15	0·8969	0·8044	25	0·8347	0·6967	35	0·7595	0·5768
40	0·9858	0·9718	10	0·9446	0·8922	20	0·8962	0·8032	30	0·8339	0·6954	40	0·7585	0·5753
50	0·9853	0·9708	15	0·9441	0·8913	25	0·8956	0·8021	35	0·8331	0·6940	45	0·7576	0·5739
10° 0′	0·9848	0·9698	20	0·9436	0·8904	30	0·8949	0·8009	40	0·8323	0·6927	50	0·7566	0·5724
10	0·9843	0·9688	25	0·9431	0·8895	35	0·8943	0·7997	45	0·8315	0·6913	55	0·7557	0·5710
20	0·9838	0·9678	30	0·9426	0·8886	40	0·8936	0·7986	50	0·8307	0·6900	41° 0′	0·7547	0·5696
30	0·9832	0·9668	35	0·9421	0·8877	45	0·8930	0·7974	55	0·8298	0·6886	5	0·7537	0·5681
40	0·9827	0·9657	40	0·9417	0·8867	50	0·8923	0·7962	34° 0′	0·8290	0·6873	10	0·7528	0·5667
50	0·9822	0·9647	45	0·9412	0·8858	55	0·8917	0·7951	5	0·8282	0·6859	15	0·7518	0·5653
11° 0′	0·9816	0·9636	50	0·9407	0·8849	27° 0′	0·8910	0·7939	10	0·8274	0·6846	20	0·7509	0·5638
10	0·9810	0·9625	55	0·9402	0·8840	5	0·8903	0·7927	15	0·8266	0·6832	25	0·7499	0·5624
20	0·9805	0·9614	20° 0′	0·9397	0·8830	10	0·8897	0·7915	20	0·8258	0·6819	30	0·7489	0·5609
30	0·9799	0·9602	5	0·9392	0·8820	15	0·8891	0·7903	25	0·8249	0·6805	35	0·7480	0·5595
40	0·9793	0·9591	10	0·9387	0·8811	20	0·8883	0·7892	30	0·8241	0·6792	40	0·7470	0·5580
50	0·9787	0·9579	15	0·9382	0·8802	25	0·8877	0·7880	35	0·8233	0·6778	45	0·7460	0·5566
12° 0′	0·9781	0·9568	20	0·9377	0·8793	30	0·8870	0·7868	40	0·8225	0·6765	50	0·7451	0·5551
10	0·9775	0·9556	25	0·9372	0·8783	35	0·8863	0·7856	45	0·8216	0·6751	55	0·7441	0·5537
20	0·9769	0·9544	30	0·9367	0·8774	40	0·8856	0·7844	50	0·8208	0·6737	42° 0′	0·7431	0·5523
30	0·9763	0·9532	35	0·9362	0·8764	45	0·8850	0·7832	55	0·8200	0·6724	5	0·7422	0·5508
40	0·9756	0·9519	40	0·9356	0·8754	50	0·8843	0·7820	35° 0′	0·8191	0·6710	10	0·7412	0·5494
50	0·9750	0·9507	45	0·9351	0·8745	55	0·8836	0·7808	5	0·8183	0·6696	15	0·7402	0·5479
13° 0′	0·9744	0·9494	50	0·9346	0·8735	28° 0′	0·8829	0·7796	10	0·8175	0·6683	20	0·7392	0·5465
7½	0·9739	0·9484	55	0·9341	0·8725	5	0·8822	0·7784	15	0·8166	0·6669	25	0·7382	0·5450
15	0·9734	0·9475	21° 0′	0·9336	0·8716	10	0·8816	0·7772	20	0·8158	0·6655	30	0·7373	0·5436
22½	0·9729	0·9465	5	0·9330	0·8706	15	0·8809	0·7760	25	0·8149	0·6641	35	0·7363	0·5421
30	0·9724	0·9455	10	0·9325	0·8696	20	0·8802	0·7748	30	0·8141	0·6628	40	0·7353	0·5407
37½	0·9719	0·9445	15	0·9320	0·8686	25	0·8795	0·7735	35	0·8133	0·6614	45	0·7343	0·5392
45	0·9713	0·9435	20	0·9315	0·8677	30	0·8788	0·7723	40	0·8124	0·6600	50	0·7333	0·5378
52½	0·9708	0·9425	25	0·9309	0·8667	35	0·8781	0·7711	45	0·8116	0·6586	55	0·7323	0·5363
14° 0′	0·9703	0·9415	30	0·9304	0·8657	40	0·8774	0·7699	50	0·8107	0·6573	43° 0′	0·7313	0·5349
7½	0·9698	0·9404	35	0·9299	0·8647	45	0·8767	0·7686	55	0·8099	0·6559	5	0·7304	0·5334
15	0·9692	0·9394	40	0·9293	0·8637	50	0·8760	0·7674	36° 0′	0·8090	0·6545	10	0·7294	0·5320
22½	0·9687	0·9384	45	0·9288	0·8627	55	0·8753	0·7662	5	0·8082	0·6531	15	0·7284	0·5305
30	0·9681	0·9373	50	0·9283	0·8617	29° 0′	0·8746	0·7650	10	0·8073	0·6517	20	0·7274	0·5291
37½	0·9676	0·9362	55	0·9277	0·8607	5	0·8739	0·7637	15	0·8064	0·6503	25	0·7264	0·5276
45	0·9670	0·9352	22° 0′	0·9272	0·8597	10	0·8732	0·7625	20	0·8056	0·6489	30	0·7254	0·5262
52½	0·9665	0·9340	5	0·9266	0·8587	15	0·8725	0·7612	25	0·8047	0·6475	35	0·7244	0·5247
15° 0′	0·9659	0·9330	10	0·9261	0·8576	20	0·8718	0·7600	30	0·8038	0·6462	40	0·7234	0·5232
5	0·9655	0·9323	15	0·9255	0·8566	25	0·8711	0·7588	35	0·8030	0·6448	45	0·7224	0·5218
10	0·9652	0·9315	20	0·9250	0·8556	30	0·8703	0·7575	40	0·8021	0·6434	50	0·7214	0·5203
15	0·9648	0·9308	25	0·9244	0·8546	35	0·8696	0·7563	45	0·8012	0·6420	55	0·7203	0·5189
20	0·9644	0·9301	30	0·9239	0·8535	40	0·8689	0·7550	50	0·8004	0·6406	44° 0′	0·7193	0·5174
25	0·9640	0·9293	35	0·9233	0·8525	45	0·8682	0·7538	55	0·7995	0·6392	5	0·7183	0·5160
30	0·9636	0·9286	40	0·9227	0·8515	50	0·8675	0·7525	37° 0′	0·7986	0·6378	10	0·7173	0·5145
35	0·9632	0·9278	45	0·9222	0·8505	55	0·8667	0·7513	5	0·7977	0·6364	15	0·7163	0·5131
40	0·9628	0·9271	50	0·9216	0·8494	30° 0′	0·8660	0·7500	10	0·7969	0·6350	20	0·7153	0·5116
45	0·9624	0·9263	55	0·9211	0·8484	5	0·8653	0·7487	15	0·7960	0·6336	25	0·7143	0·5102
50	0·9620	0·9256	23° 0′	0·9205	0·8473	10	0·8646	0·7475	20	0·7951	0·6322	30	0·7132	0·5087
55	0·9617	0·9248	5	0·9199	0·8463	15	0·8638	0·7462	25	0·7942	0·6308	35	0·7122	0·5073
16° 0′	0·9613	0·9240	10	0·9194	0·8452	20	0·8631	0·7449	30	0·7933	0·6294	40	0·7112	0·5058
5	0·9608	0·9232	15	0·9188	0·8442	25	0·8624	0·7437	35	0·7925	0·6280	45	0·7102	0·5044
10	0·9604	0·9225	20	0·9182	0·8431	30	0·8616	0·7424	40	0·7916	0·6266	50	0·7092	0·5029
15	0·9600	0·9217	25	0·9176	0·8421	35	0·8609	0·7411	45	0·7907	0·6252	55	0·7081	0·5014
20	0·9596	0·9209	30	0·9171	0·8410	40	0·8601	0·7399	50	0·7898	0·6238	45° 0′	0·7071	0·5000
25	0·9592	0·9201	35	0·9165	0·8399	45	0·8594	0·7386	55	0·7889	0·6224	5	0·7061	0·4985
30	0·9588	0·9193	40	0·9159	0·8389	50	0·8587	0·7373	38 0′	0·7880	0·6210	10	0·7051	0·4971
35	0·9584	0·9185	45	0·9153	0·8378	55	0·8579	0·7360	5	0·7871	0·6195	15	0·7040	0·4956
40	0·9580	0·9177	50	0·9147	0·8367	31° 0′	0·8572	0·7347	10	0·7862	0·6181	20	0·7030	0·4942