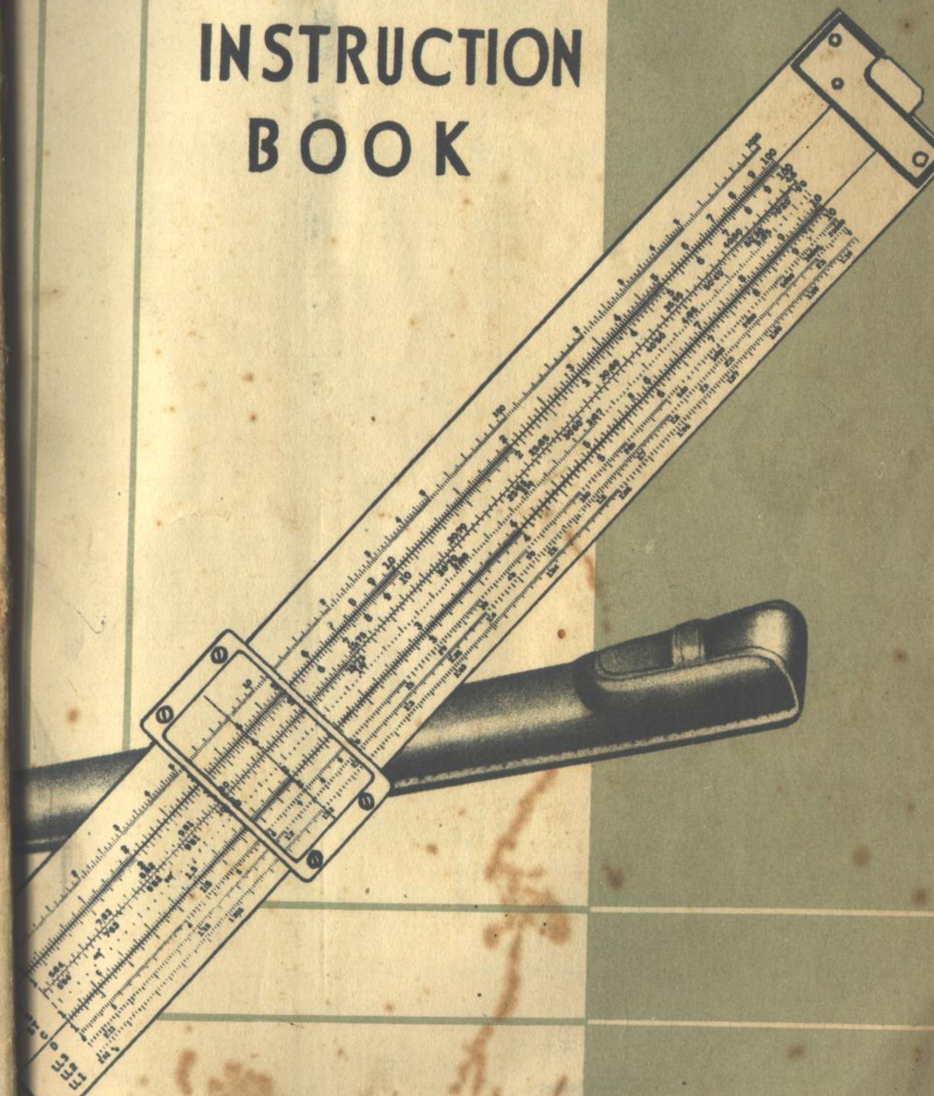
Hemmi Bamboo Slide Rule

INSTRUCTION BOOK



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INSTRUCTIONS

FOR THE USE OF

HEMMI BAMBOO DUPLEX SLIDE RULES

Nos. 250, 251, 255, 275, 256,
259, 279, 154, 153, 200

PUBLISHED BY

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Slide Rule Diagram

In order to explain the method of operation of a slide rule, diagram under the following assumption is used.

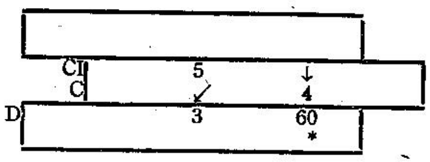
"Setting mark" ✓ Set the figure on a scale here.

"Indicator mark" ↓ Set indicator at this point.

"Answer mark" * Answer is found here.

Following diagram explains an operating procedure, such as "Set 5 on *CI* scale to 3 on *D* scale, Opposite 4 on C scale, find answer 60 on *D* scale under the hair line of an indicator".

Note: Extreme right or left vertical line in diagram indicates position of index lines of *C* and *D* scale, instead of actual end.



CONTENTS

INSTRUCTION OF HEMMI SLIDE RULE (DUPLEX TYPE)

I. Classification of new Duplex Slide Rule.

There are two major classifications for the new designed Hemmi Slide Rule according to it's usage, one is business man's use and other is engineer's use. Furthermore the latter may be divided into classification of general engineer's, mechanical engineer's and electrical engineer's use.

(a) Slide Rule for business man.

There are two kinds of slide rules, No. 250 (10 inches) and No. 200 (16 inches) of which scales are so designed that multiplication, division and proportion are conveniently made to meet with daily business calculation.

No. 250 is the new designed slide rule based on old No. 2600 rule and in addition to the $\sqrt{10}$-folded scales, a new *CIF* scale is provided. It is the aim of this new designed scale that specific $\sqrt{10}$-folded system in our country is operated just in the same way as π-fold system in United States of America. The fact that Trigonometric function scales in line with *SI* and *TI* which are the speciality of our company extremely raised up an efficiency of general calculation would be no exaggeration to say, this is the highest quality out of this kind of slide rules.

No. 200 is only one slide rule with high accuracy capable of four digits computation for multiplication and division.

(b) Slide Rule for engineers.

Slide rule for engineers as stated previously are divided into classification of general engineer's mechanical engineer's and electrical engineer's use. These slide rules include No. 251 (10 inches) for mechanical engineer, No. 259 (10 inches) and No. 279 (20 inches) for expert mechanical engineer, No. 153 for electrical engineer No. 255 (10 inches) No. 275 (20 inches), No. 154 (20 inches) for expert electrical engineer and No. 256 (10 inches) for electric communication engineering.

And also for general engineering use (civil architect, surveying etc) No. 250 stated above will do as well as for business calculations.

On basis of the recent trend of design the slide rule for engineers provides Log Log scale and tried to extend the range of computation. Almost all the kinds of slide rule stated in the previous paragraph provide *LL* scale with positive exponent and *CF*, *DF* and *CIF* scales for multiplication and division are operated in line with the operating practice of π-fold in U. S. A.

Detailed remarks are explained in the following chapter.

II. Fundamental Calculations.

Multiplication, division and proportion as fundamental operation of slide rule are explained in this chapter.

These operations are available for all new Duplex slide rules in common, except No. 153 (10 inches) No. 154 (20 inches) and No. 200 (16 inches) which are explained in another chapter.

Elementary method of operation, such as reading graduations and placing decimal point is explained in Appendix (1) " Explanatory Remarks for Biginners ".

(1) Multiplication and Division

There are two kinds of operations of slide rule which are the slide and indicator operation. Practically the following three items are preferable to get high efficiency of computation.

(a) Always find answer on the stock.

(b) Treatment of one value should be made by one operation.

(c) Movement of the Slide should be lessened as much as possible.

Improvement of slide rule is at least now progressing to the direction to meet with the need of above items, even though the function of any present slide rule is not enough to satisfy these items.

Concrete procedure of operation is as follows:

Set indicator to multiplicand or dividend on the stock.

Calculation is made firstly by the slide operation.

And then, if necessary, calculate by indicator operation. By repeated operating procedure of Slide operation→Indicator operation, computation can be made quickly and greatly increase the operating efficiency.

It will be an immovable principle that answer is located by the following operating procedures:

(i) Always read the answer on the stock, opposite the index of *C* scale by the slide operation.

(ii) Always read the answer on the stock, under the hairline by the indicator operation.

Multiplication—joint operation of *DF*, *CF*, *CIF*, *CI*, *C*, *D* scales—If Multiplication and division are worked out by a slide rule which has six scales of *DF*, *CF*, *CIF*, *CI*, *C*, and *D*, answer will not fall beyond the end of a scale. Therefore it can be said this is the most well developed slide rule. Slide rules of No. 250, No. 251, No. 255 (No. 275) No. 256, and No. 259 (No. 279) have the same scale arrangements of these systems, and in case of handling these scales classification including two systems should be taken into consideration; i. e.

DF, *CF*, *CIF* (folded scale system)

D, *C*, *CI* (normal scale system)

In the slide rule operation the following procedure is considered:

(1) Slide operation (use the same system of scale)→Read answer on *D* scale, opposite index on *C* scale.

(2) Indicator operation (use any system of scale)→Read answer on the particular scale of stock, under the hairline of an indicator.

Multiplication by the slide operation:

Set multiplicand on *D* or *DF* scale.

To the indicator line set multiplier on inverted scale of the particular scale system, *CI* or *CIF*.

Read answer always on *D* scale, opposite index of *C* scale.

Multiplication by the indicator operation:

Set multiplier (on *C* or *CF* scale) of any scale system.

Always read answer on the same system of scale on stock; i. e. *D* or *DF* scale.

Care should be taken with regard to the calculating efficiency which depends upon the first multiplicand is set on scale *D* or *DF*.

Set on *D* scale, if effective number of multiplicand falls between 2~5.

Set on *DF* scale for others.

This means indicator should be located at about middle part of slide rule as much as possible.

Example 1

$3 \times 4 \times 5 = 60$

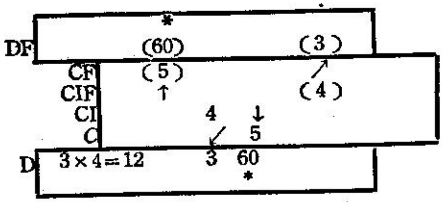


Fig. 1

Figure in parenthesis indicates the use of folded scale system.

Successive multiplication of three numbers is shown in Fig. 1. Multiplication of two numbers such as 3×4=12 is found out only by the Slide operation.

By the use of this procedure, answer will never fall beyond the end of scale by the second multiplication, i. e. indicator operation, because it will be carried out by the operation of some other scale system any time. In order to reduce the shifting of slide in case of multiplication, attention should be called for that product of two numbers to be used for the slide operation should be selected as close as possible to the significant figure 1 (10, 100, 0.1, 0.001 etc.).

Slide rule No. 154 without *CIF* scale and No. 200 which is different form without *CI* and *CIF* scale will be explained in the respective chapter.

Division

Division is carried out entirely the same procedure as multiplication, but on basing of its function, *C* or *CF* scale should be used, instead of *CI* or *CIF* scale which was used in case of multiplication.

In order to reduce the movement of the slide for division, significant figure of dividend and divisor for slide operation should be selected as close as possible.

Example 2 $8 \div 5 \div 4 = 0.4$

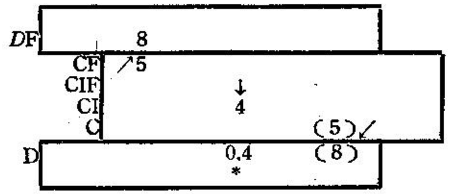


Fig. 2

Combination of Multiplication and Division

Example 3 $\frac{3 \times 6}{5} = 3.6$

There are two procedures to solve this example. One is to work out 3÷5×6. In this case note that you never use inverted *CI* and *CIF* scales. Other is to work out 3×6÷5. In this case note that you never use *C* and *CF* scales. Less movement of the slide decides which procedure is adopted. (i) in Fig. 3 indicates the former procedure and (ii) indicates the latter procedure.

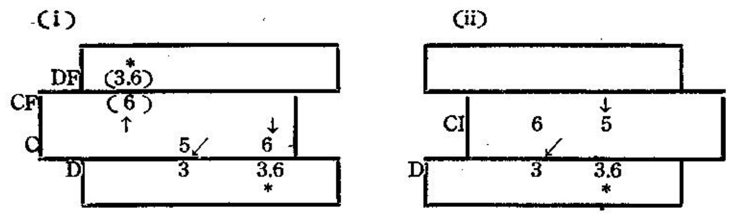


Fig. 3

Characteristics of Folded Scale

Generally speaking, when folded scale *DF* with folding point *m* is provided, folding point *m* on this scale is set opposite the right or left index on *D* scale.

Therefore if *a* is set on *D* scale. value on *DF* scale opposite *a* becomes always *ma*. On the contrary, if value *a* is set on *DF* scale, value on *D* scale opposite this point always becomes $\frac{a}{m}$.

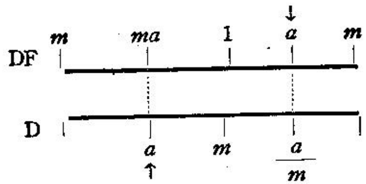


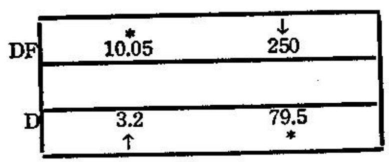
Fig. 4

Folded point *m* is selected based on the purpose of the slide rule. $m=\sqrt{10}$ for our No. 250 and $m=\pi$ for our No. 251 No. 255 (No. 275) No. 256, No. 259 (No. 279) and No. 154 are adopted.

Advantage of $\sqrt{10}$ fold will be explained in the chapter of No. 250. With regard to folded scale of π-fold, if diameter of circle *d* is set on *D* scale, circumference πd is found on *DF* scale, opposite point *d* on *D*. And also it's reverse operation is possible. Relation between diameter and circumference of circle is found out by indicator operation only.

Example 4 Find circumference of circle with 3.2 m diameter.

Find diameter of circle with circumference of 250 cm.



Ans. 10.05 m
79.5 cm

Fig. 5

(2) Proportion

How to work out proportion as a special case of multiplication and division is explained in the following article.

Generally speaking, if the slide is set at one point, there exists certain mathematical relation between the scales on the slide and stock. Series of computations based on this relation is made by the shifting of indicator only, with reference to scales concerned. This kind of operation is called "to use slide rule as reference scale method."

Proportion and inverse proportion referred to the multiplication and division is worked out by the reference scale method.

(a) Proportion

Proportion is found by the use of "reference scale method" with reference to *C* and *D* scales (or *CF* and *DF* scales). This method is widely applied to conversion, indexes, proportional division, percentage and also sale and purchase of commodity.

Example 5

Fill the following blanks, given 127 kg=280 lbs

Lbs	45	63	(50.7)	(150)	180
Kg	(20.4)	(28.6)	23	68	(81.6)

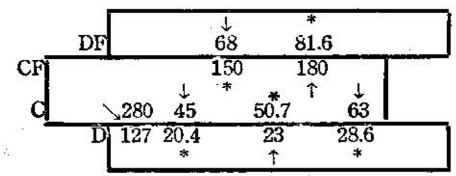


Fig. 6

For calculation referred to proportion relation between any concrete number or quantity and scale should be fixed until the operation finished. Above example indicates "lbs" is set on *C* or *CF* scale and kg on *D* or *DF* scale with fixed relation.

If any answer falls beyond the extremity of reference scales *C* and *D*, it may be read on the "reference scales" *CF* and *DF*.

In the same way, if answer falls beyond the extremity of *CF* and *DF* scales, it may be read on the "reference scales" *C* and *D*.

Example 6

Find % in the following table.

	Amount	%
A	$ 3,250	59.10
B	$ 1,180	21.43
C	$ 1,070	19.47
Total	$ 5,500	100.00

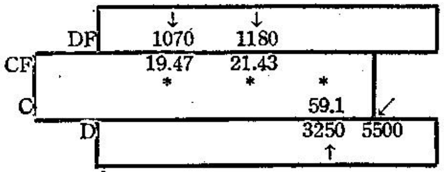


Fig. 7

If total sum of % doesn't come up to 100.00 %, it may be practical to adjust figures of the largest % in order to get exact 100 % in total sum.

Example 7

How much is 25 pcs of certain commodity 13 dollars per dozen? And how much pcs can be purchased at 30 dollars?

Ans. $27.10
27.7 pcs.

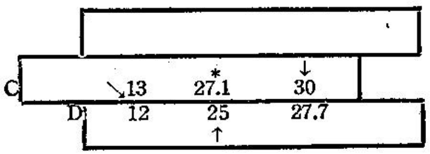


Fig. 8

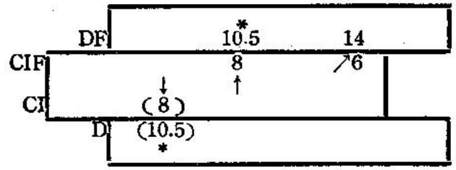
(b) Inverse Proportion

Inverse proportion is performed without any answer falling beyond the end of scale by the use of "reference scales" *D* and *CI* (or *DF* and *CIF*).

In case of inverse proportion on the contrary to proportion, operation may be made freely with regardless of concrete number and scales concerned, after the "reference scales" once set up.

Example 8

What is the depth of land area with width of 8 m, "equivalent to the area with width of 6 m and depth of 14 m"?



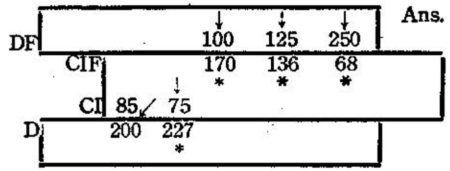
Ans. 10.5 m

Fig. 9

Example 9

The gear wheel *A* with 85 teeth revolves 200 per minute. Find number of teeth of pinion *B* to make it's revolution of 125 and 250 per minute respectively.

Find number of revolution per minute, if number of teeth of pinion *B* is 100 or 75 respectively.



Ans. 136. 68 teeth
170. 227 revolution

Fig. 10

III. No. 250 (10 inches) Duplex Slide Rule

(For General Engineering and Business use)

(1) General Description (See figure on page 104)

Characteristics

This slide rule is so designed that high efficiency of business calculation such as multiplication, division and proportion may be given. To do so old slide rule No. 2600 was modified adding a new *CIF* scale to the $\sqrt{10}$ folded scales in order to get the same operating function as other rules which have π-fold scales. Beside this, reduction of the Slide shifting through "cross operation" which can't be imitated by other slide rule has been made possible.

Furthermore, for engineer's sake, this slide rule is provided with inverted scales of trigonometric functions *SI* and *TI* and solution of right triangle is simplified greatly. It would not be too much to say this slide rule is the highest class one among this kind.

Arrangement of scales and usage

Front face:

DF, *CF*, *CIF* ($\sqrt{10}$-fold)
D, *C*, *CI* }............Multiplication, division and proportion

L ....Common logarithm

Rear face:

K ....Cube and cube root

A, B..Square and square root

TI_2, TI_1 (Tangent)
SI (Sine)
D, *DI* }Trigonometric function

Trigonometric function scales are based on degree and minute system. And complementary angles are lettered in red figure.

(2) Multiplication and Division

Multiplication, division and proportion are performed in the same way as in the chapter of fundamental computation. Since this slide rule has a folded scale of $\sqrt{10}$-fold, so called "cross operation" which is never seen in other products is possible.

As a result of this function, shifting of the slide which is one of 3 principal items of multiplication and division is adequately reduced.

Example 1

$3 \times 4 \times 9 = 108$

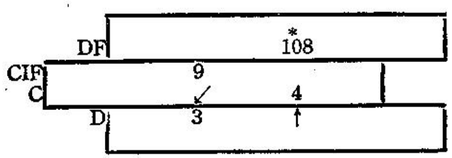


Fig. 11

Fig 11 indicates that by cross operation of normal scale and folded scale system movement of the slide has been reduced. This kind of operation is called "cross operation" and an advantage given only by folded scale of $\sqrt{10}$-fold.

In case of cross operation, if no. of use of folded scales (*DF*, *CF*, *CIF*) is odd number, answer is given on *DF* scale, and if it is even number answer is given on *D* scale. In example 1 since No. of use of folded scale is once for *CIF* scale, answer is found out on *DF* scale.

Example 2 $1.5 \div 8 \div 7 = 0.0268$

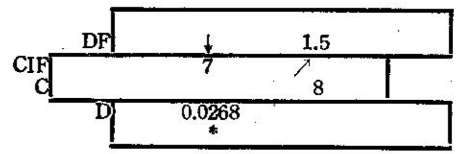


Fig. 12

(Note) *CIF* scale is absolutely necessary on π-fold scale in order to make successive operation without any answer falling beyond the end of scales. Slide rule with $\sqrt{10}$-fold scale could be used without *CIF*. But it would be much better if *CIF* is granted.

(3) Square and Cube

Square

To extract square and square root, answer may be found out with *A* and *D* scales in the reference relation.

A scale is considered that two *D* scales which have been reduced exactly 1/2 of original length are tied together.

Therefore the characteristics, is just the same as *D* scale and multiplication and division may be available in joint use with *B* scale.

Example 3 $5^2 = 25$ $\sqrt{64} = 8$

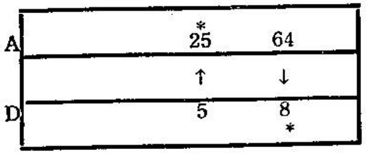


Fig. 13

Method of placing decimal point for square value is the same as that of multiplication.

In case of square root, given number is divided into several groups with two digits per group, counting from decimal point to the direction of the first significant figure of given number.

If the value in the top group is less than 10, given number is set between 1–10 on A scale. If it is over 10, set between 10–100.

Place the decimal point of answer, taking one digit per group.

Multiplication and division including square & square root

Form $a \times b^2 = c$ or $a \div b^2 = c$ are solved as in the following examples. It should be noted that answer for calculation of any form including square value always lies on A scale.

Example 4 $\qquad 15 \times 2.2^2 = 72.6$

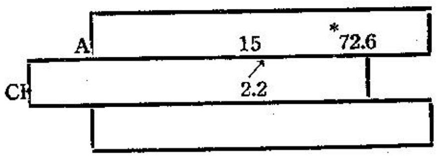


Fig 14

Example 5 $\qquad 8 \div 1.3^2 = 4.72$

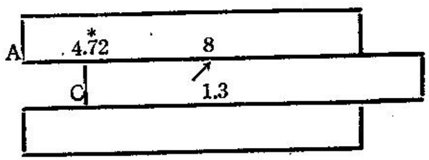


Fig. 15

Example **6** $\qquad 23^2 \div 65 = 8.14$

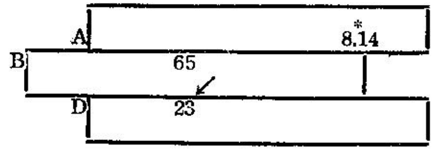


Fig. 16

Multiplication and division including square root such as $\sqrt{a} \times b$ or $\sqrt{a} \div b$ are worked out as follows:

Example **7** $\qquad 18 \times \sqrt{20} = 80.5$

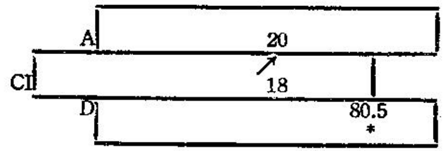


Fig. 17

Example **8** $\qquad \sqrt{12} \div 6.5 = 0.533$

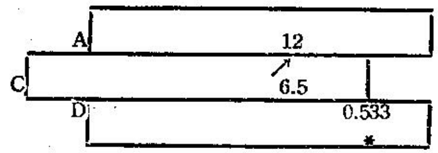


Fig. 18

Example **9** $\qquad 8.5 \div \sqrt{24} = 1.735$

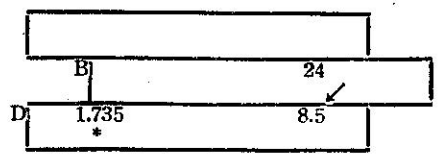


Fig. 19

How to use Gauge mark "c"

There is a gauge mark "c" at point 1.128……on *C* scale, which is used for relation of circle diameter and area. It was derived from the following formula.

area of circle $a=\frac{\pi}{4}d^2$ $\qquad d$= diameter of circle

Changing the form, $a=\left(\sqrt{\frac{\pi}{4}}\,d\right)^2=\left(d/\sqrt{\frac{4}{\pi}}\right)^2$

Denominator $\sqrt{\frac{4}{\pi}}$ in parenthesis corresponds to the value of *c*.

Example 10 Find area of circle with diameter of 2.3 m.

Ans. 4.15 m²

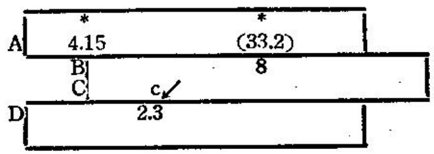


Fig. 20

If the significant figure of a diameter is larger than $\sqrt{10}$, it may be set on *DF* scale, instead of *D* scale.

If the volume of cylinder with 2.3 m in diameter and 8 m in length is required, follow the procedure shown by Fig. 20, move indicator to 8 on *B* scale and volume 33.2 m³ is read on *A* scale under the hairline of indicator.

Reference scale

A, *B* and *C*, *D* or *CI* scales may be used as the "reference scales". Calculation of circle areas from many given diameters by the use of the gauge mark *c* at the same time is shown in the following example.

Example 11

Find circle area, given diameters of 2.5 m, 3.2 m, and 5.8 m respectively.

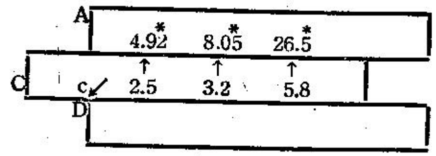


Fig. 21

Cube and Cube root

K scale is provided for calculation of cube and cube root. Since *K* scale consists of three reduced scales tied together which is exactly the one third of *D* scale reduced, its characteristics is the same as *D* or *A* scale.

Calculation of cube or cube root with reference to *K* and *D* scale is shown by Fig. 22.

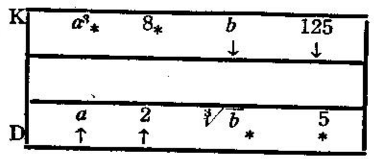


Fig. 22

In case of calculating cube root, given number is divided into several groups with 3 digits per group, counting from decimal point to the direction of the first significant figure. According to the significant figure in the first group being one, two and three, given number is set in the section 1~10, 10~100, and 100~1000 of *K* scale respectively.

Method of placing decimal point is determined on basis of one digit per group.

Multiplication and division including cube or cube root and also method of using slide rule as the "reference scales" are carried out as explained in the article of square calculation and *K* scale may be used in place of *A* scale.

(4) Logarithm

Common Logarithm

Common logarithm with base of 10 includes integer (Characteristic) and decimal fraction (mantissa). Equally graduated scale L is provided in order to find mantissa of common logarithm.

Characteristic is mentally calculated from the following formula:

(number of places above decimal point of given number)−1

i, e, one less than the number of figures at the left of decimal point.

If given number is of m places under decimal point, characteristic is $\bar{m}$ and simply put at the left of decimal point. $\bar{m}$ may be considered as symbolic expression of $-m$.

Example 12 $\log_{10} 2.5 = 0.398$ $\log_{10} 0.025 = \bar{2}.398$

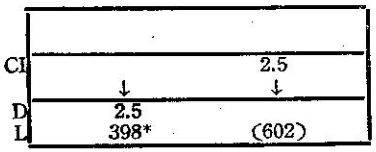


Fig. 23

As shown by the above example, $\bar{2}.398$ of $\log_{10} 0.025$ means that characteristic is negative and mantissa is positive, which is not allowed to multiply or divided as it is. In this case $0.398-2=-1.602$ should be first computed. If given number is set on *CI* scale you can directly find out the complementary number of 0.398 *i, e,* 0.602 on *L* scale.

Natural Logarithm

Natural logarithm $\log_e a$ with base e, may be found out by multiplying 2.3026 to common logarithm from the following formula.

$$\log_e a = 2.3026 \log_{10} a$$

Anti-logarithm

Form of $\log_{10}{}^{-1} x = a$ is given by the perfect reverse procedure of the preceding example.

Take mantissa only out of given number x and set on L scale. Significant figure of number of logarithm a is given on the scale D, opposite this point.

One is added up to characteristic of logarithm x to place decimal point.

Exponent

Computation of form A^n needs following three steps of operation.

(1) Find $\log_{10} A$ (Use D and L scales)

(2) Calculate $n \log_{10} A$ (Use D and CI scales)

(3) Answer is given by $\log_{10}{}^{-1} (n \log_{10} A)$ (Use L and D scales)

(5) Trigonometric Function

This slide rule has trigonometric function scales such as *SI* (6°–90°) for sine, TI_1 (6°–45°) and TI_2 (45°–84°) for Tangent.

These scales are graduated with degree and minute system and it is so called inverted scale, graduated from right to left direction.

An idea of using inverted scale of trigonometric function is the speciality of our rules and has never been used for foreign slide rule, while this system will prove high efficiency for the computation of a vector and complex number.

Trigonometric function and its multiplication and division

Location of value of each trigonometric function is shown by Fig. 24, when index of *D* scale is set on θ of *SI* or *TI* scale.

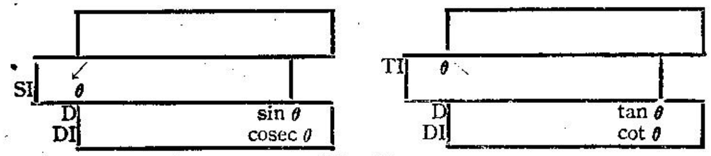


Fig. 24

Since angle in red on these scales represents complementary angle $(90°-\theta)$, Cos θ and Sec θ are found out in the same way. Multiplication and division are worked out with joint use of *C, D, CI* or *CF, DF* on the front face. Calculation of Sine is explained in the following example and Tangent is also given in the same way, except that *TI* scale is used instead of *SI* scale.

Example 13 $\sin 35° = 0.574$ $\dfrac{\sin 35°}{2.8} = 0.205$

$5.6 \sin 35° = 3.21$ $\mathrm{cosec}\, 35° = 1.745$

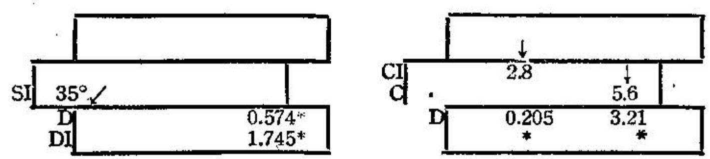


Fig. 25

For small angle of $\theta<6°$, unit of Radian which is converted from these degree is used, assuming $\tan\theta \doteqdot \sin\theta$. ρ'', ρ' and ρ° on C scale are conversion gauge marks respectively when θ is expressed in second, minute and degree.

Set θ on scale D, referred to these gauge marks.

Opposite index of C scale, read answer on D scale.

Method of placing decimal point is estimated as follows:

1 second$\doteqdot$0.000005

1 minute$\doteqdot$0.0003

1 degree$\doteqdot$0.02

Sine Proportion

Calculation of Sine proportion is carried out with the "reference scales" of SI and DI, but it is rather far desirable to meet with operating procedure by the use of following formula of inverse proportion than to use ordinary method of proportion.

$$a \cdot \sin B = b \cdot \sin A$$

$$b \cdot \sin C = c \cdot \sin B$$

Example 14

An angle of elevation θ 35° is given, looking at top of tree from point A on the ground, an angle of elevation 60° is given at point B, walking 40 m from A to B. Find the height of tree.

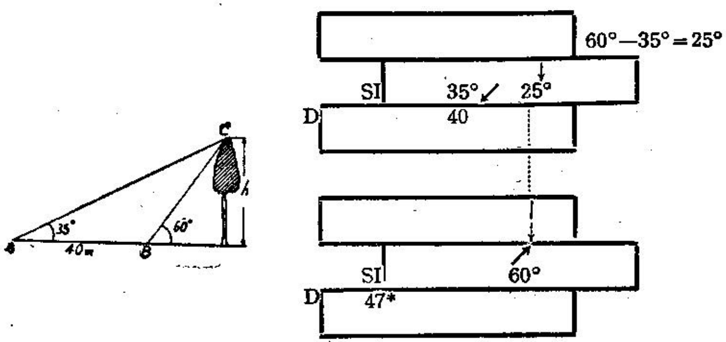


Fig 26

Solution of right triangle

By the use of SI and TI scales, solution of right triangle is greatly simplified.

Fig. 27 indicates operating relation of base angle θ and sides a, b, c.

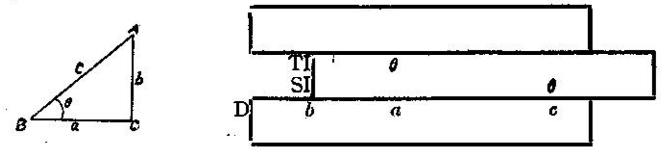
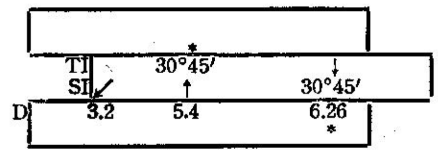


Fig. 27

Example 15 Find θ and c, given $a=5.4$, $b=3.2$ in right triangle.



Answer $\theta=30°45'$
$c=6.26$

Fig. 28

This calculation is also applied to the form of $c=\sqrt{a^2\pm b^2}$, which is so called Pythagorean theorem.

Example 16 $\sqrt{1.8^2+3.5^2}=3.93$

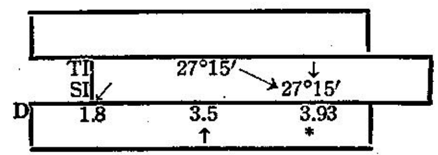


Fig. 29

Joint use of this method to A and B scales may be applicable to calculation of standard deviation of the science of statistics.

Example 17 Find standard deviation of 375, 368, 370.

Find sample mean of 371 first and deviation is expressed as +4, −3, −1

Standard deviation $S=\sqrt{\dfrac{4^2+3^2+1^2}{3}}=2.94$

The answer is given by the operation of Fig. 30.

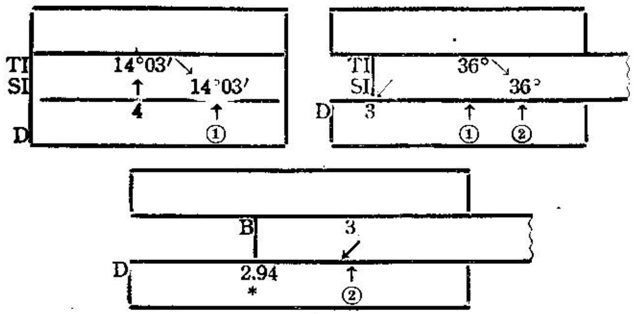
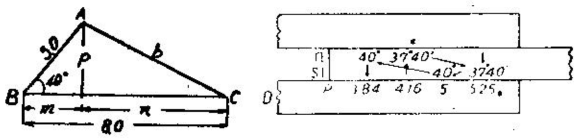


Fig. 30

Solution of Oblique Triangle

With application of relation as shown in Fig. 27, in use of this kind of slide rule, solution of oblique triangle is easily made giving two sides and the included angle.

Example **18** Find ∠C and b of triangle in Fig. 31



Ans. ∠C=37°40′
b=5.25

Fig. 31 m=3.84 8.0−3.84=4.16

Since this oblique triangle is divided into two right triangles and perpendicular p is in common, answer will be given in the same way as preceding problem.

In other words, **Example 18 seems** to be a solution of subtraction of two vectors, 5.0 and 8.0 with phase angle difference of 40°.

To find addition of these two vectors, answer will be easily given in the same way, making 8.0 + 3.84 instead of 8.0 − 3.84. In this case it should be noted that DF scale is used in place of D scale.

Consequently the use of scale SI and TI makes computation of right triangle much easier, which will be applied to the conversion of coordinates in case of computation of vector or complex number. **Detailed explanation is** indicated in the instruction of No. 255 slide rule.

Gauge marks

Gauge marks on this slide rule are as follows:

c (1.128 on C scale) For computation of circle area

π (3.1416 on C and D scale) {Ratio of circumference of circle to its diameter.

ρ'' (20626 on C scale)
ρ' (3437.7 on C scale)
ρ° (57.29 on C scale)
} For conversion of small angle into Radian.

IV. No. 251 (10 inches) Duplex Slide Rule

(For Mechanical Engineers)

(1) General Description (See figure on page 105)

Characteristics

This slide rule is an improved modification of our old No. 150 slide rule with additional Log Log scale and so designed that not only computation of multiplication and division at high efficiency, but also exponential calculation which frequently appear in empirical formulas, are possible for the use of mechanical engineers.

Folded scales of this slide rule are of π-fold system and convenient for circle calculation.

For the computation of trigonometric functions, trig scales S, T and ST of Rietz system are adopted, and calculation of very small angle below 6° can be easily performed. And also it is the feature of this slide rule that complementary angle in red figures is added upon S, T and ST scales and it is used to the best advantage.

All the angles of this slide rule are represented in degree and minute system.

Arrangement and Usage of Scales

Front Face:

DF, CF, CIF (π-fold) *D, C, CI*	Multiplication, division and proportion
L	Common logarithm
DI	Reciprocal (mainly used for trigonometric function)

Rear Face:

K	Cube
A, B	Square
S (Sine), *T* (Tangent), *ST* (Small angle), *D*	Trigonometric function
C, LL_3, LL_2, LL_1	Exponent

(2) Multiplication and division

Multiplication, division and proportion are worked out in the same way as explained in the chapter II fundamental calculation.

But cross operation explained in the chapter III-(2) may not be possible, because the folded scale of this slide rule is π-fold.

Since the values of $\sqrt{10} \doteqdot 3.16$ and $\pi = 3.14$ are too closed each other, it looks like there isn't much difference and cross operation is possible.

It must be noted that we can't do that.

But calculation of circle is easily made by this slide rule and it is the reason why there is no ordinary π gauge mark on this scale.

Example 1

Find circumference of circle with diameter of 2.3m

Answer 7.22m (Fig. 32)

Find diameter of circle with circumference of 12.5m

Answer 3.98m

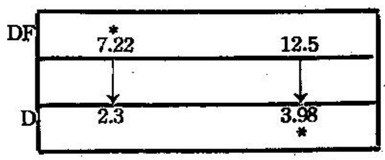


Fig. 32

(3) Square and Cube

Square and Cube operation is made in the same way as explained in the chapter III-(3). Since it seems that this kind of forms are frequently solved in engineering computation, an efficient method of operation will be explained mostly in case of multiplication and division referred to square and cube.

Combined multiplication and division including Square value

The following notes are the point of operation for combined form of multiplication and division including square value.

(1) Start with division

(2) Use *CI* scale skilfully.

(3) Complicate form is broken down and worked out with *C, D, CI, CF DF* and *CIF*.

Example 2 $\dfrac{3.82 \times 1.57^2}{2.62} = 3.6$

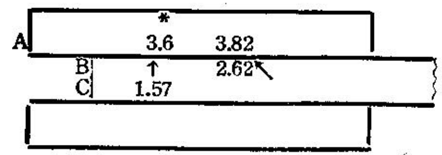


Fig. 33

Example 3 $\dfrac{8.65}{2.56 \times 4.85^2} = 0.143$

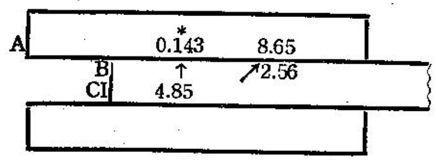


Fig. 34

Calculation of a form includes square value in denominator sholud be made in the arrow direction as shown in Ex. 3. a^2/bc will not be able to calculate without falling beyond the end, if A, B scales are used. This form is dissolved into $\frac{a \times a}{b \times c}$ form and applied ordinary multiplication and division procedure.

Combined multiplication and division including Square root

For computation of multiplication and division, including square root, it may be an efficient point of operation to follow the rules as in the follwing items.

(1) Set square root in numerator on A scale first.

(2) Start with division of square root, in case of denominator inclusive of square root.

(3) Be careful about the part of A and B scales to which the value in square root is set on.

Example 4 $\frac{\sqrt{9.66} \times 3.23}{5.68} = 1.77$

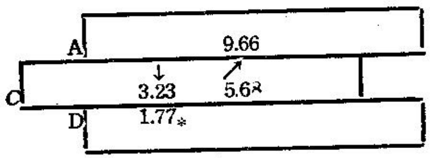


Fig. 35

Example 5 $\frac{7.22}{3.88 \times \sqrt{4.63}} = 0.865$

←

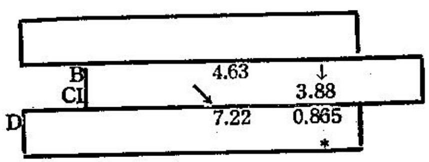


Fig. 36

Note that calculation should be started with division of $\sqrt{4.63}$.

Multiplication and division including cube and cube root

It is very difficult to work out free calculation of these forms because K is only one scale on the stock for cube and cube root computation and there is no corresponding scale on the slide.

Several examples are explained as follows:

Example 6 $1.25 \times 3.62^3 = 59.2$

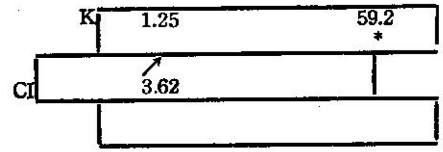


Fig. 37

Example 7 $250 \div 3.2^3 = 7.62$

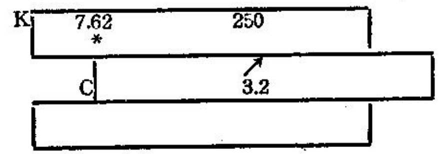


Fig. 38

Example 8 $8.55 \times \sqrt[3]{16.5} = 21.8$

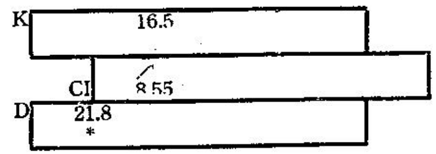


Fig. 39

Example 9 $25.3 \div \sqrt[3]{268} = 3.92$

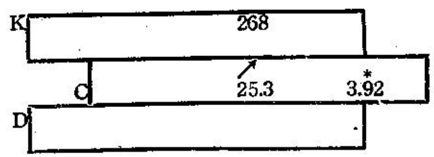


Fig. 40

This example shows that firstly calculate $\sqrt[3]{268} \div 25.3$ and get reciprocal of it.

$\frac{3}{2}$ power and $\frac{2}{3}$ power

The slide rule with this kind of arrangement of scales may be able to calculate $a^{\frac{3}{2}}$ and $a^{\frac{2}{3}}$ with the "reference scale" of K and A.

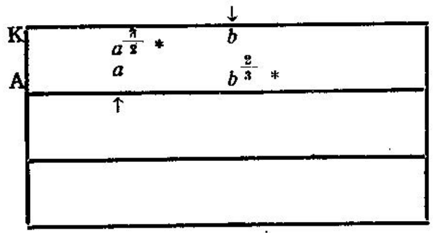


Fig. 41

It is noted that $\frac{3}{2}$ power means 1.5 power or cube of square root$(\sqrt{a})^3$. And also $\frac{2}{3}$ power means square of cube root $(\sqrt[3]{a})^2$

(4) Logarithm and Exponent

Common Logarithm

To get common logarithm through the use of L scale is done in the same way as in the chapter III-(4).

This slide rule provides LL_1, LL_2, LL_3 scales which are three portions of folded scales of a log log scale in the range of 1.01–20,000 folded at the value of e and $e^{0.1}$. There e is the base of natural logarithm. Common logarithm can be found out by the use of these scales also. However, in this case not only mantissa, but also characteristic are given at the same time.

Example **10** $\quad \log_{10} 35 = 1.544$

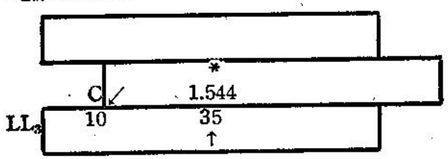


Fig. 42

To get anti-logarithm, at first, set the index of C to 10 on LL_3, and then opposite given value of logarithm on C, read the answer on LL scale. If it falls beyond the end of scale it will be easily found out by changing index of C scale from left to right.

Natural logarithm

To find natural logarithm by this slide rule, set the hairline to the given number a on LL scale, then we can read $\log_e a$ under the hairline on D scale.

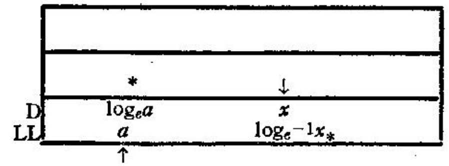


Fig. 43

The method of placing decimal point on D scale is as follows:

(1) a on LL_3 scale one digit at the left of decimal point. (For instance: 1.5, 2.3 etc.)

(2) a on LL_2 scale one digit at the right of decimal point. (For instance: 0.15, 0.23 etc.)

(3) a on LL_1 scale two digits at the right of decimal point. (For instance: 0.015, 0.023 etc.)

Exponent

In case of exponential computation A^n which is frequently used in an empirical formula, if n is positive, it is simply computed by the use of LL scale.

Example **11** $\quad 3.2^{6.5} = 1910$

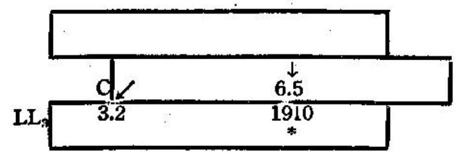


Fig. 44

Example 12 $\sqrt[4.3]{845}=4.79$

Fig. 45

Example **13** $e^{1.5}=4.48$

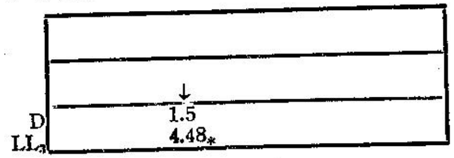


Fig. 46

If n is negative, reciprocal of A^n should be found out. In this case, it may be advisable to get answer by the use of D and DI scales on the front face.

Following this procedure hyperbolic functions are computed from the following relations:

Sin $hx=(e^x-e^{-x})/2$

Cos $hx=(e^x+e^{-x})/2$

(5) Trigonometric Function

Arrangement of trigonometric scales of this slide rule are adopted T, S and ST of so called Rietz system. Angle is graduated at degree and minute system and complementary angle is written in red. ST scale is used for computation of Sin θ and Tan θ of small angle below 6°. The following procedure should be taken to get trigonometric function by this kind of scale arrangement.

Trigonometric Function and its Multiplication & division

(a) If it is requested to find only Sin θ, tan θ and so on, set the slide exactly in the stock and set indicator on S or T and read answer on D scale under the hairline. Read on DI scale on front face, when its reciprocal is required.

Example **14** sin 32° = 0.53

tan 40° = 0.84

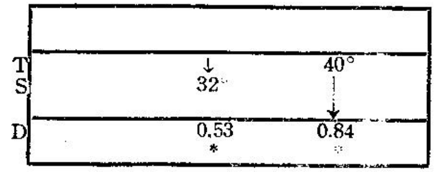


Fig. 47

If cos θ is required, answer may be given in the same way as in above explanation referred to angle in red on S scale.

(b) Multiplication and division, including trigonometric function is determined by combination of operation of S or T and C, D, CI scales.

Example **15** 5.3 × sin43° = 3.62

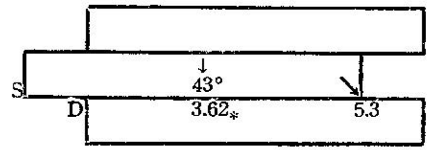


Fig. 48

Example **16** 2.55 ÷ sin50° = 3.33

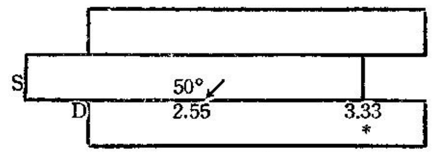


Fig. 49

Example **17** sin32° ÷ 0.67 = 0.791

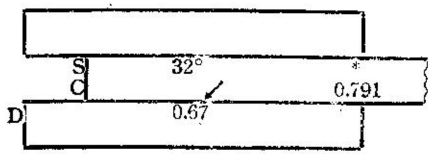


Fig. 50

Explanation referred to Sin θ has been made in the previous paragraph. And also that of cos θ or tan θ is the same, except that complementary angle of S scale should be used referred to cos θ.

Sine Proportion

Sine Proportion is performed through the use of "reference scale method" of S and D scales.

Example 18 Find b and c, in **Fig. 51** Ans. $b=6.96$ $c=8.52$

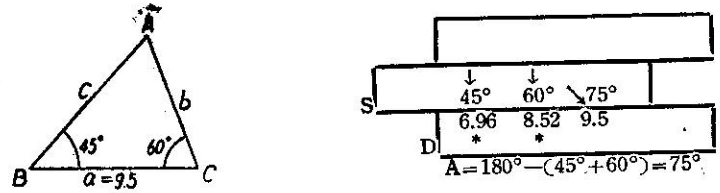


Fig. 51

Find angle A from $A=180°-(B+C)$ and follow the procedure shown in the above Fig.

Solution of Right Triangle

Solution of right triangle and also to solve the vector and complex number by this slide rule effeciently might be carried out by joint use of S, T scales and DI scale.

Example 19

Given $a=8$, $b=5$, of right triangle in Fig. 52

Find θ and c.

Ans. $\theta=32°$ $c=9.43$

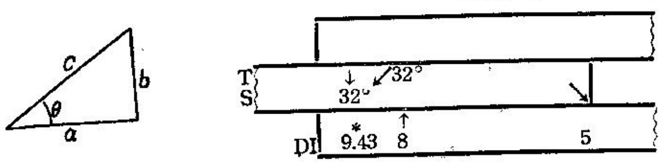


Fig. 52

If it happens to be $c>10$ in the above example, operation should be followed as in Fig. 53

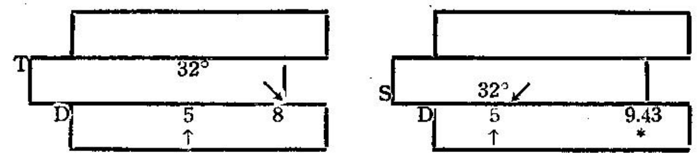


Fig. 53

Example 20 $3.25+6.63i=7.38\ \underline{|63°50'}$

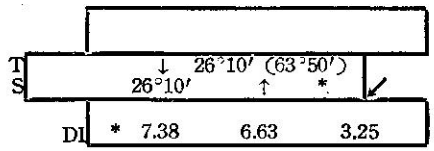


Fig. 54

Note: 63°50′ in parenthesis indicates the reading of red figure on T scale.

Gauge marks:

There are following gauge marks on this slide rule.

c (1.128 on C scale)...calculation of circle area.

R (57.29 on C scale on rear face)...conversion of degree and Radian.

V. No. 255 (10 inches) / No. 275 (20 ,,) Duplex Slide Rule

(For Expert electrical engineer)

(1) General description (See figure on page 105)

Characteristics

This slide rule is specially designed for arrangement of scales to meet with the requirement of expert electrical engineers. Calculations of higher exponent frequently arise in empirical formula and form of e^n necessary for the theory of electricity will be made with additional series of Log-Log scales. And also calculation of hyperbolic function which is necessary in design of long distance electrical transmission line by the use of hyperbolic log scales Sh & Th is available. Furthermore the inverted scales of trigonometric functions SI & TI on basis of Radian are provided for vector computation and conversion of degree to Radian is available by the use of "reference scale" of x and θ.

These features illustrate the epoch-making excellence of this kind of slide rule compared with the former one for electrical engineer's use.

There are two kinds, No. 255 (10 inches) and No, 275 (20 inches) in line with above mentioned system and the latter is convenient to be used in laboratory and test room.

Arrangement and usage of scales.

Front Face:

DF, CF, CIF (π–fold) *D, C, CI*	Multiplication, division and proportion.
L	Common logarithm
K	Cubes and Cube roots
X, θ	Conversion of angle

Rear Face:

Sh_1, Sh_2, *Th*	Hyperbolic functions
A, B	Squares and Square roots
C, LL_3, LL_2, LL_1	Exponent
TI_1, TI_2, *SI, D*	Trigonometric Function & Vector

(2) Multiplication & Division

Inasmuch as the folded scale of this slide rule is based on π–fold, the use of scale is made just the same as in II, Fundamental Calculation and Supplementary Note in III–(2). π, the ratio of circumference of circle to its diameter is frequently applied to the theory of alternating current in the field of electrical engineering. There are many advantages to eliminate complicated procedure of various calculations by the use of characteristics of folded scale and ordinary scale.

Example 1

Find inductive reactance in an alternating current circuit with 25 *mH* inductance under 50 cycle frequency.

Ans. 7.85 ohms

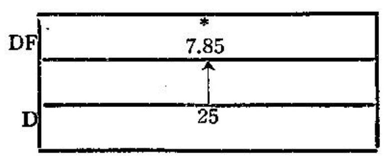


Fig. 55

In case of $f=50$ cycle no operation of slide is necessary as shown in the above fig., because of $2f=100$, but if $f=60$ cycles, one more operation to multiply $2f=120$ through *CIF* scale has to be done.

Example 2

Find condensive reactance in an alternating current circuit with static capacity of 1.5 μF under 50 cycles frequency.

Ans. 0.212×10^{-4}

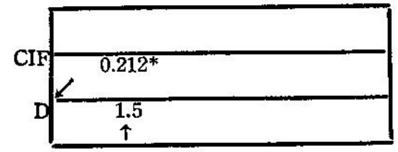


Fig. 56

Condensive reactance X_c is calculated from the formula $X_c = \frac{1}{2\pi fc}$ ohm, as shown in the above Fig. If frequency $f=60$ cycles, answer of 0.177×10^{-4} is to be given by the operation indicated in Fig. 57.

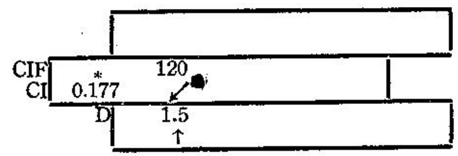


Fig. 57

(3) Square and Cube

Calculations of Square, Square roots, Cube and Cube roots are made entirely the same way as explained in III–(3) and IV–(3).

But if resonance frequency *f* is required for self inductance *L* and static capacity *C*, answer will be given by the use of folded scale as in the following example.

Example 3

Find series resonance frequency *f* of alternating current circuit with inductance of $L=200$ *mH* and static capacity $C=380\mu F$.

Answer 18.25 cycles (Fig. 58)

Frequency f is calculated from the formula $f=\frac{1}{2\pi\sqrt{LC}}$ without taking into consideration π. You might proceed to find out the reciprocal of a reading on *DF* scale.

In case of setting L and C on scale A and B, note that it is square roots.

Care should be taken that the right part of B scale is used for 200 $mH=0.20\ H$ and also the left part of A scale is used for $380\mu F=0.00038F$

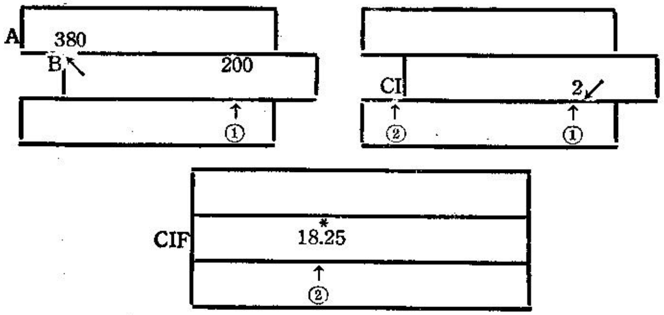


Fig. 58

(4) Logarithm and Exponent

Logarithms and exponential calculations are carried out in the same way as in III-(4) and IV-(4). Couple of examples relating electrical engineering will be explained in the following paragraph.

Example 4

What would be the iron loss of a transformer which has iron loss of 450 w at rated primary voltage of 3,300 v, if the voltage were raised up to 3,500 v, assuming the iron loss is proportional to 1.9 th power of applied voltage?

Answer 502 w (Fig, 59)

Solution: Since iron loss may be computed from the formula $450\times\left(\frac{3,500}{3,300}\right)^{1.9}$, Firstly, we get $3,500\div3,300=1.06$ by the Scales C and D. Then $1.06^{1.9}$ is worked out on LL_1, LL_2 and C scales, and answer 1.1173 is given as shown by Fig. 59.

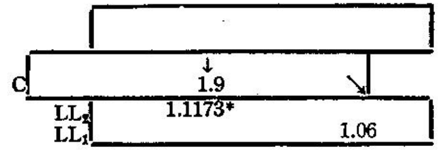


Fig. 59

Finally we get requested answer through the use of scales D and CI from $1.1173\times450=502$.

Example 5

How much is the weight of vertical type alternator made by certain manufacturer, assuming the following empirical formula can be used?

$w=8.2\left(\frac{\text{K. V. A.}}{\text{r. p. m.}}\right)^{0.65}$

in which

K. V. A. = capacity = 12,000

r. p. m. = No. of revolution = 450

w = Weight in ton

Answer 69.4 tons (Fig. 60)

As explained in the preceding example,

compute first $12,000\div450=26.65$

and next $26.65^{0.65}=8.45$, as shown by Fig. 60.

Finally find $8.2\times8.45=69.4$ using *CIF* and *DF* scales.

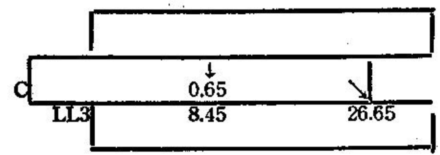


Fig. 60

Example 6

How long is the life of tungsten incandescent lamp at 110 V, which has the life of 2,000 hrs. at rated voltage of 100 V, assuming life is inversely proportional to 13.5th power of the applied voltage?

Answer 551 hrs. (Fig. 61)

Since $110\div100=1.1$ is easily found out by guess, compute first $1.1^{13.5}=3.63$, as shown by Fig. 61. And then answer may be given as $2,000\div3.63=551$ on C and D scales.

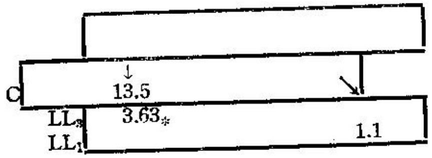


Fig. 61

(5) Trigonometric Function and Vector

Trigonometric function and vector computation are done entirely the same procedure as in chapter III-(5), since inverted scales TI_1, TI_2 and SI were arranged on this slide rule with features of Radian graduation. Conversion from degree to Radian or its reverse process is made by the use of θ and x scales.

Example 7

Find Radian for 13.5° $x=0.235$ Radian

Find degree for 0.87 Radian $\theta=49.8°$

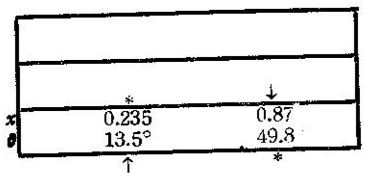


Fig. 62

Example 8

Find impedance of alternating current circuit with resistance $r=0.85$ ohm, inductive reactance $x=0.335$ ohm Answer 0.914 ohm (Fig. 63)

Calculation may be made by the operation of slide rule as shown in Fig. 63 from formula $Z=\sqrt{r^2+x^2}$

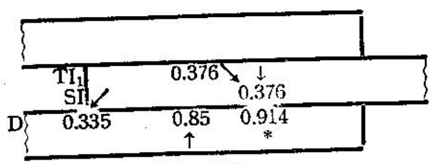


Fig. 63

(Note) Angle 0.376 on TI_1 scale is only used as a "parameter" in this calculation, even though this angle indicates phase angle.

Example 9

Find the resultant current I in an alternating current circuit as shown by Fig. 64

Answer $I=25$ amps (Fig. 64)

Since it is evident that phase angle difference between resistance and reactance circuits is right angle, resultant current I may be given by the operation of the slide rule from $I=\sqrt{20^2+15^2}$ as the right side diagram.

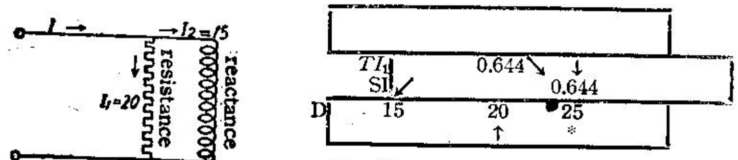


Fig. 64

Example 10

Find the resultant current $\dot{I}$ of $\dot{I}_1=3+j5$ and $\dot{I}_2=8+j2$ in polar coordinate.

Answer $\dot{I}=13.05\ \underline{|0.567}$ (Fig. 65)

$\dot{I}=(3+8)+j(5+2)=11+j7$, then absolute value is $\dot{I}=\sqrt{11^2+7^2}$

The use of DF scale is more convenient than to use D scale, taking into consideration this absolute value.

The operation of the slide rule is done as indicated in Fig. 65.

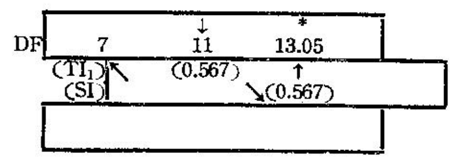


Fig. 65

(Note) Letters and figures in parenthesis indicate that these are on rear face of the slide rule.

Example 11

Change vector 7.5 $\underline{|0.55}$ indicated by the Polar coordinate to that of rectangular coordinate.

Answer $6.4+j3.92$

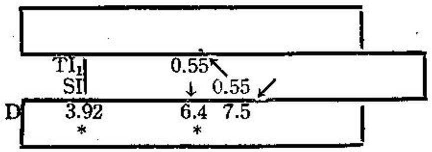


Fig. 66

Example 12

Find current $\dot{I}$ in an alternating current circuit with impedance $\dot{Z}=2+j3$ when voltage $\dot{E}=50+j15$ is applied to the circuit.

Answer $\dot{I}=11.2-j9.18$

We can solve this problem using the formula $\dot{I}=\frac{\dot{E}}{\dot{Z}}=\frac{50+j15}{2+j3}$, and firstly have to change the coordinates of $\dot{E}$ and $\dot{Z}$ into polar system.

Hence $\dot{E}=52.2\ \underline{|0.292}$

$\dot{Z}=3.61\ \underline{|0.981}$

we get $\dot{I}=\frac{52.2}{3.61}\ \underline{|0.292-0.981}=14.45\ \overline{|0.689}$

Again by changing it to rectangular coordinate,

$\dot{I}=11.2-j9.18$

The following procedure is advisable that in the midst of calculation, an angle of vector $\dot{Z}$ is rotated 90° for convenience and to get the complementary angle 0.589. And then, subtracting 0.589 from $\pi/2\doteqdot1.57$, get the value 0.981.

(6) Hyperbolic Function

The computation relating to hyperbolic function is frequently necessary for the study of alternating current theory and long distance electric power transmission line. This slide rule has logarithmic scales of hyperbolic function Sh_1, Sh_2 and Th on the stock and inverted scales of trigonometric function TI_2, TI_1 and SI on the slide, in order to compute their multiplication and division conveniently. By the use of these arrangement, we can also make possible to calculate hyperbolic function of complex angle.

Hyperbolic Function

If only the hyperbolic function is requested, the "reference scales" of D scale to *Th* or *Sh* scale are to be used.

Example 13

$\sinh 0.345=0.352$

$\tanh 0.345=0.332$

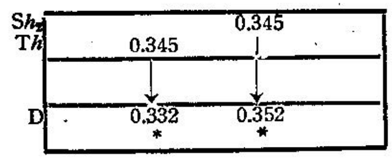


Fig. 67

Example 14

$\cosh 0.345=1.06$ (Fig. 68) (Fig. 69)

It is easily found by $\cosh x=\frac{\sin hx}{\tan hx}$, but the special procedure is considered to be advisable by the use of this slide rule as shown by Fig. 68 and 69.

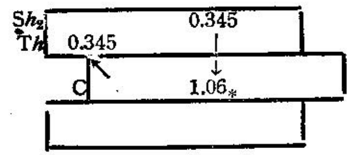


Fig. 68

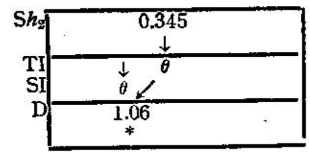


Fig. 69

Hyperbolic Function of Complex Angle

Hyperbolic function of complex angle $a+jb$ may be calculated in the following two ways:

(a) Use of rectangular coordinate system.

(b) Use of polar coordinate system.

(a) Calculation by rectangular coordinate system

$$\sin h(a+jb)=\sin h\ a\ \frac{\sin b}{\tan b}+j\cosh a\cdot\sin b$$

$$\cosh(a+jb)=\cosh a\ \frac{\sin b}{\tan b}+j\sin h\ a\cdot\sin b$$

From above relations, they are computed as shown by the following examples.

Example 15 $\quad \sinh(0.82+j1.2)=0.331+j1.263$ (Fig. 70, Fig. 71)

Calculation of real quantity : (Fig. 70)

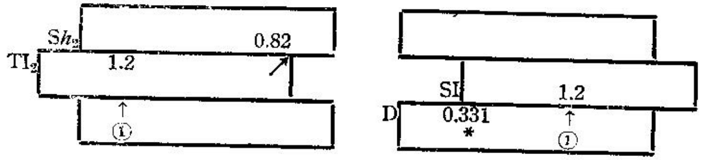


Fig. 70

Calculation of imaginary quantity : (Fig. 71)

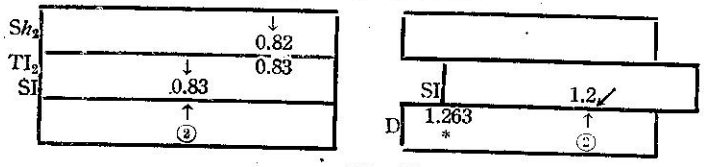
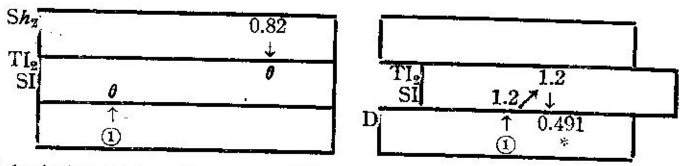


Fig. 71

Example 16 $\quad \cosh(0.82+j1.2)=0.491+j0.853$

Calculation of real quantity.



Calculation of imaginary quantity.

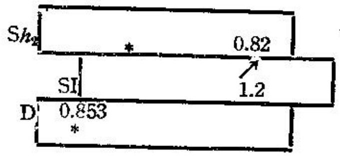


Fig. 72

(b) Calculation by polar coordinate system.

Following formulas are used for computation.

$\sinh(a+jb)=\sqrt{\sinh^2 a+\sin^2 b}\quad \Big|\tan^{-1}\left(\dfrac{\tan b}{\tan ha}\right)$

$\cosh(a+jb)=\sqrt{\sinh^2 a+\cos^2 b}\quad \Big|\tan^{-1}(\tan b\cdot\tan ha)$

$\tanh(a+jb)=\sqrt{\dfrac{\sinh^2 a+\sin^2 b}{\sinh^2 a+\cos^2 b}}\quad \Big|\tan^{-1}\dfrac{\sin 2b}{\sin h2a}$

(c) Calculation of $\tan h^{-1}(a+jb)$

This calculation is quite often made in finding out position angle at particular point of long electric power transmission line. The procedure is as follows:

Let $\tan^{-1}(a\pm jb)=Z=x\pm jy$ then, $\dfrac{e^z-e^{-z}}{e^z+e^{-z}}=\dfrac{e^{2z}-1}{e^{2z}+1}=a\pm jb$ and $e^{2z}=e^2(x+jy)$ $=\dfrac{1+(a\pm jb)}{1-(a\pm jb)}=A\underline{|\theta}$ substituting $y=\dfrac{\theta}{2}$, $x=\dfrac{1}{2}\log_e A=1.1513\log_{10} A$

The answer may be found out from above equations.

Example 17

$\tan h^{-1}(2.5-j0.57)=0.396+j\left(\dfrac{\pi}{2}+0.101\right)$

Find e^{2z} first, and then answer is given as:

$1+(a-jb)=3.5-j0.57=3.545\,\overline{|0.161}$

$1-(a-jb)=-1.5+j0.57=1.6\underline{|\pi-0.363}$

$A\underline{|\theta}=\dfrac{3.545\,\overline{|0.161}}{1.6\,\underline{|\pi-0.363}}=2.21\,\underline{|\pi-0.202}=2.21\,\underline{|\pi+0.202}$

$y=\dfrac{\pi}{2}+0.101\qquad x=\dfrac{1}{2}\log_e 2.21=\dfrac{0.792}{2}=0.396$

Gauge mark

Gauge mark on this slide rule is "c" (1.128 on C scale on rear face) which is used for computation of circle area.

VI. No. 256 (10 inches) Duplex Slide Rule (For electric communication engineers)

(1) General Description (See figure on page 106)

Characteristics

This slide rule is used not only for the convenience of calculation on electric communication engineering: for instance frequency and wave length; decibel and impedance; problem of resonance frequency, but also ordinary multiplication and division with scale in which the most superior system is adopted. This slide rule has been originaly designed by Tokyo Shibaura Electric Co., but we have designed a new duplex slide rule after several improvements have been made upon arrangement of various kinds of scales.

Remarkable feature of this slide rule is that formula and index so as to easily can place decimal point have been provided.

Arrangement and use of scales

Front face:

db (*L*)	Decibel and logarithm
Neper	Neper calculation
DF, *CF*, *CIF* (π-fold) / *CI*, *C*, *D*	Multiplication, division and proportion
LL_3, LL_2, LL_1	Exponential calculation

Rear Face:

A	Square, Square roots and Surge impedance
$\dot{L}$	Inductance
$\dot{C}$	Capacitance
TI, *SI*, *D*	Trigonometric function and vector
λ / *F*	" Reference scale " of wave length and frequency

(2) General Calculation

Inasmuch as the folded scale of this slide rule is π-fold, which is the same as most of other kinds, general calculation such as multiplication and division are made just the same way as indicated in the explanation of fundametnal calculation in the chapter II and IV, V-(2).
But we do not place an emphasis upon the calculation of Square and Cube relation. Since only *A* scale is provided on rear face, the calculation of square relation has to be done as shown in the following procedure.

Example 1 $1.5\times2.2^2=7.26$

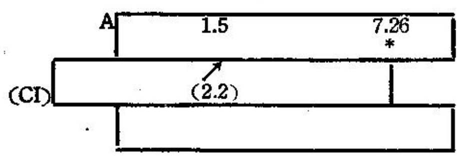


Fig. 73

(Note) Letters and numbers in the parenthesis indicate that those are on opposite face.

Example 2 $8\div1.3^2=4.73$

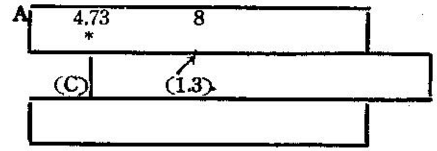


Fig. 74

Example 3 $18\times\sqrt{20}=80.5$

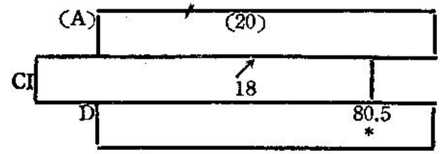


Fig. 75

Example 4 $\sqrt{12}\div6.5=0.533$

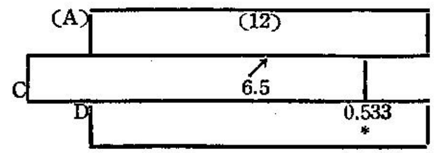


Fig. 76

Form of $a^2\div b$ and $a\div\sqrt{b}$ are not operated as easy as the above examples. $a^2\div b$ is changed to the form $\frac{a\times a}{b}$ which may be calculated as an ordinary multiplication and division. $a\div\sqrt{b}$ is given on *C* scale in the form of reciprocal of result obtained through the operation of $\sqrt{b}\div a$.

Example 5 $8.5\div\sqrt{24}=1.735$

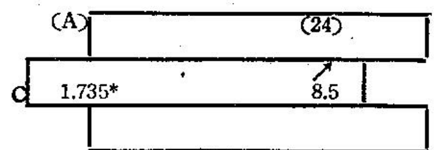


Fig. 77

There is no K scale to solve cube relation . Therefore a^3 has to be found out in the form of $a \times a^2$.

To extract cube root by the use of Log Log scale explained in the following chapter is much easier than to use A and CI scales.

(3) Logarithm and Exponent.

Logarithm and exponent are solved by the use of L and LL scales in the same way as explained in the chapter III, IV and V-(4)

It should be noted that L scale on this slide rule is the modification of db scale explained in the succeeding chapter, adding red lettering on the scale.

(4) Trigonometric function and Vector

Inverted scale, SI and TI are adopted so called Deci-trig system which indicate the angle with degree and its decimal fraction. Therefore the calculation of trigonometric function and vector is done in the same operation as in the chapter III-(5) and V-(5), but there is no scale for small angles below 6° for sine and tangent.

Operation may be made as treatment of small angle by the use of gauge mark R, exactly same as explained in the method of $\rho°$ in the chapter III-(5)

(5) Calculation of Decibel

In the electric communication circuit, let voltage and current be V_1 and I_1 respectively at input side, and also those at output side be V_2 and I_2. Decibel for voltage ratio and current ratio are as follows:

$$db(V)=20\log\frac{V_2}{V_1}$$

$$db(I)=20\log\frac{I_2}{I_1}$$

When V_1, V_2 or I_1, I_2 are given, find $\frac{V_2}{V_1}$ or $\frac{I_2}{I_1}$ by the use of C and D scale, and db is read on db scale, opposite the index of C scale.

If "Neper" is requested instead of db, read the value on Neper scale.

Conversion of db and Neper are easily carried out each other by the use of db and Neper scale, moving indicator only, because of db and Neper scale being the reference scales.

Let power ratio of input and output side W, db corresponding to W is.

$db(W)=10\log W$

In this case calculate power ratio first by C and D scale and take db reading on L scale.

(6) Calculation of Resonance Frequency

Resonance frequency F of alternating current circuit with inductance L and capacity C is expressed by the formula:

$$F=\frac{1}{2\pi\sqrt{LC}}$$

This kind of calculation may be done with joint use of $\dot{L}$, $\dot{C}$ and F scales. Generally speaking, mistaken result would be given, if L and C were not adequately set, because this calculation includes Square roots. However, section to set the value of L and C and placing decimal point are clearly shown by indexes in red on this slide rule, which prevents perfectly these mis-operations.

symbolic law of placing decimal point: $\dot{F}=\dot{L}+\dot{C}$

Example 6

Find the resonance frequency F, if $L=7\mu H$ and $C=50pF$ are given.

Answer 8.5 mc/s (Fig. 78)

Method Set red line F of the slide to $7\,\mu H$ (index 0) on $\dot{L}$ scale.

Set indicator to $50pF$ (index 1) on $\dot{C}$ scale, under the hairline read off 85 on scale F.

According to the law of placing decimal point, from $F=0+1=1$ answer is found as 8.5 mc/s

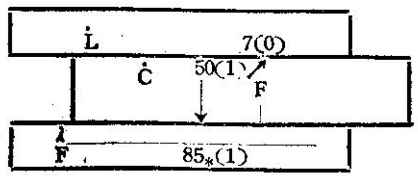


Fig. 78

Example 7

Given $F=500\ kc/s$ and $C=300\ pF$, find L.

Answer $L=336\ \mu H$ (Fig. 79)

To 500 KC (index 2) on F scale, set 300 pF (index 1) of C scale.

Opposite red gauge line F of the slide, read 336 on L scale. According to the law of placing decimal point $L=2-1=1$, answer is 336 μH.

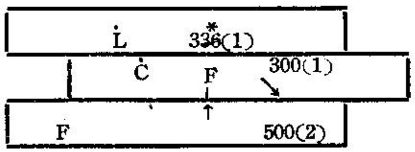


Fig. 79

(7) Calculation of Surge Impedance.

Let inductance and Capacitance per unit length of the line be L and C, then surge impedance is solved from the formula:

$$X=\sqrt{\frac{L}{C}}$$

This formula is also available for the calculation of constant K type filter. To solve this problem by this slide rule, we use in convenience $\dot{L}$, $\dot{C}$ and A scales.

Symbolic law of placing decimal point: $\dot{X}=\dot{L}-\dot{C}$

Example 8

$L=250\ mH$ and $C=350\ pF$ are given.

Find surge impedance X.

Answer $X=26.8\ K\Omega$ (Fig. 80)

To 250 mH (index 3) of $\dot{L}$ scale, set 350 pF (index 1) on $\dot{C}$ scale.

Opposite red gauge line F of the slide, read significant figure 268 on A scale. According to the law of placing decimal point, $X=3-1=2$

Answer is given as 26.8 $K\Omega$

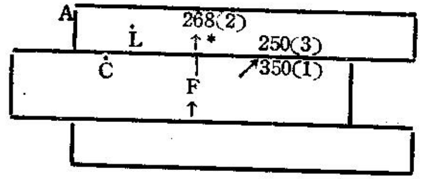


Fig. 80

(8) Calculation of Reactance

Inductive reactance X_L and capacitive reactance Xc on basis of inductance L and capacity C are calculated from the following formula:

$$X_L=2\pi FL \qquad Xc=\frac{1}{2\pi FC}$$

where F=resonance frequency

Law of placing decimal point is as follows:

$\dot{X}_L=2\dot{L}-\dot{F}$

$\dot{X}c=\dot{F}-2\dot{C}$

Example 9

Given $F=500\ kc/s$ and $L=10\ mH$

Find X_L

Answer $X_L=31.4\ K\Omega$ (Fig. 81)

Computation may be made through the groups of scales for multiplication and division on front face of the slide rule and decimal point is adequately placed.

However, the following operating procedure shows easy way of placing decimal point: find C to meet with resonance condition from the value of F and L. X_L may be found out from C & L.

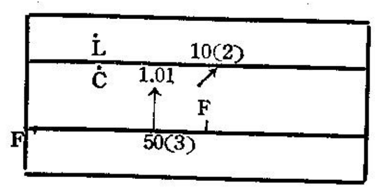


Fig. 81

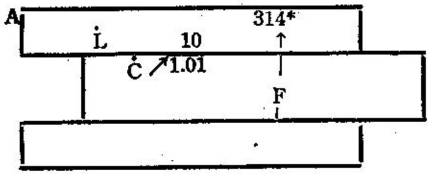


Fig. 81

To 10 mH (index 2) on $\dot{L}$ scale, set red gauge line F first. Opposite 500 kc/s on F scale, read 1.01 on $\dot{C}$ scale. To 10mH on $\dot{L}$ scale, set 1.01 on $\dot{C}$ scales.

Opposite red gauge line F of the slide, read significant figures 314 on A scale.

Answer is 31.4 $K\Omega$ by the law of placing decimal point $\dot{X}=2\times2-3=1$

(Note) In this case value of C is only used for "parameter" and it is not necessary to know its index value.

(9) Relation between Wave Length and Frequency.

Inasmuch as frequency scale F and wave length scale λ are provided for this slide rule as "reference scales" relation between frequency and wave length is easily found out.

Law of placing decimal point is as follows:

$F(Kc/s)\times\lambda(M)=3\times10^5$

As a reference, the relation is shown in the following table.

F	100KC	1000KC	10MC	100MC	1000MC	10000MC
λ	3000M	300M	30M	3M	30CM	3CM

Gauge marks

Following gauge marks are provided on this slide rule.

C(1.128 on C scale of front face)	Calculation of circle area.
F(on rear face of the slide)	Impedance, reactance and frequency.
R(57.29 on C scale of front face)	Conversion of degree to Radian

VII No. 259 (10 inches) / No. 279 (20 inches) Duplex Slide Rule

(For Expert Mechanical Engineer)

(1) General Description (See figure on page 106)

Characteristics

Various kinds of scales are so specially designed in arrangement that to aim at the convenient use by the expert mechanical engineer. Above all, function of Log Log scale which is necessary for exponential calculation frequently needed in an empirical formula has been enlarged such as extention of value of A for A^x to 1.001~22000, and also LLo, LL_1, LL_2, LL_3 scales in case of positive value of x exponent and LL/o, $LL/1$, $LL/2$, $LL/3$ scales in case of negative value of x exponent are provided.

Trigonometric function scales are designed with "Rietz" system and graduations of angle are furnished with so called Deci-trig system which is expressed in degree and its decimal fraction, and also complementary angles are lettered in red. We believe that this slide rule has an epoch-making excellence over ordinary slide rule for mechanical engineer. There are two kinds No. 259 (10 inches) and No. 279 (20 inches) and the latter may be well designed for test room or laboratory use.

Arrangement and use of scale

Front face:

DF, CF, CIF (π-fold), D, C, CI	Multiplication and division
L	Common logarithm
K	Cube and Cube roots
LLo, LL/o	Exponential calculation

Rear Face;

$LL/1$, $LL/2$, $LL/3$	Exponential calculation with negative exponent
C, LL_1, LL_2, LL_3	Exponential calculation with positive exponent
A, B	Square and Square roots
T, S, ST, D	Trigonometric function

(2) Multiplication and Division

Operation of slide rule for multiplication, division and proportion are done in the same way as in the fundamental calculation of II and III, IV, V-(2), because π-fold scales are provided on this slide rule.

Following examples show the multiplication and division of combination of 3 or more numbers and proves how high its operating efficiency is.

Example 1

$$\frac{18\times45\times37}{23\times29}=44.9$$

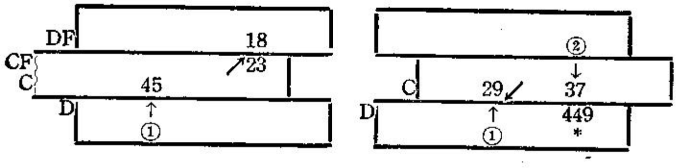


Fig. 82

Mental fatigue will be far more diminished by alternative operation of multiplication and division, even though there is no priority or definite procedure.

Indicator might as well be operated at about middle part of the slide rule. Placing the decimal point for answer may be made much easier in using round up figures and compute mentally as follows:

$$\frac{20\times45\times40}{20\times30}\doteqdot50$$

And comparing with this to the significant figure 449 which obtained by the slide rule, the answer is decided as 44.9. This method is so called "comparison" or "Mental survey" method which has been mainly developed in U.S.A.

Example 2

$$1.843\times92\times2.45\times0.584\times365=88600$$

This equation is changed to the form of $\dfrac{1.843\times2.45\times365}{(1/92)\times(1/0.584)}$ and may be operated as in Fig. 83.

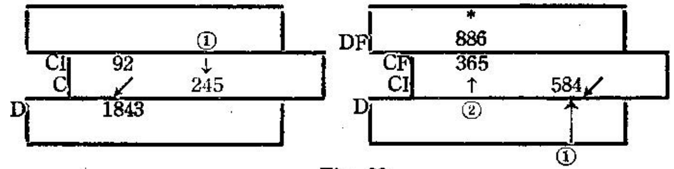


Fig. 83

Method of placing decimal point is roughly estimated as follows:

$$2\times100\times2.5\times0.6\times300=90,000$$

Hence answer is decided as 88,600

Example 3

$$\frac{0.873\times46.5\times6.25\times0.75}{7.12}=26.7$$

In the same way as in Example 2, the equation may be changed to the form of $\dfrac{0.873\times46.5\times0.75}{7.12\times(1/6.25)}$ and operated as in Fig. 84.

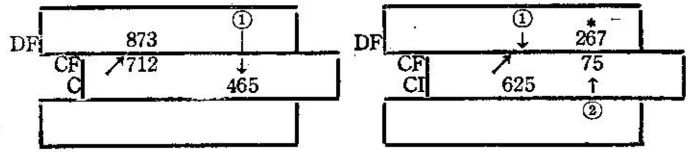


Fig. 84

Rough estimation in placing decimal point is made in the form of

$$\frac{1\times42\times6\times1}{7}=36$$

Hence the exact answer is 26.7.

(3) Square and Cube

Multiplication and division relating to square or cube are available by the use of *A*, *B* and *K* scales in the same way as chapter III, IV-(3). The following example shows the calculation of a little complicated form.

Example 4

$$\frac{0.286\times652\times\sqrt{2350}\times\sqrt{5.53}}{785\times\sqrt{1288}}=0.755\quad\text{(Fig. 85)}$$

It is observed that ordinary multiplication and division may be applied, after square roots are computed first through the use of *A* and *B* scales. And then quicker operation can be performed in the following way.

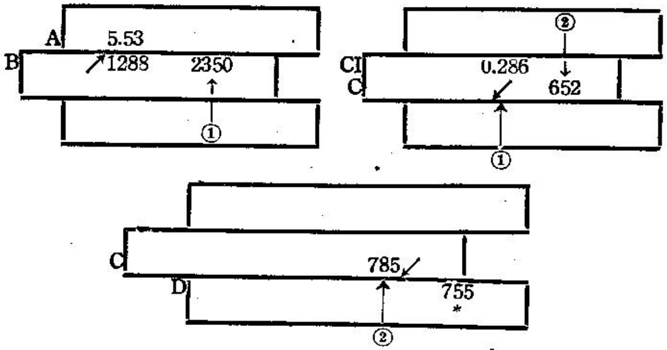


Fig. 85

Placing the decimal point easily be made by eliminating similar number in numerator and denominator as follows.

$$\frac{0.3\times(700)\times(\sqrt{2000})\times\sqrt{6}}{(800)\times(\sqrt{1000})}\doteqdot 0.75$$

By inspection, answer is decided 0.755. This method of pointing off is called "elimination".

Example 5

$$\frac{\pi^2\times 875\times 278}{72.2^2\times 0.317^2}=4580$$

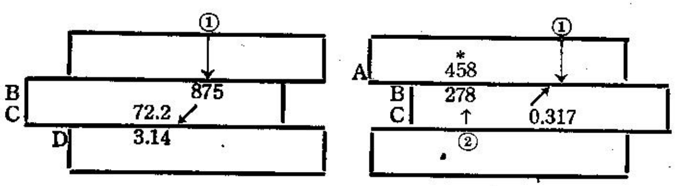


Fig. 86

Placing decimal point is made by the use of round up figures.

$\frac{10\times 1000\times 300}{5000\times 0.1}\doteqdot 6000$ Answer is evidently 4580 by observation of significant figures.

Example 6 $\quad \frac{5.37\times\sqrt[3]{0.0835}}{\sqrt{52.5}}=0.324$

This is converted to the following form:

$$\frac{x}{5.37}=\frac{\sqrt[3]{0.0835}}{\sqrt{52.5}}$$

And it is convenient to operate as calculation of proportion. Figure in parenthesis indicates that it is located on rear face.

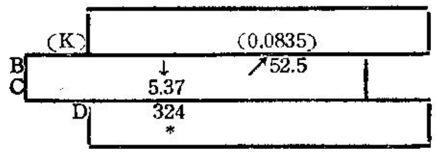


Fig. 87

Placing decimal point is done by the use of round up figures from the relation $\frac{x}{5}=\frac{0.4}{7}$, in comparison with figures on the Slide Rule. Answer is decided as $x=0.324$.

Example 7 $\quad \frac{1.736\times 6.45\times\sqrt{8590}\times\sqrt[3]{581}}{\sqrt{27.8}}=1643$

Procedure of calculation is started with cube root first, then square root next and finally ordinary multiplication and division. It may be made much easier than we expected.

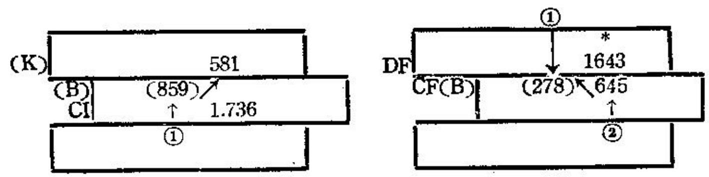


Fig. 88

As indicated in the above operation, it should be noted that $\sqrt{8590}$ and $\sqrt{27.8}$ were both intentionally set at the left hand side of *B* scale in order to avoid answer falls off scale.

Placing decimal point of answer is worked out with regard to the value on slide rule from the following round up figures and **1643 is decided as** answer.

$$\frac{2\times5\times100\times8}{5}=1600$$

(4) Logarithm and Exponent.

Logarithm

Using L or LL scale, common and natural logarithms are found in the same way as in chapter III, IV-(4).

Exponential function

Log log scales to be used for exponential calculation include not only $LL/1$, $LL/2$, $LL/3$, reciprocal scales of LL_1, LL_2, and LL_3 which are provided on other kinds of slide rule in order to use them for the calculation of A^n in which exponent n is positive or negative, but also addition of LLo, and $LL/_0$ so as to enlarge the range of their application.

These scales have broad range of value A which covers 1.001–22000 and 0.00005–0.999 respectively.

Next to LLo or LL/o scale might be considered common logarithmic scale, D. Therefore we are not speaking too much, even though we say this slide rule provides complete Log Log scale on realistic basis.

As shown by Fig. 89, these Log Log scales are divided into several groups extending to right or left from middle point zero, with base of natural logarithm e, $e^{0\cdot1}$ and $e^{0\cdot01}$.

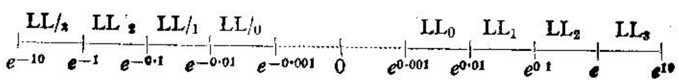


Fig. 89

LL and $LL/$ scales which have the same numbering are in the relation of reciprocal each other as shown in the above Fig.

Example 8 $e^{0\cdot36}=1.433$

$e^{-0\cdot36}=0.698$

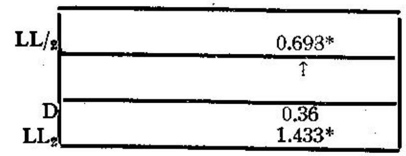


Fig. 90

Example 9 $8.32^{\frac{7\cdot2}{2\cdot8}}=232$

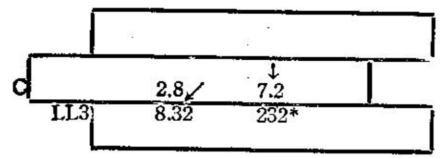


Fig. 91

Example 10 $\sqrt[5\cdot1]{0.8^{6\cdot4}}=0.8^{\frac{6\cdot4}{5\cdot1}}=0.756$

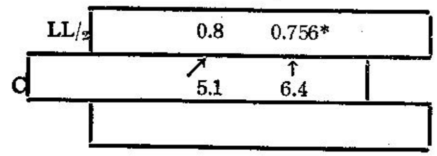


Fig. 92

(5) Trigonometric Function and Vector

Since trigonometric function scales of this slide rule are S, T, and ST, of Rietz system, operation of the rule is carried out in the same way as in chapter IV-(5). But we would like to call attention that calculation on an application of graphical figure should be made by other method, because of no DI scale.

Of course, the same operation as in chapter III-(5) may be made by taking the slide out of the stock and put in reverse direction. Specific operation of this slide rule is explained in the following applied example.

In this case the fact that angle is expressed in "deci-trig" system with degree and decimal fraction should be bear in mind.

Complementary angle is written in red on *S*, *T*, and *ST* scales in the same as No. 251.

Solution of triangle

Example 11   Find solution of triangle of Fig. 93

Answer $b=35.5$ $c=53.3$ $C=75°$

$C=180°-(40°+65°)=75°$

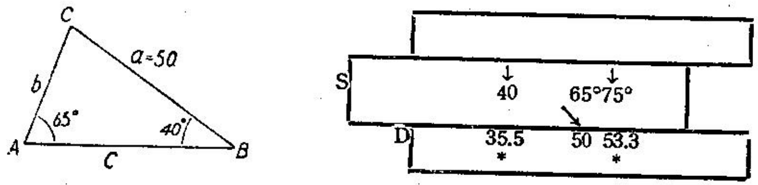


Fig. 93

Example 12   Find solution of triangle of Fig. 94

Answer $b=55.8$

$c=80.7$

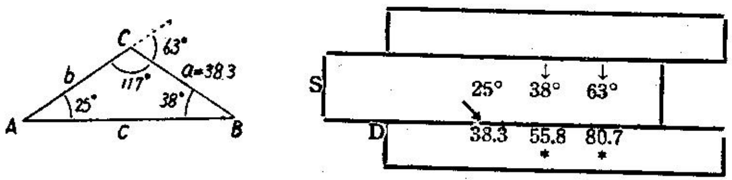


Fig. 94

Example 13   Find *c* and θ of Fig. 95

Answer $\theta=36.9°$ $c=5$

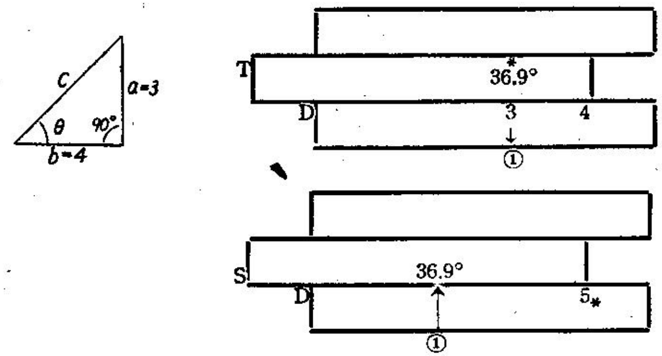


Fig. 95

Example 14

Given three sides of a triangle $a=15$, $b=18$, $c=20$, find angle *A*, *B*, *C*.

From the formula $\cos A=\dfrac{b^2+c^2-a^2}{2bc}$, find angle *A*, that is

$$\cos A=\frac{18^2+20^2-15^2}{2\times18\times20}=\frac{499}{720}$$

By the operation of "proportion" $A=46.1°$ (Red figure on *S* scale). Then by sine proportion through the use of *S* and *D* scales, we get $B=59.9°$ $C=74.0°$

Multiplication & Division including Trigonometric Function

Example 15

$$\frac{4\sin 38°}{\tan 42°}=2.735$$

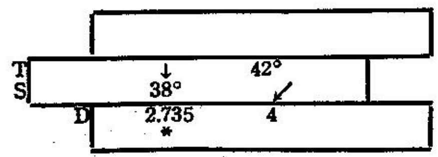


Fig. 96

Example 16 $\dfrac{6.1\sqrt{17}\,\sin 72°\,\tan 20°}{2.2}=3.96$

To 17 on the right half of *A* scale, set indicator hairline. Set 2.2 on *C* scale under the hairline and move indicator to 20° on *T* scale.

Set 6.1 on *CI* scale under the hairline of indicator. Move indicator to 1 at the left end of scale.

Opposite the hairline of indicator, set the right index of *C* scale and move indicator to 72° on *S* scale.

Opposite indicator, read 3.96 on *D* scale.

Vector Calculation

Example 17 $3.6+j1.63=3.95\ \underline{|24.4°}$

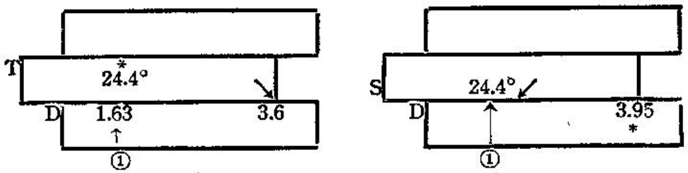


Fig. 97

Example 18 Analyze the following vector.

Answer $x=21.1$ $y=16.5$

$90°-38°=52°$

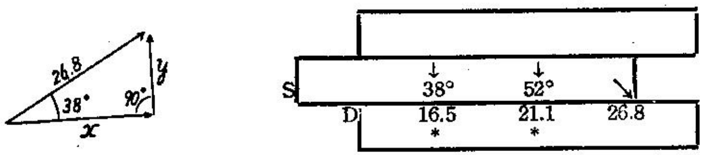


Fig. 98

(Note) Practically it is much better to use red figures on *S* scale, instead of calculating complementary angle.

Example 19 Calculate Vectors in Fig. 99.

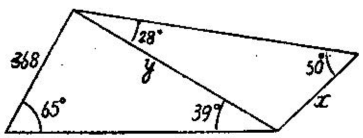


Fig. 99

Answer $x=325$

$$\frac{368}{\sin 39°}=\frac{y}{\sin 65°} \qquad \frac{y}{\sin 50°}=\frac{x}{\sin 28°}$$

Solving two proportional equation of sin, find $x=325$

(6) Hyperbolic Function

To obtain hyperbolic function by this slide rule, *LL* and *LL/* scales are used with the application of the following formula:

$$\text{Sin}\,hx=\frac{e^x-e^{-x}}{2} \qquad \cosh\,x=\frac{e^x+e^{-x}}{2}$$

$$\tan\,hx=\frac{e^x-e^{-x}}{e^x+e^{-x}}=\frac{e^{2x}-1}{e^{2x}+1}$$

Example 20 $\sin h1.04=1.238$

$\cos h1.04=1.592$

$\tan h1.04=0.777$

In line with the procedure shown in Fig. 100, find $e^{1.04}=2.83$, $e^{-1.04}=0.354$ $e^{2.08}=8.0$

From these values we get

$$\sin h\,1.04=\frac{2.83-0.353}{2}=1.239$$

$$\cos h\,1.04=\frac{2.83+0.353}{2}=1.592$$

$$\tan h\,1.04=\frac{8.0-1}{8.0+1}=\frac{7}{9}=0.777$$

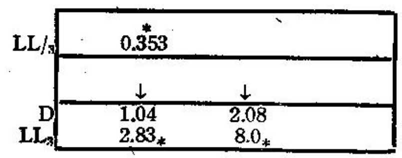


Fig. 100

Gauge Marks

Following gauge marks are provided on this slide rule.

c (1.128 on *C* scale on rear face)...Calculation of circle area.

R (57.29 on *C* scale on rear face)...Conversion of degree and radian.

VIII. No. 154 (20 inches) Duplex slide rule
(For expert electrical engineer)

(1) General description (See figure on page 108)

(Characteristics)

This slide rule has been designed for use of expert electrical engineer and provides different arrangement of scales and graduations from No. 255 or No. 275.

Especially, purpose of this slide rule lies in simplification of calculation for Vector and hyperbolic function relating to the calculation for theory of electricity or electric power transmission line.

Feature of graduation of this scale is that for vector computation purpose by the use of un-logarithmic scales *P* and *Q* which are never seen in any other slide rule, calculation of form of $\sqrt{a^2 \pm b^2}$ may be possible to be worked out just the same as an ordinary multiplication and division. This slide rule also provides two kinds of trigonometric function scales with angle unit of degree or Radian.

This is one of the remarkable feature of this slide rule in which these two units are left to user's choice.

(Arrangement and usage of scale)

FRONT FACE:

DF, CF (π-fold) *CI, D*	Multiplication and division
P, P', Q	Absolute value of vector
S θ° (referred to *A* scale)	Vector angle (degree)
A	
K	Cube

REAR FACE:

Sx, Tx (referred to *D* scale)	Trigonometric function (Radian)
T θ° (〃)	tan θ (degree)
Th, Sh	Hyperbolic function
D, C	Multiplication and division
L, X	Logarithm, conversion of degree and Radian

(2) Multiplication and Division

Calculation is worked out by joint use of π-fold scales *CF, DF* and ordinary scales *CI, C* and *D*. The operating method is almost the same principle as in the chapter II (Fundamental Calculation) or IV-(4)

But this slide rule has no *CIF* scale. Therefore certain procedure in operation should be taken into consideration. If division is performed by indicator operation, be careful to avoid the divisor falling off *CI* scale.

Example 1 $3.5 \div 2.2 \div 1.5 = 1.06$ (Fig. 101)

If the calculation is made in the order of above equation through *C D, CI* scales, division by 1.5 falls off scale. In order to avoid falling off scale, the following operation is recommended.

It is noted that:

Set dividend 3.5 on *DF* scale.

Divide it by 1.5 on *CF* scale, taking into consideration the movement of the slide.

Move indicator to 2.2 on *CI* scale.

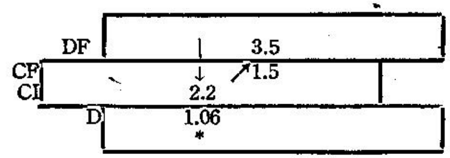


Fig. 101

(3) Square, Cube, Logarithm

Square Calculation has to be done in the same way as in chapter VI-(2)

Cube calculation is made in the same way as in chapter IV-(3) by the use of *K* scale. Common logarithm is found by the use of *L* scale in the same way as in chapter III-(4).

Typical examples of various forms are explained in the following paragraph briefly.

Example 2 $18.4^2 = 338.6$

$\sqrt{15.32} = 3.915$

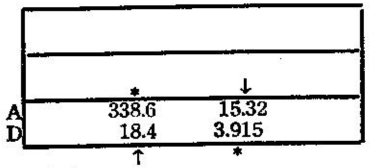


Fig. 102

Example 3 $3.26^3 = 34.6$

$\sqrt[3]{86.2} = 4.417$

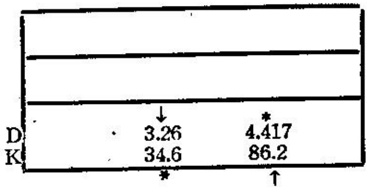


Fig. 103

Example 4 $\log_{10} 385 = 2.5855$

Opposite 385 on *D* scale, read mantissa 0.5855 on *L* scale. The characteristic is found by the usual rule, taking one less than the number of figures at the left of the decimal point of given number as 3—1=2

Answer is 2.5855

Example 5 $\log_{10} \mathrm{Sin}\ (0.348) = \bar{1}.5328$

Opposite 0.348 on S_x scale, read mantissa 0.5328 on *L* scale. Since the decimal point precedes the first digit of Sin (0.348), characteristic is decided as $\bar{1}$.

Example 6 $\log_{10} \tan 30° = \bar{1}.7615$

Angle is given in degree.

Opposite 30 on $T\theta°$ scale, read mantissa 0.7615 on *L* scale. The characteristic is the same as previous example.

Example 7 $\log_{10} \tan h\ (0.348) = \bar{1}.5245$

Opposite 0.348 on *Th* scale, read 0.5245 on *L* scale. In the same way as in Ex. 6, characteristic is $\bar{1}$.

(4) Trigonometric Function and Vector

Trigonometric Function

Trigonometric function scales which are classified four kinds:

S_x (Radian vs. Sine)
T_x (Radian vs. Tangent) } —With referred to *D* scale.
$T\theta°$(Degree vs. Tangent)
$S\theta°$(Degree vs. Sine) —With referred to *A* scale.

Accordingly, trigonometric function of given angle, either in degree or radian is calculated directly. $T\theta°$ and $S\theta°$ scales are lettered their complementary angles in red figure. Using these angles $\cot\theta$ and $\cos\theta$ can also be computed.

Example 8 $\sin 19.5° = 0.334$

$\cos 19.5° = 0.943$

Insert the slide exactly into the stock, and opposite 19.5° in black figure and 19.5° in red figure on $S\theta°$ scale, read 0.334 and 0.943 respectively on *A* scale.

Example 9 $\tan 23.4° = 0.433$

$\cot 23.4° = 2.314$

Set indicator to 23.4 on $T\theta°$ Scale.

Read 0.433 and 2.314 on *D* and *CI* scale under the hairline.

Example 10 $\sin (0.345) = 0.3385$

Opposite 0.345 on S_x scale, read 0.3385 on *D* scale.

Example 11 $\tan (0.486) = 0.5285$

Opposite 0.486 on T_x scale, read 0.5285 on *D* scale.
Furthermore, conversion from degree to Radian or its reverse operation is possible with use of *L* and *X* scales as "reference scale."

Vector

To find out the absolute value of vector, this slide rule provides unlogarithmic scale *P*, *Q* and extended scale *P'*.

By the use of these scales, form of $\sqrt{a^2 \pm b^2}$ is computed in the same operation as ordinary multiplication and division.

Example 12 $\sqrt{3.52^2+5.86^2}=6.836$

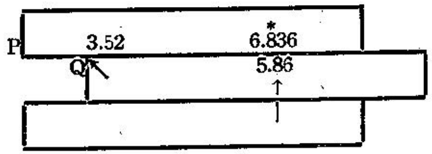


Fig. 104

Example 13 $\sqrt{8.63^2-3.38^2}=7.94$

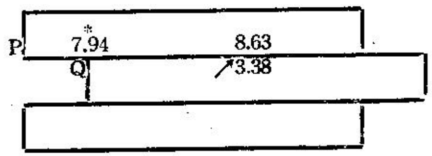


Fig. 105

Example 14 $\sqrt{6.85^2+9.56^2}=11.76$

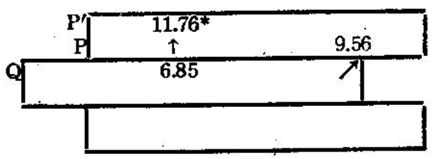


Fig. 106

Example 15 $\sqrt{10.75^2-8.22^2}=6.928$

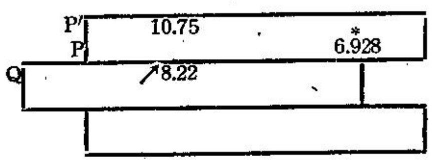


Fig. 107

Example 16 $\sqrt{21.6^2-8.6^2}=19.82$

If given numbers do not lie within the range of P and P' scales, individual number multiplied by 1/2 and then computed in the form of $\sqrt{10.8^2-4.3^2}=9.91$ as in the previous example.

Finally, answer is given by mentally as

$$9.91 \times 2 = 19.82$$

Example 17 $6.8\,e^{j25.6°}=6.13+j\,2.94$

The following relation is used, because this is nothing but the conversion of vector in polar coordinate system to rectangular coordinate system.

$$Ae^{j\theta}=A\cos\theta+jA\sin\theta$$

Set right index of the slide to 6.8 on A scale as shown by Fig. 108. Move indicator to 25.6° in red and 25.6° in black on $S\theta°$ scale. Read $A\cos\theta=6.13$ and $A\,\mathrm{Sin}\,\theta=2.94$ under the hairline of indicator. Then Answer is $6.13+j2.94$.

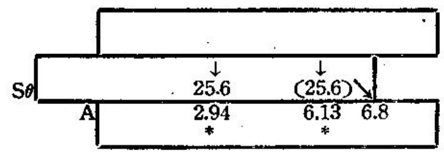


Fig. 108

(Note) Figures in parenthesis indicate red figures in the above diagram.

Example 18 $7.8\,\underline{|0.38}=7.245+j\,2.895$

Since angle is given in Radian, set 7.8 on CI scale to 0.38 on S_x scale.

Opposite the left index on C scale, read the imaginary part 2.895 on D scale.

Set 2.895 on Q scale to 7.8 on P scale.

Opposite zero on Q scale, read the real part 7.245 on P scale.

Example 19 $3.45+j8.24=8.933\,\underline{|67.3}\,°$

This is the example of conversion of $a+jb$ to polar coordinate system.

Computation may be done from $A=\sqrt{a^2+b^2}$ and $\theta=\tan^{-1}\dfrac{b}{a}$.

Find $\sqrt{3.45^2+8.24^2}=8.933$ by the use of P and Q scales.

Find $\frac{a}{b}$ on D scale as shown by Fig. 109.

Opposite the value of $\frac{a}{b}$ on D scale, read angle $\theta=67.3°$ in red on $T\theta$ scale.

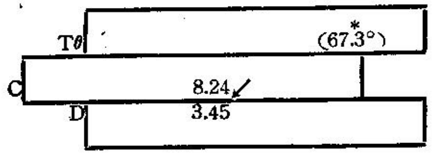


Fig. 109

(Note) Since it is noted to solve this example that $\frac{b}{a}>1$ and $\theta>45°$, answer is read in red figure.

Example 20 $6.32+j\,2.56=6.82\ \underline{|22.05°}$

Different way of operation is explained in this example. Set two indicators, attached to this slide rule to 6.32 and 2.56 on A scales.

The slide is slowly moved to the left until angle $S\theta°$ in red and black figure which are given under the hairline of indicators reach the same value. This same angle 22.05° is the phase angle of vector. Opposite the right index of $S\theta°$, read 6.82 on A scale which is the absolute value of the vector.

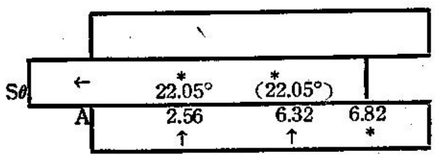


Fig. 110

(5) Hyperbolic Function

Hyperbolic Function

Logarithmic scales of hyperbolic function, Th and Sh are provided on the slide for computation of hyperbolic function.

Method of operation of this slide rule is almost the same as No. 255. Since these scales are on the slide, answer is given by the shifting of indicator with referred to C scale, or by setting the slide exactly in the stock the answer is given by the shifting of indicator with referred to D scale.

Example 21 $\tan h\,(0.522)=0.4792$

$\sin h\,(0.428)=0.4415$

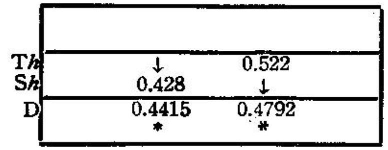


Fig. 111

To find $\cos hx$, solution be given from the formula $\cos hx=\frac{\sin hx}{\tan hx}$

If $\sin hx<10$, answer will be given simply by the use of P and P' scales as example 22.

Example 22 Let $\sin hx=0.48$, find $\cos hx$.

Ans. 1.1092

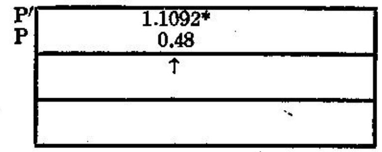


Fig. 112

Hyperbolic function of complex angle

Hyperbolic function of complex angle $a+jb$ is given from the following formula:

$$\sin h\,(a+jb)=\sin ha\cdot\cos b+j\cos ha\cdot\sin b$$

$$\cos h\,(a+jb)=\cos ha\cdot\cos b+j\sin ha\cdot\sin b$$

To find in the form of polar system.

$$\sin h\,(a+jb)=\sqrt{\sin h^2a+\sin^2 b}\ \underline{|\tan^{-1}(\tan b/\tan ha)}$$

$$\cos h\,(a+jb)=\sqrt{\sin h^2a+\cos^2 b}\ \underline{|\tan^{-1}(\tan b.\tan ha)}$$

Example 23 $\sin h\,(0.82+j\,1.2)=0.3314+j\,1.263$

Find complementary angle $\frac{\pi}{2}-1.2=0.3708$ of angle 1.2

Set the right index of C scale to 0.3708 on Sx scale.

Opposite 0.82 on Sh scale, read real part 0.3314 on D scale.

Then opposite 0.82 on Sh scale, read $\sin h\,(0.82)=0.915$ on C scale.

Find $\cos h\,(0.82)=1.3654$ by the use of P and P' scales.

Set the right index of C scale to 1.2 on Sx scale.

Opposite 1.3554 on C scale read imaginary part 1.263 on D scale.

Example 24 $\qquad \cos h\,(0.82+j1.2) \fallingdotseq 0.491+j0.8525$

From the above equation

$$\cos h\,(0.82+j1.2)=\cos h\,(0.82)\cos(1.2)+j\sin h(0.82)\sin 1.2$$
$$=\cos h\,(0.82)\sin(0.3708)+j\sin h(0.82)\sin 1.2$$
$$=1.3554\times 0.3627+j0.915\times 0.933$$
$$=0.491+j0.8525$$

Computation of Line constants

As an example of application of hyperbolic function in electrical engineering, computation of admittance y, impedance Z per unit length is shown under the following assumption.

One end of line opened.

Admittance Y_o measured from the other end of a line.

Impedance Z_s measured from the other end with one end short circuited.

In case of above condition, relation exists as follows:

$$Y_o=\frac{\tanh \xi D}{R},\quad Z_s=R\tanh \xi D.$$

From the formula $y=\frac{\xi}{R}\quad Z=\xi R$

Required line constants are determined.

Example 25

Find unit line constant per km from the following values, with line length of 77.9 km under 60 cycles.

$$Y_o=212.25\times 10^{-6}\angle 90 \text{ mho}$$
$$Z_s=72.61\ \angle 71.8°$$

R and ξD are computed from above constants as follows.

$$R=\sqrt{\frac{72.61}{212.25}}\times 10^{3}\angle\overline{9.1°}=585\angle\overline{9.1°}$$
$$\sqrt{Y_oZ_s}=\sqrt{72.61\times 212.25}\times 10^{-3}\angle 80.9°=0.01964+j0.1220$$
$$e^{2(x+jy)}=\frac{1.01964+j0.1220}{0.98036-j0.1220}=\frac{1.026\ \angle 6.83°}{0.9875\angle\overline{7.10°}}=1.039\ \angle 13.93°$$
$$y=\frac{13.93°}{2}=6.965°=0.12155$$
$$x=1.1513\log_{10}1.039=0.01905$$
$$\xi D=0.01905+j0.12155=0.1231\ \angle 81.12°$$
$$\xi=1.579\times 10^{-3}\angle 81.12°$$

From these values

$$Z=R\xi=0.924\ \angle 72.02°=0.2852+j0.8795 \text{ ohms/km}$$
$$y=\frac{\xi}{R}=2.699\times 10^{-6}\angle 90.22° \fallingdotseq j2.699\times 10^{-6}\text{mho}$$

Gauge marks

This slide rule, provides the following gauge marks.

C (1.128 of C D scales on Rear face) calculation of circle area.

π (3.14159 of C D scales on Rear face) Ratio of circumference of circle to its diameter.

IX. No. 153 (10 inches) Duplex Slide Rule
(For Electrical engineers)

(1) General Description (See figure on page 107)

(Characteristics)

This slide rule has almost similar scale arrangement to No. 154 rule.

And enabling not only to perform calculations of Vector problem, Hyperbolic Functions, but also Higher exponent.

Back bone of scales on this rule is an Un-logarithmic scale P (Q), and in addition to this, it has Log Log scales for Higher exponent Calculation, and scale $G\theta$ for Hyperbolic functions as well as ordinary scales for Multiplication and Division.

Naturally the usage of this rule is more wider than that of No. 154 rule, that is: it can deal with the form $\sqrt{a^2 \pm b^2}$ which shows the absolute value of given vector, reading values of sine cosine and tangent for the angle in any measure, degree system or radian system at the same time as well as all the hyperbolic functions. Furthermore, it is able to perform Calculations of the form e^x which is very important for studies on alternating theory, characteristics of long transmission electric line and problems of electric oscillation, etc.

Scale arrangement and their usage

Front face.

L	......Logarithms
K	......Cubes and Cube roots
A B	......Squares and Square roots
CI C D	......Multiplication, Division and Proportion
T	......Trigonometric tangent and Hyperbolic sine.
Gθ	......Angle of Gudermann.

Back face:

θ Rθ	......Angles in degree or radian measure
P Q Q'	......Absolute value of Vector.
C, LL_3, LL_2, LL_1	......Higher exponent

(2) Multiplication and Division

This rule is provided *CI*, *C* and *D* scales for ordinary Multiplication and Division.

Therefore, if we consider merely above mentioned matter, this rule some what inferior comparing to other rules which are provided with folded scales *CF*, *DF* and *CIF*.

Multiplication

Example 1 $3\times4\times5=60$

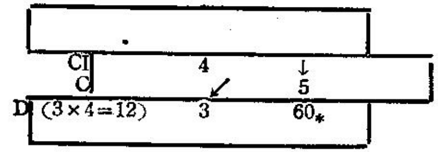


Fig. 113

Fig. 113 shows the case of successive multiplication of 3 factors, and the product of 2 factors, for instance $3\times4=12$, can only be obtained by slide operation as shown in parenthesis in the same figure.

In the second operation, you will often meet with the "Off scale". In such a case, you must set hairline to the sequence of preceding slide operation, and again proceed the calculation newly by slide operation.

Division

Example 2 $8\div5\div4=0.4$

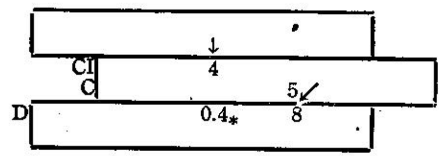


Fig. 114

For the case of "off scale" in the second operation. you must treat it just the same to the case of multiplication above mentioned.

Combined multiplication and division

Example 3 $\frac{3\times6}{5}=3.6$

There are two procedures for this calculation, in the orders of $3\div5\times6$ and $3\times6\div5$.

You should judge which procedure is more efficient from the stand point of lessening the slide shifting.

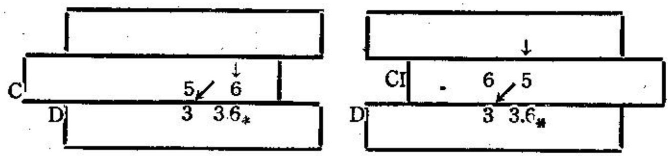


Fig. 115

As the methods of operating *C* and *D* for direct proportion and that of *CI* and *D* for inverse proportion, we have explained them for several times already in the chapters regarding instruction of other rules.

(3) Square and Cubes

This slide rule is provided with *A, B* scales for calculations of squares and square roots and *K* scale for those of cubes and cube roots. These operations are fully explained in III-(3) and IV-(3).

(4) Logarithm and Higher exponents

Operations for Common logarithm using *L* scale, and those for Natural and Common logarithms using *LL* scale are the same as mentioned in IV-(4).

And the operation for Higher exponents using *C* and *LL* scales are also the same as that in IV-(4).

(5). Trigonometric functions

This slide rule is different from other rules for calculations of trigonometric functions and has to use Angle scale θ (for degree measure) or Rθ (for radian measure) in cooperation with un-logarithmic scales, so called Square scales, *P* and *Q*. Fig. 116 shows the general relation of it.

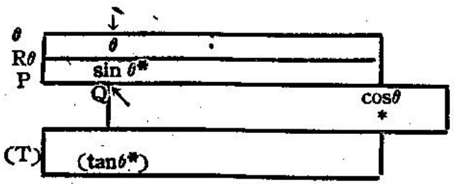


Fig. 116

Symbols in parenthesis mean that is on the opposite face of the slide rule.

Example 4 $\sin 25° = 0.422$

$\cos 25° = 0.906$

$\tan 25° = 0.466$

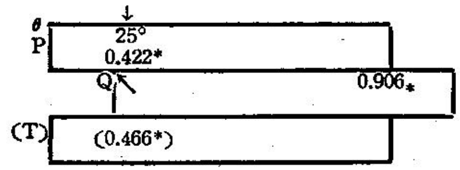


Fig. 117

Example 5 $\sin 0.9 = 0.783$

$\cos 0.9 = 0.622$

$\tan 0.9 = 1.260$

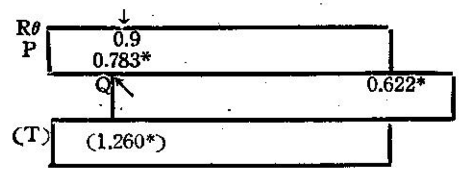


Fig. 118

(6) Vector calculations

Absolute value

Absolute value of the vector which is given in form of $A+jB$ can be obtained from the equation $\sqrt{A^2+B^2}$, This calculation will be done using *P*, *Q* and *Q'* scales as easily as that of ordinary multiplication.

Example 6 Get the absolute value of $20+j15$.

Set 0 on *Q* scale against 15 on *P* scale, then we can read off the answer $Z=25$ on *P* scale against 20 on *Q* scale.

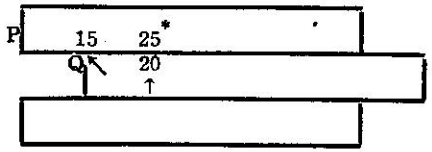


Fig. 119

Example 7 $\sqrt{38^2+95^2}=102.3$

If we perform this calculation as the example 6, then, we shall see that *P* scale becomes "Off scale".

So, we must take one of the other convenient processes as follows

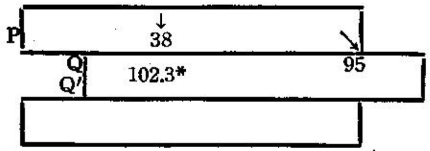
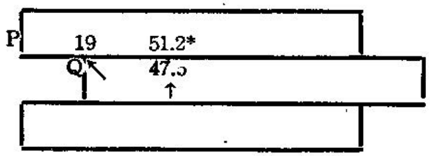


Fig. 120

In case you do not use *Q'* scale, multiply or divide the given numbers by a simple digit as 2 or 1/2 so as to be treated them in the range of *P* and *Q* scale, and its sequence must be reduced by reverse treatment to get the right answer (Ref. Fig. 121)



$38\div 2=19$

$95\div 2=47.5$

$51.2\times 2=102.4$

Fig. 121

Phase angle

The phase angle θ between the real part *A* and the absolute value of given vector which is represented by the form $A+jB$, can be solved as, $\theta=\tan^{-1}\frac{B}{A}$. For instance, in the example 6, $\theta=\tan^{-1}\frac{15}{20}=36.9°$.

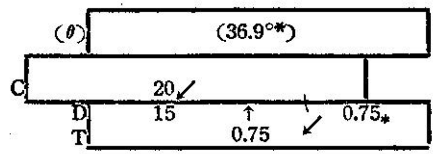


Fig. 122

Firstly, calculate $\frac{15}{20}$ using *C* and *D* and replace its sequence 0.75 on *T* scale by the aid of indicator. Then, we can read $\theta=36.9°$ on θ scale on the back face under the hair line.

Conversion of measure of angle

Using scales θ and $R\theta$ in "Reference scales", we can convert the measure of angles, degree into radian and vice versa.

Example 8 Convert 42° into radians

" 1.005 radian into degrees

Ans. 0.733 radian

57.6 degree

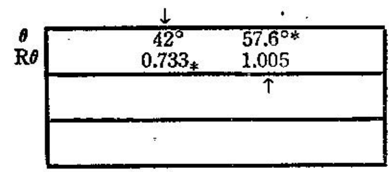


Fig. 123

Conversion of coordinates

Summerizing above mentioned techniques, we can convert the coordinates of given vector of the form $A\pm jB$ into that of $Z\underline{|\theta}$, and vice versa.

Example 9 Convert $6.85+j\,22.8$ into polar coordinates.

In convenience, multiply every term of given numbers by 2, and operate as shown in Fig. 124.

Dividing its sequence 47.6 by 2, we get the absolute value $Z=23.8$

For the Calculation of phase angle, firstly divide 22.8 by 6.85 using *C* and *D* scale.

Replacing its sequence on *T* scale we get $\theta=73.3°$ on θ scale.

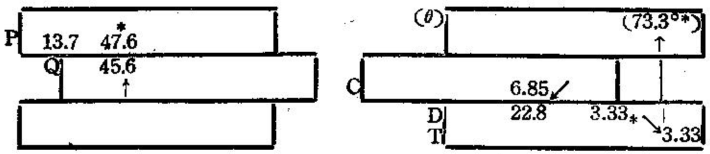


Fig. 124

Example 10 Convert $1.407 \underline{|0.594}$ into rectangular coordinates.

Calculate $\sin\theta=0.559$ and $\cos\theta=0.829$ by cooperative use of $R\theta$, P and Q and then, by ordinary multiplication using C and D, we get as the real part of the vector $1.407\times0.829=1.17$ and the imaginary part $1.407\times0.559=0.786$.

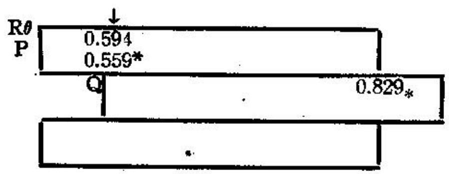


Fig. 125

(7) Hyperbolic functions

To get hyperbolic functions, $G\theta$ scale is available. If you set the given argument x on $G\theta$ scale you can get the value of sin hx on T scale, and that of tan hx on P scale under the hair line of indicator.

Example 11 $\sin h\,0.32=0.325$

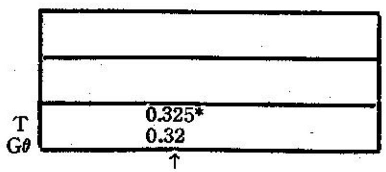


Fig. 126

Example 12 $\tan h\,0.83=0.681$

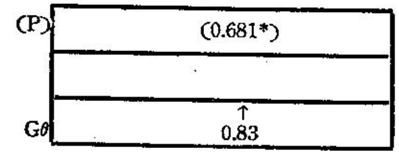


Fig. 127

Example 13 $\cos h\,0.55=1.155$

Procedure 1

Firstly using $G\theta$, get $\sin h\,0.55=0.578$ on T scale. Then, replacing this sequence on Q scale, and against it, get the answer on Q' scale as 1.155.

Procedure 2

By the aid of hair line, set the left index of C to 0.55 on $G\theta$ scale. Against the right index of P, we get sec$h$ $0.55=0.866$ on Q scale. Finally, its reciprocal $\cos h\,0.55=1.155$ can be read easily using C and CI scales.

Hyperbolic function of complex argument

Hyperbolic functions of complex argument as $\sin h\,(a\pm jb)$ and $\tan h\,(a\pm jb)$, will be obtained from formulas as shown V-(6) or VIII-(5).

(8) Application on Electrical problems

Example 14

Compute the effective value of a voltage wave which contains higher harmonics, assuming effective values of its component as follows.

The fundamental wave $E_{e1}=82$

The 3rd harmonics $E_{e3}=25$ and

The 5th harmonics $E_{e5}=12$

$E_{e0}=\sqrt{E^2_{e1}+E^2_{e3}+E^2_{e5}}=\sqrt{82^2+25^2+12^2}=86.6$

Set 0 on Q to 82 on P, move the hairline to 25 on Q, set 0 on Q to the hairline, and then, we can read the answer 86.6 on P against 12 on Q.

Example 15

compute the current in an electric circuit, which impedance is $2+j3.2$ and the potential difference between its terminals is $50+j15$.

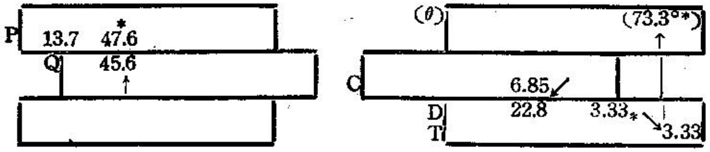


Fig. 124

Example 10　　Convert 1.407 $\underline{|0.594}$ into rectangular coordinates.

Calculate Sin θ = 0.559 and Cos θ = 0.829 by cooperative use of *R*θ, *P* and *Q* and then, by ordinary multiplication using *C* and *D*, we get as the real part of the vector 1.407 × 0.829 = 1.17 and the imaginary part 1.407 × 0.559 = 0.786.

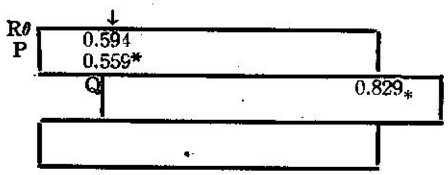


Fig. 125

(7) Hyperbolic functions

To get hyperbolic functions, *G*θ scale is available. If you set the given argument *x* on *G*θ scale you can get the value of sin *hx* on *T* scale, and that of tan *hx* on *P* scale under the hair line of indicator.

Example 11　　sin *h* 0.32 = 0.325

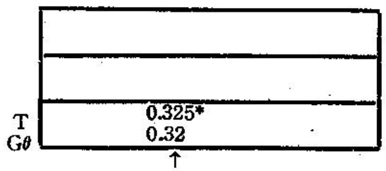


Fig. 126

Example 12　　tan *h* 0.83 = 0.681

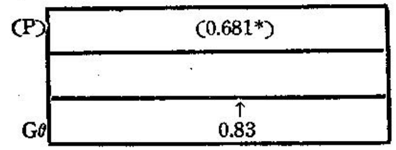


Fig. 127

Example 13　　cos *h* 0.55 = 1.155

Procedure 1

Firstly using *G*θ, get sin *h* 0.55 = 0.578 on *T* scale. Then, replacing this sequence on *Q* scale, and against it, get the answer on *Q'* scale as 1.155.

Procedure 2

By the aid of hair line, set the left index of *C* to 0.55 on *G*θ scale. Against the right index of *P*, we get sec*h* 0.55 = 0.866 on *Q* scale. Finally, its reciprocal cos *h* 0.55 = 1.155 can be read easily using *C* and *CI* scales.

Hyperbolic function of complex argument

Hyperbolic functions of complex argument as sin *h* (*a*±*jb*) and tan *h* (*a*±*jb*), will be obtained from formulas as shown V-(6) or VIII-(5).

(8) Application on Electrical problems

Example 14

Compute the effective value of a voltage wave which contains higher harmonics, assuming effective values of its component as follows.

The fundamental wave　　$E_{e1}=82$

The 3rd harmonics　　$E_{e3}=25$ and

The 5th harmonics　　$E_{e5}=12$

$E_{e0}=\sqrt{E^2_{e1}+E^2_{e3}+E^2_{e5}}=\sqrt{82^2+25^2+12^2}=86.6$

Set 0 on *Q* to 82 on *P*, move the hairline to 25 on *Q*, set 0 on *Q* to the hairline, and then, we can read the answer 86.6 on *P* against 12 on *Q*.

Example 15

compute the current in an electric circuit, which impedance is 2+*j*3.2 and the potential difference between its terminals is 50+*j*15.

$$\dot{I}=\frac{\dot{E}}{\dot{Z}}=\frac{50+j15}{2+j3.2}$$

Representing both numerator and denominator in a polar coordinates, we have

$$50+j15=52.2\ \underline{|0.29}$$
$$2+j3.2=3.77\ \underline{|1.011}$$

and then

$$\dot{I}=\frac{52.2}{3.77}\ \underline{|0.29-1.011}=13.85\ \overline{|0.721}$$

If necessary, the answer can be converted into rectangular coordinates as below:

Using P and Q scales,

Sin 0.721 = 0.661

cos 0.721 = 0.751

and multiplying on these sequences by the absolute value 13.85, we get

$$\dot{I}=10.4-j9.15$$

Gauge marks:

This slide rule has gauge marks as follows:

π, 2π (on C, D scales)...The ratio of circumf. to the diameter of a circle.

c (on C scale) ...Area of a circle.

ρ°, ρ', ρ'', (on C scale) ...Conversion between degree and radian.

X No. 200 (16 inches) Duplex Slide Rule

(Four figures slide rule)

(1) General Description (See figure on page 108)

Characteristics

This slide rule has been specially designed to meet with the requirement of approximate calculation for four figures, and the function of this rule is limited to multiplication, division and proportion. Principle of this slide rule is based on the system to improve the accuracy of calculation with "long scale" idea.

One unit length of logarithmic scale of 6×16"=96" is divided into 6 equal parts, and $D_1D_2D_3D_4D_5D_6$ scales on the stock and $C_1C_2C_3C_4C_5C_6$ on the slide are arranged.

Groups of folded scales such as $DF_1DF_2DF_3DF_4DF_5DF_6$ and $CF_1CF_2CF_3CF_4CF_5CF_6$ which are folded at the middle point of individual scales on front face respectively, are arranged on the rear face, in order to avoid reading falls off the scale.

In case of using this slide rule, it should be noted that real meaning of **approximate calculation of four figures** ought to be understood.

Approximate calculation of four figures means the fourth digit is not clear. In an extreme case such as 2×3=6, if answer is 6, it is absolutely accurate answer. But on the standpoint of approximate calculation of four figures, it may be allowable if answer is 6.0004 or 5.9995. We have to call attention that if this fact were not fully considered, misunderstanding on the accuracy of the slide rule would liable to be caused.

(2) Multiplication and Division

Example 1 65×43=2795

Set indicator to 65 on D_5 scale on front face of the stock.

Set index of the slide exactly under the hairline.

Move indicator to 43 on C_4 scale of the slide.

Read answer 2795 on D_3 scale under the hairline.

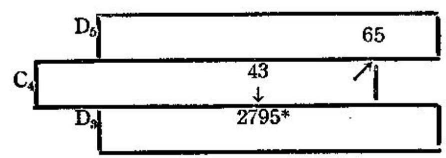


Fig. 128

Example 2 2.94×4.8=14.112

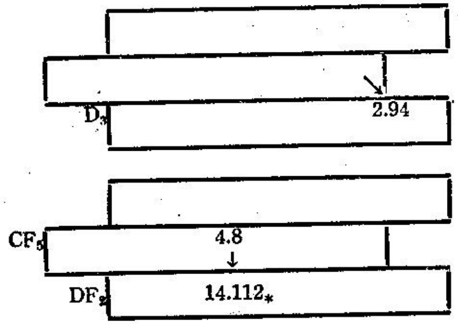


Fig. 129

In this example, rear face is used in order to avoid falling off the scale.

(Note) To find out the scale on which answer is read off, it is advisable to roughly estimate by approximate calculation by the first significant figure as a reference.

Example 3 $\qquad 97{,}968.75 \div 1{,}375 = 71.25$

Dividend is rounded up 4 figures such as 9797 and operated as shown by the following Fig.

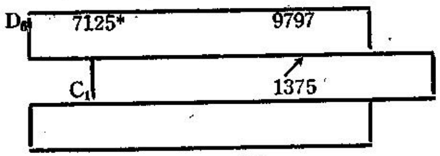


Fig. 130

Example 4 $\qquad \dfrac{6.583 \times 7.427}{4.692} = 10.420$

In the form of combination of multiplication and division, division is made at first and next the multiplication to simplify the operation.

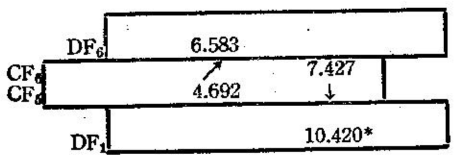


Fig. 131

(3) Proportion

Calculation of proportion which is one part of multiplication and division is nothing but of "the reference scale method". As a typical example, percentage calculation is explained as follows:

Example 5 $\quad$ Find percentage of the following areas.

Asia	43,260,000 Sq. km.	32.22%
Europe	10,190,000	7.59
North America	24,040,000	17.90
South America	18,240,000	13.58
Africa	30,000,000	22.34
Oceania	8,560,000	6.37
Total	134,290,000	100.00

At first, assume that area is set on the slide and percentage on the stock. Set 13429 on C_1 scale to the right index (100) of the stock on front face. And successive operation is available by the use of "reference scale method" through shifting the indicator.

Percentage to areas are computed by the use of both faces as shown by Fig. 132.

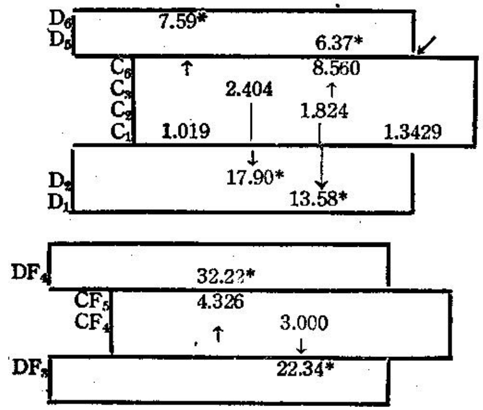


Fig. 132

APPENDIX

(1) Explanatory Remarks for Biginners

It is the purpose of this chapter to explain fundamentals of slide rule concisely for biginners.

Slide Rule is a rough estimating instrument.

Population of Tokyo prefecture is assumed about 3,490,000 in 1945.

Find what percent of the total population of 72,180,000 in all Japan is in the particular year.

$$\frac{349}{7218}=0.04835\ldots\ldots$$

This question only requires to find an approximate percentage, but not to compute many digits as shown in the previous example. Answer may be enough by 4.8%. Round up figure like this is called approximate value. Slide Rule is an instrument to make the multiplication and division of an approximate value. It would take about 15 seconds to get an answer for the previous example by the use of slide rule, while it would take one minute, if computation were made by writing. Therefore, the purpose of using a slide rule is to make calculation efficiently. That is to say "Slide rule is an instrument to perform rough estimation quickly."

Now we explain what is the construction of slide rule. A simple slide rule consists of two scales, one on the slide and the other on the stock, of which the slide moves on the stock and enables us to make computation.

An instrument to read scales on the stock and slide at the same time, so called cursor or indicator which slides freely on the stock and slide to the left or right direction is commonly provided.

Let us try to study an addition and subtraction in order to understand what is the meaning of "to compute by scale."

Now if one of two ordinary scales graduated with centimeter is slid on the other as shown in the following Fig., an addition such as $2+3=5$ is performed by the use of scales with graduation of equal division.

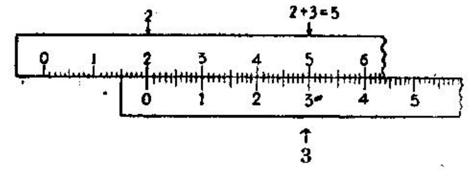


Fig. 1

You may easily understand that a subtraction can be carried out by the reverse procedure of an addition. The following Fig. shows a subtraction of $6-4=2$.

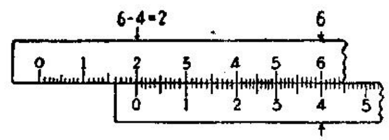


Fig. 2

Don't you ask question why a multiplication and division can't be performed in the same way as addition and subtraction. This kind of question is very invaluable and it may be said that this is the mother of slide rule invention.

The special case of multiplication and division through the use of scale with graduation of equal division is shown in the following Fig.

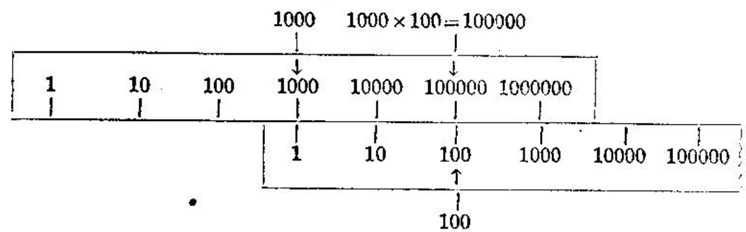


Fig. 3

As shown in the above Fig. multiplication and division of numbers corresponding to any power of 10 can be worked out by addition or subtraction of scale with graduation of equal division.

Subdivision of secondary space is necessary for ordinary multiplication and division. One of these subdivided secondary space of graduation is a fundamental of slide rule and is commonly called D scale.

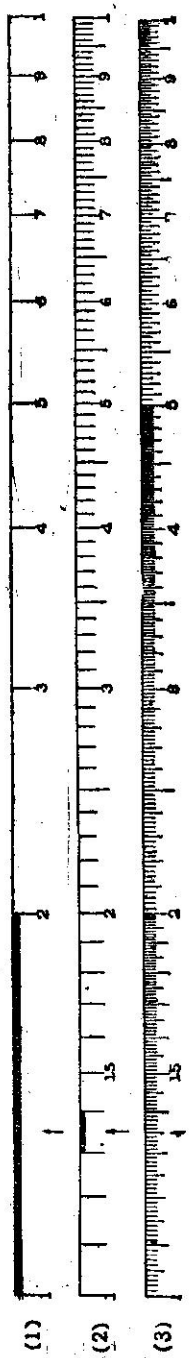


Fig. 4

Theoretically, graduation of secondary space is entirely the same in all prime spaces. It may be said that practically the prime space of 1–10 represents all other spaces.

How to read D scale:

The prime space of scale for Multiplication and division as explained in the previous article is divided into 9 sub-divisions. As in Fig. 4 graduation of scale is not equal division, but is wide at the left and gradually becomes narrow at the right. But the method of reading scale is the same as ordinary scale. (The prime space). This just like the centimeter graduation of centimeter scale. As explained in the previous article, this graduation represents also of other prime spaces which are Multiplied or divided by 10^n.

Therefore there is not definite unit. For example, graduation with numbering of 2 may be read as 2, 20, 200 or 0.2, 0.02 etc. by usage. Reading the scale without taking into consideration the decimal point is called "to read the significant figures". The first line of each prime space is numbered, and these lines represent the first significant figure. Each of these prime spaces is divided into 10 parts, which is called the secondary space.

The graduation of the secondary space corresponds to milimeter graduation of centimeter scale and this represents the second significant figure ((2) of Fig. 4).

Furthermore each of these secondary space is subdivided into several parts of tertiary space which is classified three kinds according to width of graduation.

For example the tertiary space of 10 inches slide rule is shown in Fig. 4. The secondary space between 1 and 2 of the prime space is divided into 10 parts

which represent the third significant figure; 5 parts between 2 and 5 (sometimes between 2 and 4 by kind of slide rule) which represent the third significant figure corresponding to even number: 2 parts between 5 and 10, which represent the third significant figure corresponding to 5.

In order to find significant figure such as 135 (1.35, 13.5, 0.135) by divided parts of graduation, read the first significant figure 1 between prime 1 and 2 as shown by heavy line in (1) of Fig. 4; read the second significant figure 3 between secondary 3 and 4 as shown by heavy line in (2) of Fig. 4; finally read the third significant figure 5 by tertiary line as shown by arrow in (3) of Fig. 4. In line with above mentioned reading procedure of graduation, range of space is gradually reduced until the last significant figure being decided.

If the approximate number of four digits such as 1358 is requested, the fourth significant figure 8 is determined by inspection taking approximately 80% of the space between 135 and 136. This inspection method is always applied to reading of an ordinary scale.

How to use slide rule:

The use of slide rule means computation through the shifting of the slide and indicator. Shifting of the slide and indicator is called slide operation and indicator operation respectively.

The purpose of slide operation is to bring a graduation of scale on the slide opposite a graduation of scale on the stock. And the purpose of indicator operation is to set a figure under the hairline. It is the principle of slide rule to perform multiplication and division by physical addition and subtraction of graduation through these two kinds of operations.

Elementary slide rule operation:

We explain a simple example of multiplication and division operation, according to the principle explained in previous paragraph.

Example 1

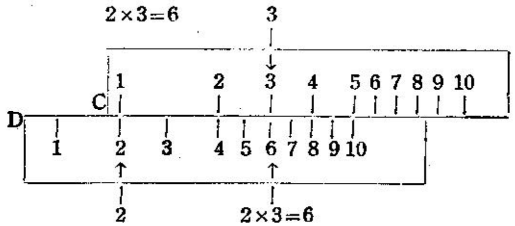


Fig. 5

Example 2

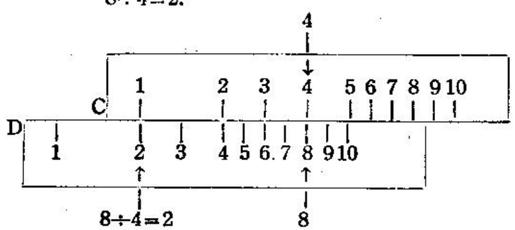


Fig. 6

Although the principle is indicated in Fig. 5 and 6, to make snap computation which is the original function of the slide rule, efficient methods of operation have been studied, as explained in series of chapters in this manual.

We would like to call reader's attention that slide rule is an instrument to work out rough estimation snappy.

The repeated practice of slide rule operation is an only way to be skillful as well as operation of other instrument. It would be no exaggeration to say there is no principle or theory. We recommend you to make repeated practices according to the next article "How to practise slide rule operation."

(2) Operating Practice of Slide Rule

(1) Should there be any person, who thinks he has already mastered the slide rule operation only by study of it's principle and theory, it would not make sense. Since a slide rule is an instrument, practice to master it is absolutely necessary. The course and method of operating practice for those who desire to master the slide rule operation to some extent in order to utilize slide rule in daily business calculation is explained in detail in this chapter.

(2) Operating practice is divided into following three kinds:

(a) To be familiar with mechanical handling of slide rule.

(b) Snap reading of graduation.

(c) To be familiar with operation to meet with form to be computed.

Item (a) will be accomplished by practising the shifting of indicator and the slide with slide rule in hand even when you do not make any calculation. We explain the method of practice in two parts; indicator and slide operations.

Indicator operation

(1) Indicator is moved with tip of thumb.

(2) For instance, to set the indicator to some graduation from right to left direction, indicator is roughly moved with tip of right thumb and then finely adjusted with the aid of left thumb as shown in Fig. 7 when the hairline approaches the graduation you wish to set.

(3) Gradually set the hairline on the point at the last step. It should be so practised that the indicator is set on the point at single movement, without moving back and forth.

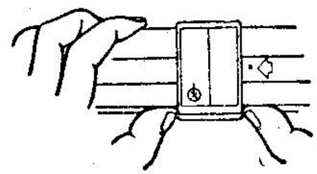


Fig. 7

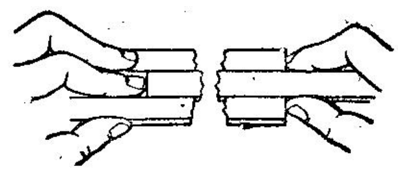


Fig. 8

Slide Operation

(1) Push one end of the slide with fore finger (thumb is used for holding slide rule) and thumb and middle fingers are slid along outside edges of the stock.

(2) At the same time another thumb and fore finger are put on one end of the stock and hold it lightly. (Fig. 8)

(3) When the position of the slide comes closely to the point, the slide is slightly moved by spring action of tip of fingers squeezing the slide by both fingers which are putting on the end of the stock.

(4) It is always necessary that finger tip which pushed the end of slide never be left hold of the slide rule all the time.

(5) Repeat this operating practice from right and left direction alternatively.

Reading of Graduation

It would be much effective for reading practice that setting or reading of graduation is made by the indicator or slide operation.

Operating practice at the dictation of instructor

We need an instructor to carry out this practice.

(1) Instructor gives an order "get set". Slide rule is put on the desk and ready for starting practice.

(2) Number is set on the scale previously determined (for instance *D* or *DF* scale) in response to the instructor's dictation. Complete full operation until the next reading begins.

(3) Instructor has to adjust the speed of reading in order to raise up the time for one operation from 5 and 3 seconds.

(4) In case of slide operation, indicator is set on 3, approximately the middle point of *D* or the center index of *DF* scale and then graduation of the scale on the slide (*C*, *CI* or *CIF*) is set under the hairline by moving the slide.

(5) It is desirable that numbers at the dictation are read starting from simple figures to complicated ones as shown in the following article.

(6) Take actual record of individual operation. It would be a great help in checking the progress of slide rule operation.

Example of dictation

(1)	145	585	168	364	206	545	152	625	282	795
(2)	1265	383	235	657	1605	572	273	742	398	827
(3)	5125	2055	6075	1437	7075	2112	6135	2728	7815	3556

Note: (3) is much useful for quick estimation by the eye.

Practice on the form of calculation:

Multiplication and Division, proportion, square, cube, trigonometric function, etc. can be worked out by the operation explained in this Instruction book.

Understanding of operating procedure only would not be useful for practical operation of slide rule.

Unless the movement of finger tip as well as physical calculating posture in quick response to the calculating form is accomplished, it can't be said that you mastered the slide rule operation.

Of course the practice is only way to be skilful in slide rule operation. But it would be more effective to repeat practice of about 5 examples of same degree than to work combination of different kinds of numbers.

Anyhow multiplication and division are the fundamental of slide rule operation. Typical examples of these operations are shown as follows:

Each problem consists of four values including answer. Practise operation until you finish each example in about 20 seconds.

(1) Successive multiplication	(2) Successive division
$4.5\times3.2\times5.8=83.6$	$8.7\div4.8\div7.7=0.235$
$1.3\times6.5\times2.8=23.7$	$2.55\div4.33\div3.5=0.1683$
$8.5\times1.35\times6.33=72.7$	$12.0\div2.78\div1.4=3.08$
$3.5\times2.55\times1.86=16.6$	$3.7\div9.8\div4.2=0.0899$
$\pi\times0.38\times1.66=1.982$	$4.22\div0.28\div3.88=3.89$

(3) Combination of multiplication and division

$$\frac{3.45\times2.87}{4.44}=2.23$$

$$\frac{1.28\times6.5}{4.85}=1.715$$

$$\frac{7.8\times1.65}{3.8}=3.39$$

$$\frac{\pi\times6.8}{4.55}=4.70$$

$$\frac{1.25\times4.87}{1.55}=3.93$$

(Note) It is desirable to operate so that the slide movement is lessened, when you compute above example by slide rule operation.

(3) How to select slide rule

Hemmi's slide rule is divided into many kinds. Since individual kind of slide rule provides specific purpose and characteristics, it is very important to select a slide rule to meet with your requirement.

If you investigate how to select a slide rule in the following order, you never regret to find that your slide rule is not adequate to your work, after purchasing it.

(i) Remember the general characteristics of symbols of scales indicated in Table 1.

There are many kinds of scales to be used for slide rule, but their purposes are definite. It is the first step in selecting slide rule that what kind of scale should be provided for the slide rule to meet with your usage. Explanation pertaining to ordinary scales is shown by Table 1.

(Table 1)

Kind of calculation	Name of Scale	Remarks
Multiplication Division and Proportion	*C D*	These are the fundamental scales of slide rule. Multiplication and division are freely available.
	CI	This is a supplementary scale for *C*, *D* scales. Speed for computation will be increased about 40% with this scale.
	CF DF	There are two kinds, one is $\sqrt{10}$-fold and the other is π-fold. Speed of computation is increased and also troublesome labour is eliminated.
	CIF	Supplementary scale of *CF* and *DF* scales.
Square and Cube	*A B*	These are used for computation of square and square root in joint use with *C* and *D* scales.
	K	This is used for computation of cube and cube root in joint use with *C* and *D* scales.
Logarithm	*L*	This is the scale to find logarithm in "reference scale" with *D* scale.
	l	This is the scale to find logarithm in "reference scale" with *A* scale

Kind of calculation	Name of Scale	Remarks
Trigonometric function	*S SI*	These are used for Sin θ computation
	T, TI_1 TI_2	These are used for Tan θ computation
	S&T ST	These are used for computation of Sin θ and tan θ of small angle.
Exponent	LL_0 LL_1 LL_2 LL_3	These are the scales to find form of A^x in joint use with *C* scale.
	LL/0 LL/1 LL/2 LL/3	These are the scales to find form of A^{-x} in joint use with *C* scale.
Vector	*P P' Q Q'*	These are used for computation of absolute value $\sqrt{a^2 \pm b^2}$ of vector.
	θ *x R*θ	These are graduated in degree and minute system, and also Radian system. These are available for supplementary use of vector computation.
		Note: Vector computation is made with use *SI* and *TI* scales also.
Hyperbolic Function	*G*θ	This is used for computation of hyperbolic function by use of Gudermanian.
	Sh_1 Sh_2	These are used for hyperbolic sine calculation.
	Th	This is used for hyperbolic tangent calculation.

(ii) Select slide rule with most convenient arrangement of scales which is useful for your computations frequently used.

Find out No. of slide rule with most suitable scale for your form of computations which is frequently used, after determination of characteristics and symbols of scales. Table 2 is indicated for this purpose. Convenience or inconvenience depends upon the arrangement of scales, even though scale is the same.

For instance, there are 5 steps of advantage priorities as shown by the Table 2 for computation of trigonometric function. Therefore you have to make a thorough study to use it.

Table 2 indicates classification by kind of form of computation and by advantage priority. In the right side of table, major slide rule with those

arrangement of scales are listed.

Table 2

Form of Computation	Advantage Priority	Scale and arrangement	Remarks	Mannheim Type 10″	Mannheim Type Pocket Type	Mannheim Type 20″	Duplex Type 10″	Duplex Type 20″
Multiplication, division and proportion	1	Provides *CD* and *CI*	Speed of computation is 40% quicker than slide rule with *CD* scales only.	50 64 80 130 2640 (8″) 40*RK*	30 32 34*RK* 74 66 86 136	70	153	
	2	Provides *CD*, *CI* and *CF*, *DF*	None off scale, calculation speed increases about 20% than (1)	2664 45 (8″)	2634			154
	3	Provides *CD*, *CI*, *CF*, *DF* and *CIF*	Highest class for multiplication and division.				250 251 255 256 259	275 279
Square	1	Provides *A* only on the stock	A little disadvantage for multiplication and division, including square and square root.	2664 2690 45 (8″)	2634		256	154
	2	Provides *A* and *B*	Convenient for computation, including square and square root.	50 64 80 130 2640 (8″) 40*RK*	30 32 34*RK* 74 66 86 136	70	250 251 255 259 153	275 279
Cube	1	Provides *K* on the Slide	Cube and Cube root computation is available, but a little Complicated.	2640 (8″)				
	2	Provides *K* on the Stock	Convenient for cube and cube root	50 64 2664 130 40*RK*	34*RK* 2634 74 136 66	70	250 251 255 256 259 153	154 275 279
Logarithm	1	Provides *L* on the rear face of the slide	Slide operation is necessary.	50 80 2664 45 (8″) 40*RK*	30 32 34*RK* 2634 86			
	2	Provides *L* on the stock	Done only by indicator operation, convenient for logarithm of decimal fraction.	64 130 2690 2640 (8″)	74 66 136	70	250 251 255 256 259 153	154 275 279

Trigonometric Function	1	Provides *S*,*T*	Trigonometric function of Sin (6°–90°) and tan (6°–45°) are found.	50 80 130 40*RK* 2640 (8″)	30 32 34*RK* 86 136			154
	2	Provides *S*, *T*, *S* & *T*	Trigonometric function of angle from 6° to 35′ is also found.	64 2690	74 66	70	251 259	279
	3	Provides *SI* and *TI*	Computation including trigonometric function. Very convenient for solution of triangles and vectors.	45 (8″)			256	
	4	Provides TI_2, TI_1, SI_1	Tangent in item 3 is extended for angle from 45° to 84°	2664	2634		255	275
	5	Provides *SI*, *TI*, *DI*	The same as item 3. Especially convenient for sine proportion				250	
Exponent	1	Provides LL_3, LL_2, LL_1	Convenient for computation of N^x or $N^{\frac{1}{x}}$	130 80	86 136		153 251 255 256	275
	2	Provides LL_3, LL_2, LL_1, LL_0, $LL/3$, $LL/2$, $LL/1$, $LL/0$	Range of computation in above item enlarged to N^{-x} and $N^{-\frac{1}{x}}$				259	279
Hyperbolic Function	1	Provides *Gθ*	Sin*hx* and tan*hx* are found, but multiplication and division of these values are not available				153	
	2	Provides Sh_1, Sh_2, *Th*	Convenient for multiplication and division to hyperbolic function				255	154 275

(iii) Final decision should be made after investigating type and dimension suitable for usage.

The slide rule, which has most convenient arrangement of scales for the purpose of own use, is not limited to one kind. For example, there are difference in dimension and types such as slide rule for laboratory use with high accuracy or for pocket use which is convenient as portable type. Major Hemmi's Slide rules are shown in Table 3. Make your final decision what kind of slide rule you select. Cuts of typical slide rules out of Table 3 are shown in Appendix 4. Those may be used for reference in selecting your slide rule.

(Table 3)

Type	Size	Cat. No.	Scale: Front face	Scale: Rear face	Characteristics
Mannheim Type	10″	50	A B CI C D K	S L T	Mannheim slide rule for general engineer
		64	K A B CI C D L	S S&T T	Rietz system with extension of red graduation on C, D, A, B.
		80	LL_2 A B CI C D LL_3	S L T	For electrical engineer with efficiency and voltage drop scales.
		2690	L D sin cos, $\cos^2$ C D A	S S&T T	For civil surveying use with Stadia scales
		130	L K A B CI C D cos sin tg	LL_1 LL_2 LL_3	Darmstadt slide rule with cos scale and log log scales
		2664	K DF CF CI C D A	TI_1 TI_2 L SI	$\sqrt{10}$-fold CF, DF and inverted trig. scales
		40RK	A B CI C D K	S L T	No. 50 Propagation type
	8″	2640	L A B CI C D	S K T	No. 50 for Student
		45	DF CF CI C D A	TI L SI	No. 2664 " "
	6″	86	LL_2 A B CI C D LL_3 K	S L T	No. 80 for Pocket use
		66	K A B CI C D L	S S&T T	No. 64 for Pocket use
		136	L K A B CI C D cos sin tg	LL_1 LL_2 LL_3	No. 130 for pocket use

	20″	70	K A B CI C D L	S S&T T	64 for laboratory.
	5″	34RK	A B CI C D K	S L T	No. 50 for Pocket use
		2634	K DF CF CI C D A	TI_2 TI_1 L SI	No. 2664 for pocket use
		74	K A B CI C D L	S S&T T	No. 64 for pocket use
	4″	30	A B CI C D	S L T	Smallest type of No. 50
		32	A B CI C D	S L T	Indicator with lens attached.
	1m	100	DF CF CI C D A	TI L SI	For Professor's use
		101	l A B CI C D	S K T	
Duplex type	10″	250	DF CF CIF CI C D L	K A B TI_1 TI_2 SI D DI	$\sqrt{10}$-fold scale and SI, TI are provided. For general engineering and business use.
		251	L DF CF CIF CI C D DI	K A B T S ST C D LL_3 LL_2 LL_1	π-fold scale is provided. Capable of Exponential Computation. For mechanical engineer
		255	L K DF CF CIF CI C D X θ	Sh_1 Sh_2 Th A B TI_2 TI_1 SI C D LL_3 LL_2 LL_1	Capable of hyperbolic function and exponential computation. For expert electrical engineer
		256	db Neper DF CF CIF CI C D LL_3 LL_2 LL_1	A L̇ Ċ SI TI D λ F	Excellent for electric communication engineer
		259	L K DF CF CIF CI C D LL/0 LL_0	LL/1 LL/2 LL/3 A B T S ST C D LL_3 LL_2 LL_1	Higher exponential computation is possible in broad range. For expert mechanical engineer
		153	L K A B CI C D T Gθ	θ Rθ P Q Q′ C LL_1 LL_2 LL_3	Capable of vector and hyperbolic function. For electrical engineer's use
	20″	275	L K DF CF CIF CI C D X θ	Sh_1 Sh_2 Th A B TI_2 TI_1 SI C D LL_3 LL_2 LL_1	For expert electrical engineer
		279	L K DF CF CIF CI C D LL/0 LL_0	LL/1 LL/2 LL/3 A B T S ST C D LL_3 LL_2 LL_1	For expert mechanical engineer
		154	DF P P′ Q CF CI Sθ A D K	Sx Tx Tθ Th Sh_1 Sh_2 C D L X	For electrical engineering Specialist
	16″	200	C_1 C_2 C_3 C_4 C_5 C_6 D_1 D_2 D_3 D_4 D_5 D_6	CF_1 CF_2 CF_3 CF_4 CF_5 CF_6 DF_1 DF_2 DF_3 DF_4 DF_5 DF_6	Multiplication and division of 4 digits

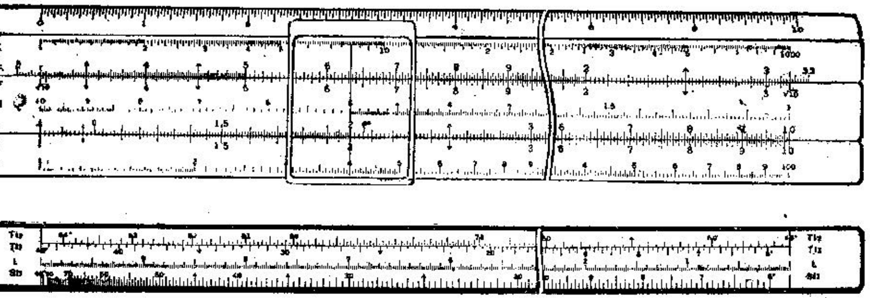
(4) Typical Hemmi's Slide Rule

Cuts indicate arrangement of scale and size for various kinds of typical Hemmi's slide rule are shown below as your reference in selecting slide rule.

(A) Mannheim type slide rule:

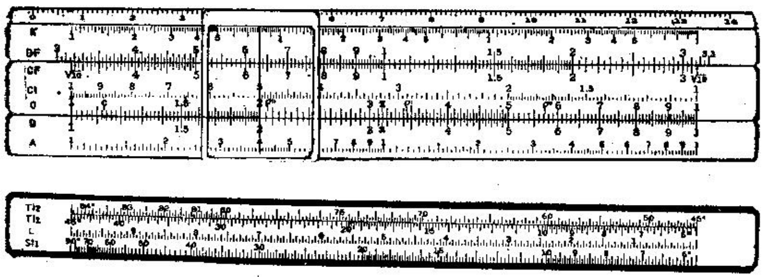
(a) Slide rule for general business use.

No. 2664 Hemmi Bamboo 10″ slide rule



Advantageous feature is that by the use of this slide rule falling off scale is avoided in case of multiplication and division and proportion for general business use.

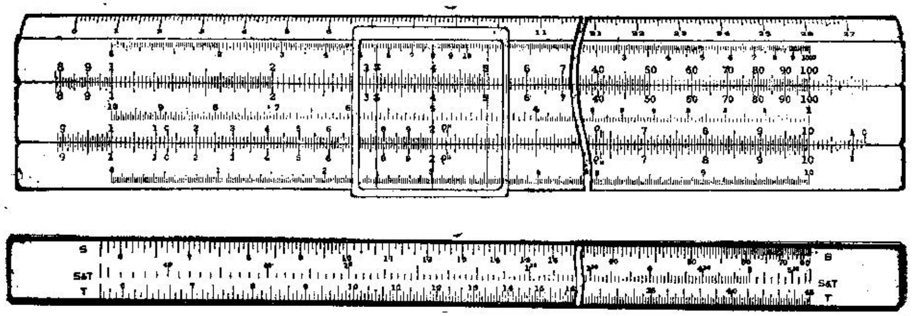
No. 2634 Hemmi Bamboo 5″ slide rule



Pocket type of No. 2664

(b) Slide rule for general engineer's use

No. 64 Hemmi Bamboo 10″ Slide rule

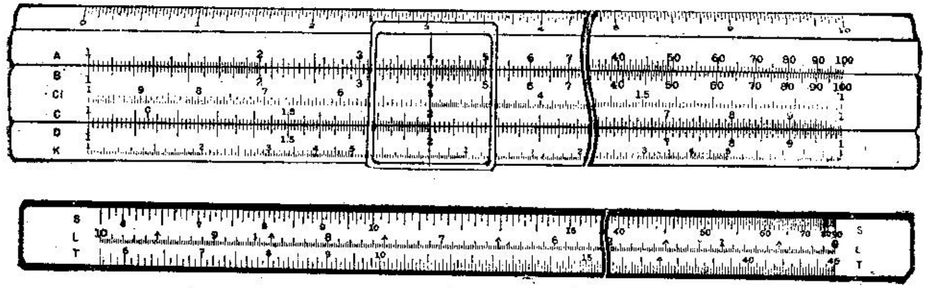


This is so called "Rietz" system with extended red scale on A, B, C, D, and also trigonometric function of small angle is found on S & T scale.

(Similar type)

No. 66 Hemmi Bamboo 6″ slide rule   Pocket tye of No. 64

No. 50 Hemmi Bamboo 10″ slide rule



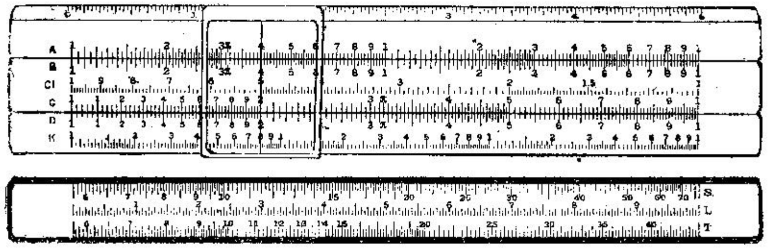
This is advanced type of typical Mannheim slide rule and convenient for square calculation besides multiplication and division.

(Similar type)

No. 40*RK* Hemmi Bamboo 10″ slide rule

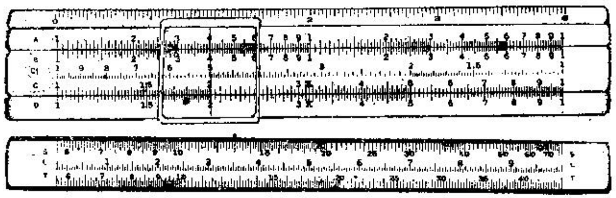
Slide Rule for practice of No. 50

No. 34RK Hemmi Bamboo 5″ slide rule



Pocket type of No. 50

No. 30 Hemmi Bamboo 4″ slide rule



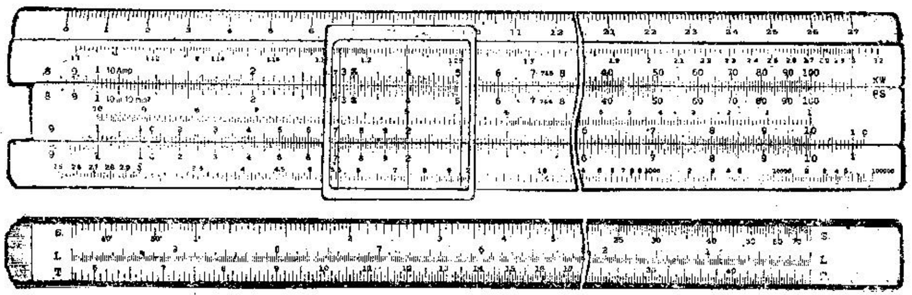
Smallest pocket type No. 50

(Similar type)

No. 32 Hemmi Bamboo 4″ slide rule

No. 30 slide rule with indicator which has lens attachment.

No. 80 Hemmi Bamboo 10″ slide rule (For Electrical Engineer)

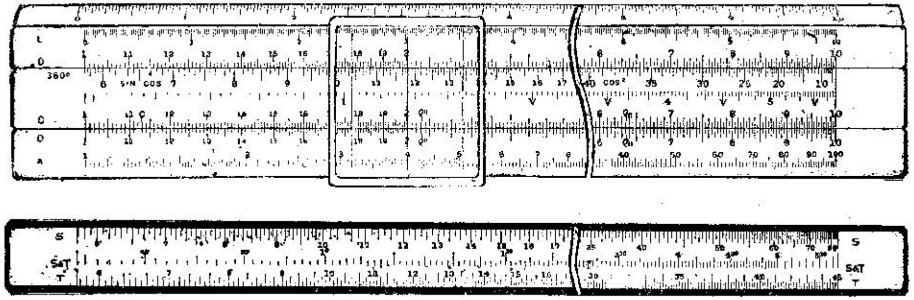


With efficiency and voltage drop scales.

(Similar type)

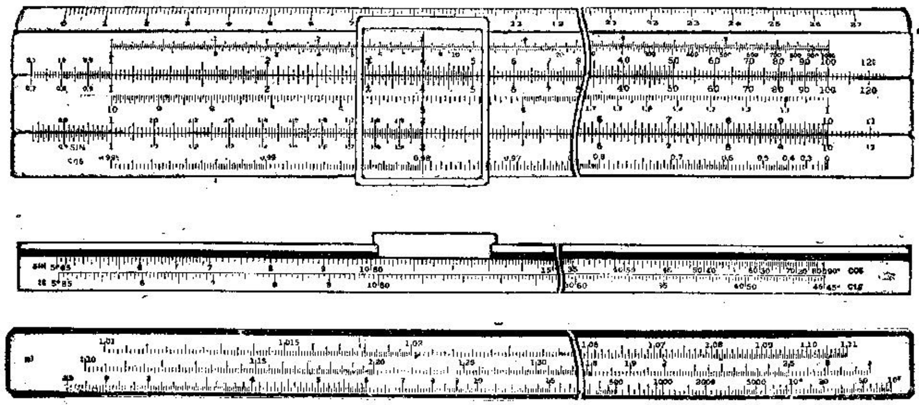
No. 86 Hemmi Bamboo 6″ slide rule. Pocket type of No. 80 with *K* scale.

No. 2690 Hemmi Bamboo 10″ slide rule



Slide rule with Stadia scale for civil surveying use.

No. 130 Hemmi Bamboo 10″ slide rule



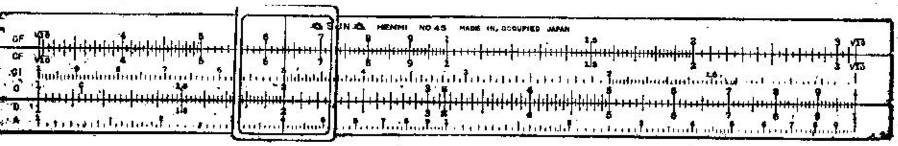
This is popularly used in Europe and is called "Darmstadt" slide rule. $\cos\theta$ scale, besides e^x scale for exponential computation, is advantageous feature of this slide rule.

(Similar type)

No. 136 Hemmi Bamboo 6″ slide rule. Pocket type of No. 130 10″ Slide Rule

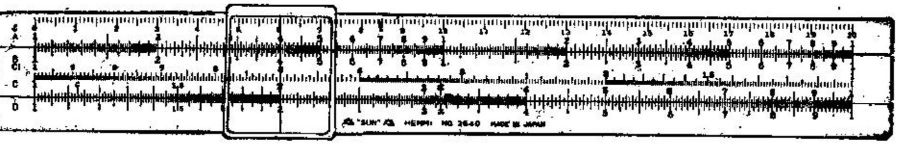
(c) **Beginner's slide rule.**

No. 45 Hemmi Bamboo 8″ slide rule



Beginner's slide rule for No. 2664

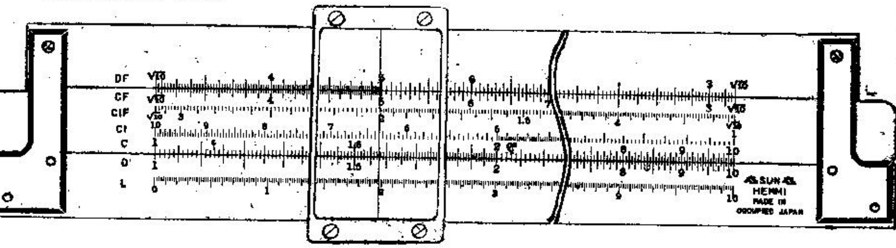
No. 2640 Hemmi Bamboo 8″ slide rule

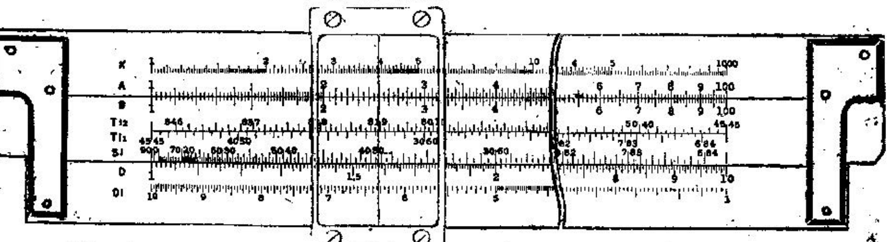


Beginner's slide rule for No. 50

(B) Duplex Slide Rule

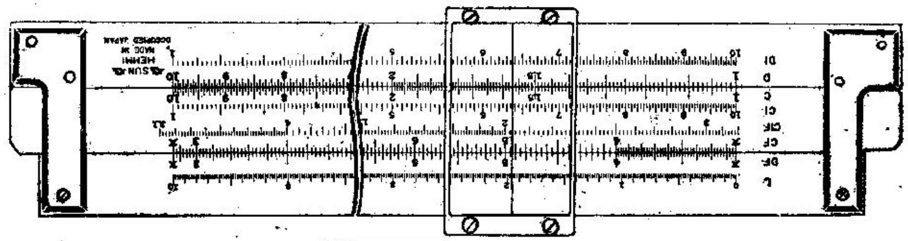
No. 250 Hemmi Bamboo 10″ Duplex slide rule (For general engineering and business use.)

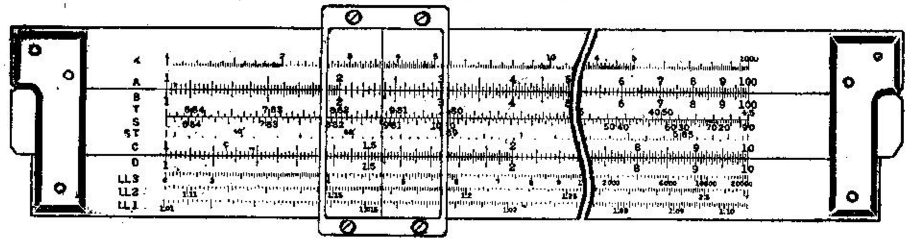




$\sqrt{10}$-fold scale and *SI*, *TI* scales are provided

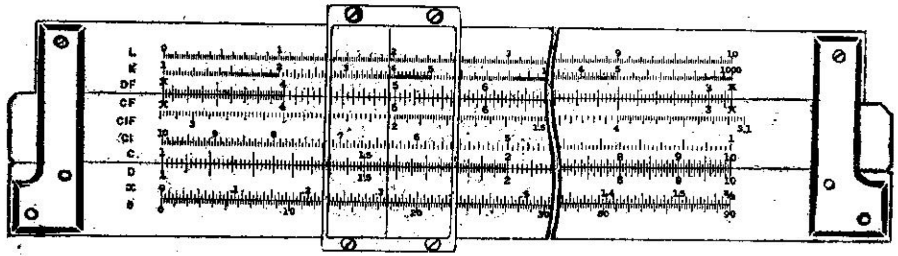
No. 251 Hemmi Bamboo 10″ Duplex slide rule (For mechanical engineer)

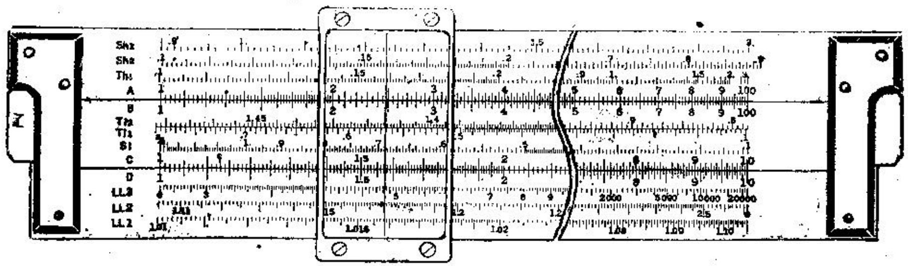




π-fold scale is provided and capable of exponential computation.

No. 255 Hemmi Bamboo 10″ Duplex slide rule (For Expert Electrical Engineer)



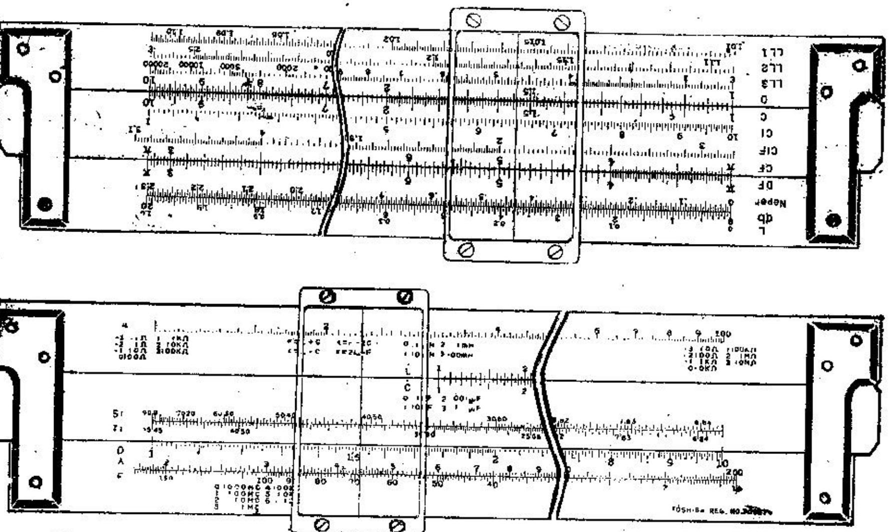


Convenient for hyperbolic and exponential computation.

(Similar type)

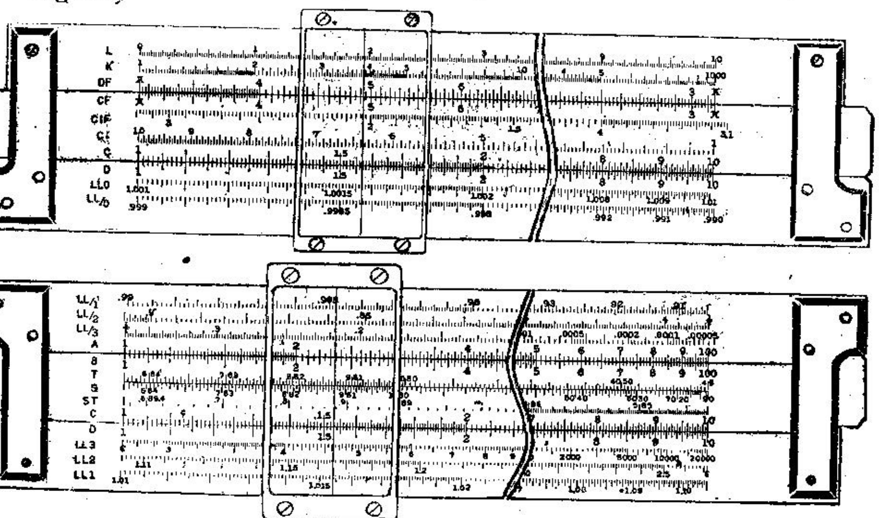
No. 275 Hemmi Bamboo 20″ Duplex slide rule. 20″ slide rule of No. 255.

No. 256 Hemmi Bamboo 10″ Duplex slide rule (For Electric Communication)



Various kinds of scales for expert electrical communication engineer.

No. 259 Hemmi Bamboo 10″ Duplex slide rule (For Expert Mechanical Engineer)

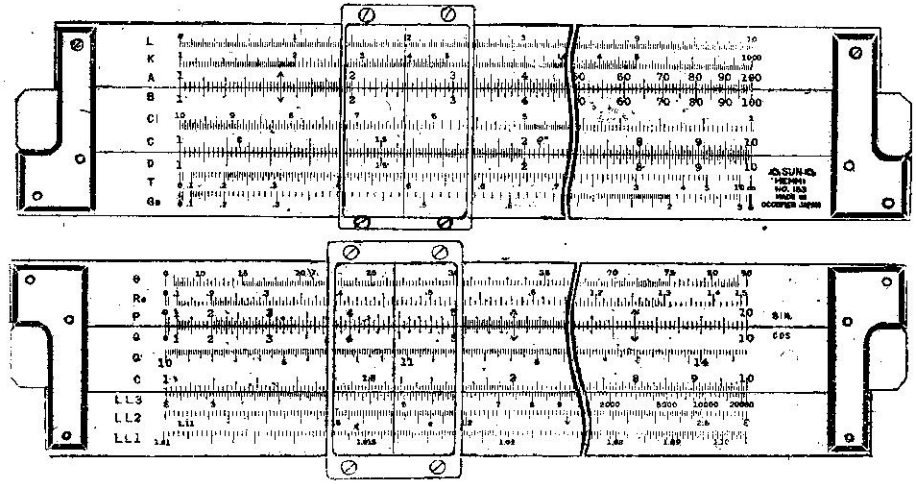


Very convenient for exponential computation.

(Similar type)

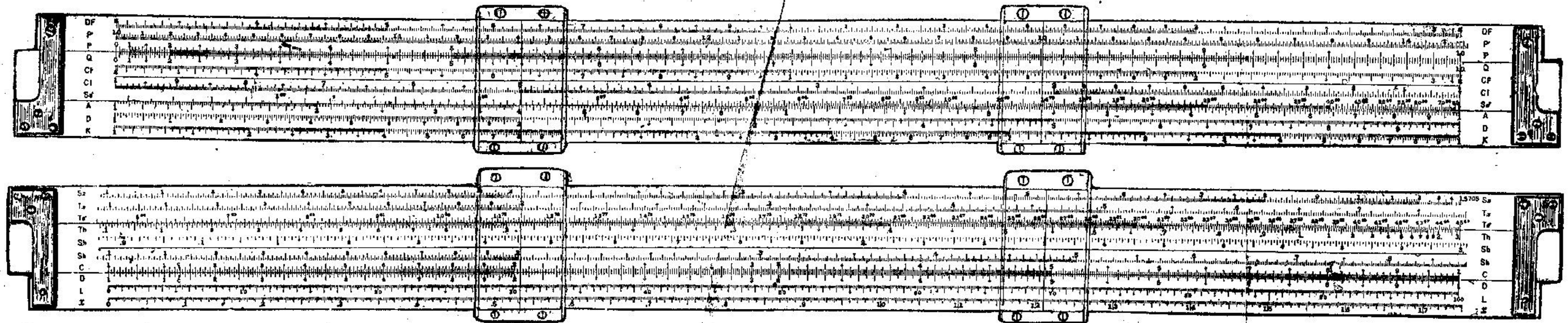
No, 279 Hemmi Bamboo 20″ Duplex slide rule.   20″ slide rule of No. 259.

No. 153 Hemmi Bamboo 10″ Duplex slide rule (For Electrical Engineer)



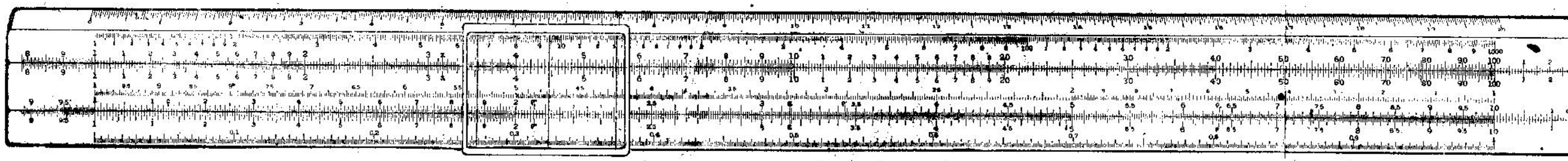
Capable of hyperbolic function and exponential computation.

No. 154 Hemmi Bamboo 20″ Duplex slide rule (For Advanced Electrical Engineer)



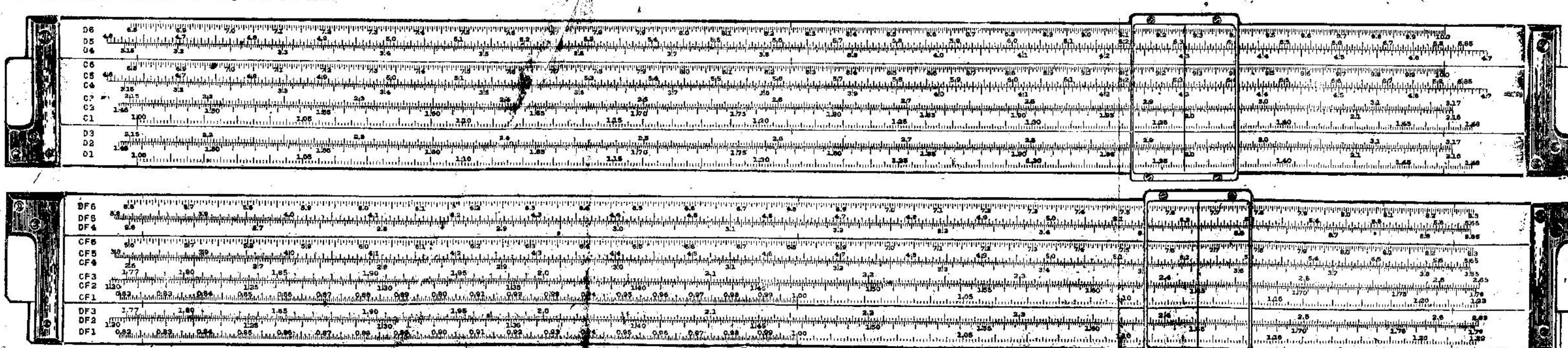
Slide rule which is very suitable for transmission line design.

No. 70 Hemmi Bamboo 20″ Mannheim slide rule



20″ type of No. 64 Slide Rule for laboratory use.

No. 200 Hemmi Bamboo 16″ Duplex Slide Rule



Multiplication and division of 4 digits is possible.