FORMULAS FOR AREA

Rectangle—base × altitude.
Parallelogram—base × altitude.
Triangle—½ base × altitude.
Trapezoid—½ sum of parallel sides × altitude.
Parabola—⅔ base × altitude.
Ellipse—product of major and minor diameters × 0.7854.
Regular polygon—½ sum of sides × perpendicular distance from center to sides.
Lateral area of right cylinder = perimeter of base × altitude.
Total area = lateral area + areas of ends.
Lateral area of right pyramid or cone = ½ perimeter of base × slant height.
Total area = lateral area + area of base.
Lateral area of frustum of a regular right pyramid or cone = ½ sum of perimeters of bases × slant height.
Surface area of sphere = square of diameter × 3.1416.

FORMULAS FOR VOLUME

Right or oblique prism—area of base × altitude.
Cylinder—area of base × altitude.
Pyramid or cone—⅓ area of base × altitude.
Sphere—cube of diameter × 0.5236.
Frustum of pyramid or cone—add the areas of the two bases and add to this the square root of the product of the areas of the bases; multiply by ⅓ of the height: $V = \frac{1}{3} h (B + b + \sqrt{B \times b})$.

IMPORTANT CONSTANTS

$\pi = 3.1416$. $\pi^2 = 9.8696$.
$\sqrt{\pi} = 1.7724$. $1 \div \pi = 0.3183$.
Base of natural logarithms = $e = 2.71828$.
$M = \log_{10} e = 0.43429$.
$1 \div M = \log_e 10 = 2.3026$.
$\log_e N = 2.3026 \times \log_{10} N$.
Number of degrees in 1 radian = $180 \div \pi = 57.2958$.
Number of radians in 1 degree = $\pi \div 180 = 0.01745$.

WEIGHTS AND MEASURES

Avoirdupois Weight
27 11/32 grs. = 1 dram
16 drams = 1 ounce
16 ounces = 1 pound
25 pounds = 1 quarter
4 quarters = 1 cwt.
2,000 lbs. = 1 short ton
2,240 lbs. = 1 long ton

Mariners' Measure
6 feet = 1 fathom
120 fathoms = 1 cable length
7½ cable lengths = 1 mile
5,280 ft. = 1 stat. mile
6,085 ft. = 1 naut. mile

Troy Weight
24 grains = 1 pwt.
20 pwt. = 1 ounce
12 ounces = 1 pound
Used for weighing gold, silver and jewels.

Apothecaries' Weight*
20 grains = 1 scruple
3 scruples = 1 dram
8 drams = 1 ounce
12 ounces = 1 pound
*The ounces and pound in this are the same as in Troy weight.

Long Measure
12 inches = 1 foot
3 feet = 1 yard
5½ yards = 1 rod
40 rods = 1 furlong
8 furlongs = 1 stat. mile
3 miles = 1 league

Square Measure
144 sq. in. = 1 sq. ft.
9 sq. ft. = 1 sq. yd.
30¼ sq. yds. = 1 sq. rod
40 sq. rods = 1 rood
4 roods = 1 acre
640 acres = 1 sq. mile.

Cubic Measure
1,728 cu. in = 1 cu. ft.
128 cu. ft. = 1 cord wood
27 cu. ft. = 1 cu. yd.
40 cu. ft. = 1 ton (shpg.)
2,150.42 cu. inches = 1 standard bushel
231 cubic inches = 1 standard gallon
1 cubic foot = about four-fifths of a bushel

Dry Measure
2 pints = 1 quart
8 quarts = 1 peck
4 pecks = 1 bushel
36 bushels = 1 chaldron

Liquid Measure
4 gills = 1 pint
2 pints = 1 quart
4 quarts = 1 gallon
31½ gallons = 1 barrel
2 barrels = 1 hogshead

Surveyors' Measure
7.92 inches = 1 link
25 links = 1 rod
4 rods = 1 chain
10 sq. chains or 160 sq. rods = 1 acre
640 acres = 1 sq. mile
36 sq. miles (6 miles sq.) = 1 township

Miscellaneous
3 inches = 1 palm
4 inches = 1 hand
6 inches = 1 span
18 inches = 1 cubit
21.8 = 1 Bible cubit
2½ ft. = 1 military pace

DUAL 10"/5" SLIDE RULE

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Published in July, 1944

DUAL 10"/5" SLIDE RULE

GENERAL DESCRIPTION

This slide rule combines the convenience of the 5-inch pocket size rule with the accuracy of the standard 10-inch rule.

On the FRONT FACE ALONE it has all of the standard 5-inch scales—A, B, C, D, S, T, K; and DI. The arrangement is more convenient than on other 5-inch rules because all of these scales are on the same side; it is not necessary to turn the rule over to read sines and tangents; triangles can be solved with a single setting of the slide.

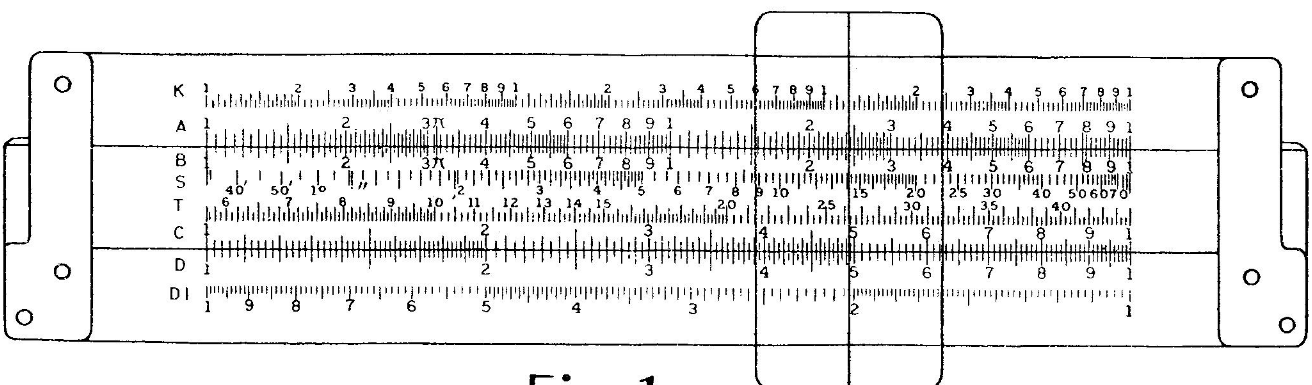


Fig. 1

On the back face there is a standard 10-inch C-D scale combination folded at $\sqrt{10}$. The two parts of C are called C_1 and C_2, and the corresponding parts of D are called D_1 and D_2. This combination gives the same accuracy as the standard 10-inch rule for all problems in multiplication and division, percentage, proportion, etc. There is also an L scale which gives logarithms with 10-inch accuracy and a 5-inch D scale. This scale, used with D_1 and D_2, gives squares and square roots with 10-inch accuracy.

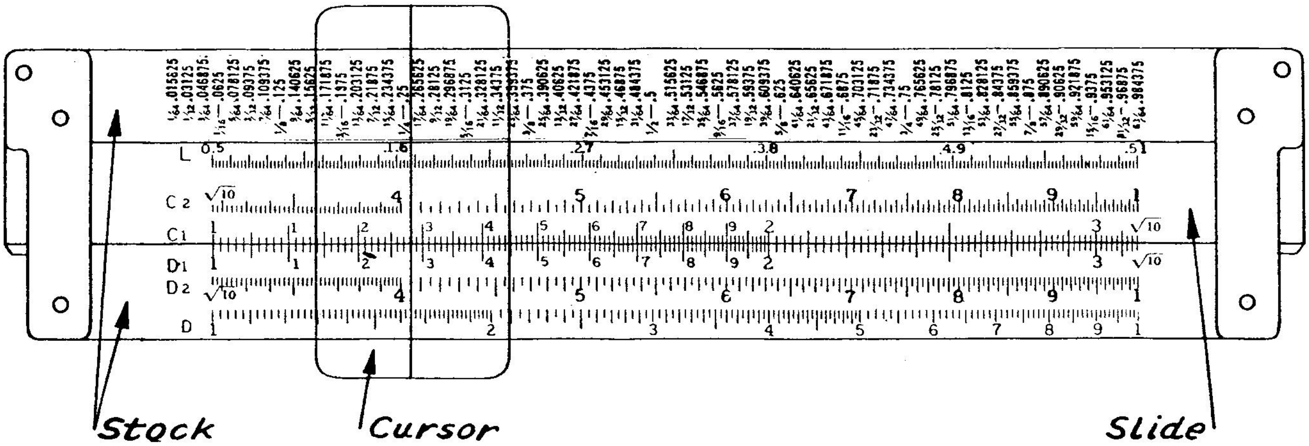


Fig 2

The names of the various parts of the rule are indicated in Fig 2. They are the BODY or STOCK; the SLIDE which moves in the grooves of the stock; and the RUNNER, or INDICATOR, or CURSOR. This cursor is molded of transparent plastic and carries a HAIRLINE or MARKER which is used in making accurate readings and settings.

Locating Numbers on the Scales

1. Reading a Slide Rule Scale. Anyone who knows how to read the scale on an ordinary ruler or yardstick can easily learn to read a slide rule scale. The only essential difference lies in the fact that the calibration marks on a slide rule scale are not uniformly spaced (except in the case of L). Fig. 3, which shows only the primary divisions of the D scale, illustrates this important point. It is much farther, for example, from 1 to 2 than it is from 8 to 9. The spacing is called "logarithmic" and it is based on the theory of logarithms. One does not need to understand this in order to use the slide rule—any more than he needs to know the theory of gasoline engines in order to drive an automobile.

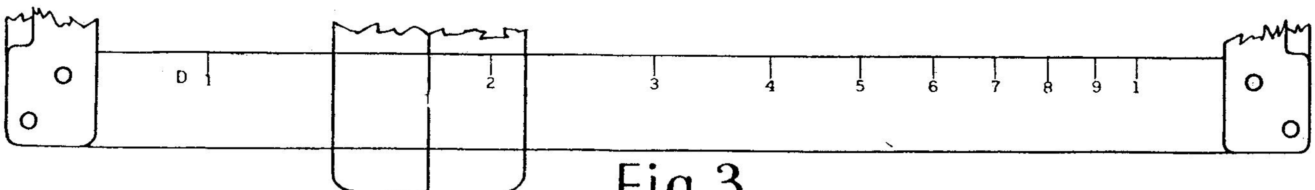


Fig.3

The part of the D scale from 1 to 2 is divided into 10 secondary divisions each representing $\frac{1}{10}$, and each of these is further divided into 5 parts as shown in Fig. 4; each smallest division then represents $\frac{1}{5}$ of $\frac{1}{10}$ which is $\frac{1}{50}$ or 0.02.

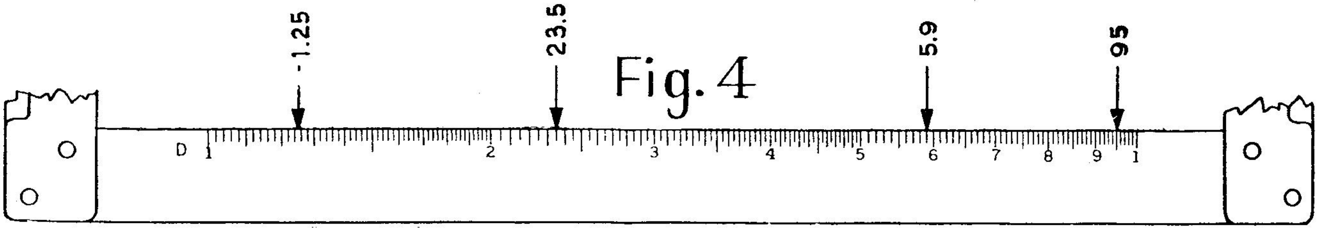
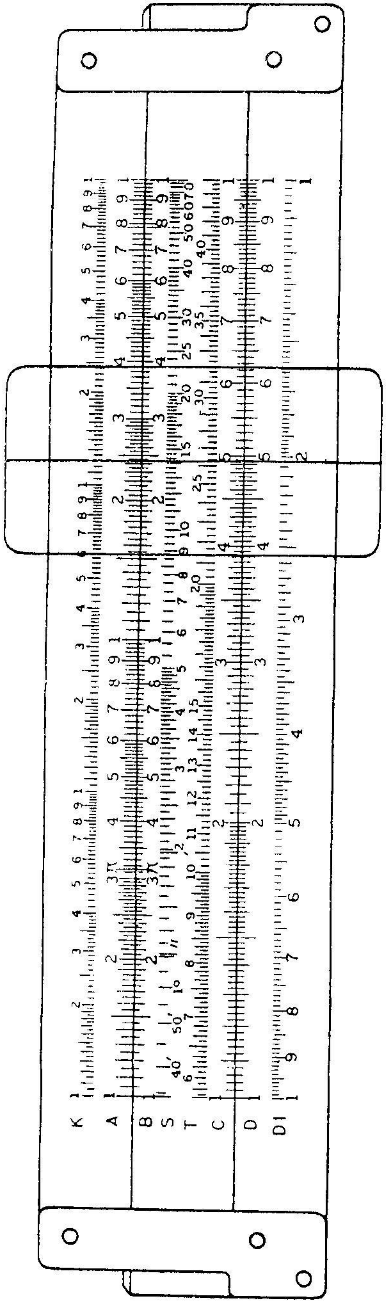


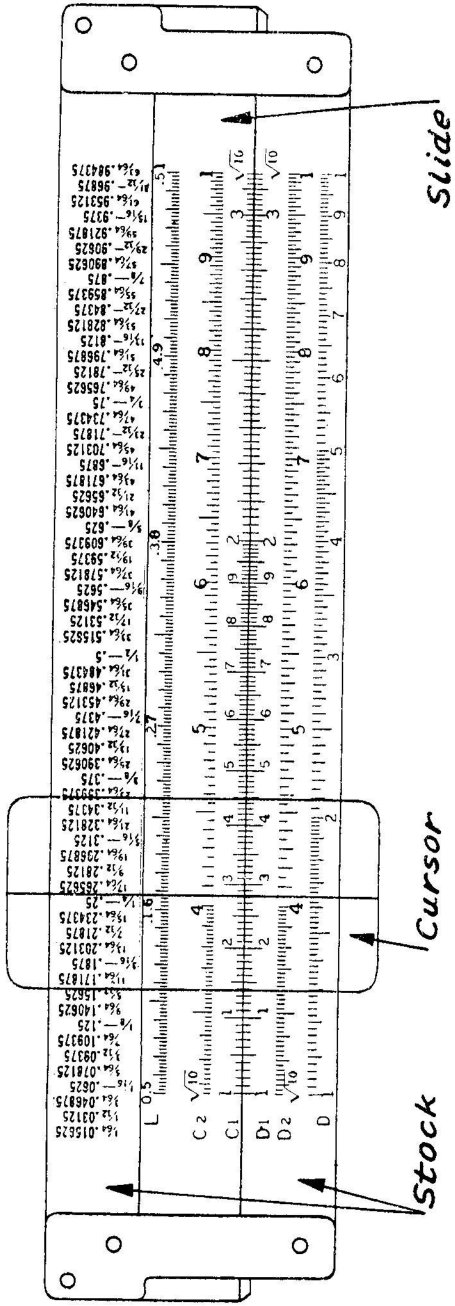
Fig. 4

Between 2 and 5 each primary division is subdivided into 10 parts and each of these is divided into 2 parts; each smallest division then represents $\frac{1}{2}$ of $\frac{1}{10}$ which is $\frac{1}{20}$ or 0.05.

Between 5 and 10 (the right-hand 1 stands for 10), each primary division is divided into 10 parts so each smallest division represents $\frac{1}{10}$ or 0.1.

When a person becomes familiar with this situation for one scale he can easily read any of them.





2. Locating a Number on the D Scale. In locating a number on the scale one disregards the decimal point entirely. Thus the same spot on the scale serves for 1.25, 0.0125, 12.5, 1250, etc. To locate this number one may regard the D scale as running from 1 to 10, the right-hand 1 standing for 10. He may then think of his number as 1.25 regardless of the actual position of the decimal point.

Since the first digit is 1, the number is between main divisions 1 and 2. Since the next digit is 2, the number is between 1.2 and 1.3 and is therefore located between the second and third secondary calibrations. Now, each smallest division in this interval represents 0.02, so the required point is half-way between the second and third of these. Figure 4 shows the location of this point and also shows several others.

Descriptions and Uses of the Various Scales

3. C and D Scales. These scales, which are exactly alike, are the fundamental scales of any slide rule. They are used for multiplication and division, and are also used with the other scales in various operations.

4. A and B Scales. These scales are also exactly alike. Either of them consists of two half-length D scales placed end to end. If an A or B scale is regarded as running from 1 to 100, then the middle 1 stands for 10 and the right-hand 1 stands for 100. They are used in finding squares and square roots and also, with S, for finding the sines of angles.

5. K Scale. This amounts to three C scales, each made to one-third scale and placed end to end. If the K scale is regarded as running from 1 to 1000, then the second 1 stands for 10 and the third 1 stands for 100. It is used in finding cubes and cube roots.

6. S and T Scales. The S scale, operating with A or B, gives the sines of angles. The T scale, operating with C or D, gives the tangents. They are of course used in solving problems of trigonometry.

7. DI Scale. This is an "inverted" D scale. The calibrations run from right to left instead of from left to right. It is used in finding the reciprocal of a number.

8. C_1, C_2, D_1, D_2 Scales. C_1 and C_2 together make a complete 10-inch C scale. C_1 runs from 1 to $\sqrt{10} = 3.1623$ and C_2 runs from $\sqrt{10}$ to 10. The same holds for D_1 and D_2. The numbers on C_1 and D_1 are printed in black while those on C_2 and D_2 are in red. This is for the purpose of simplifying the rule for determining which scale to use in certain operations.

9. L Scale. This is a uniformly graduated scale which, used with C_1 and C_2 (or D_1 and D_2), gives the common logarithms of numbers. The black numbers are used when it is operated with C_1 and the red numbers when it is used with C_2.

10. D Scale on Back Face. Used with D_1 and D_2, this scale gives squares and square roots with the accuracy of a 10-inch rule.

Slide Rule Operations

11. Multiplication using C and D. In what follows, the left-hand 1 of a scale is called its LEFT INDEX; the right-hand 1 is called the RIGHT INDEX.

We multiply two numbers as shown in the following two examples:

Example 1. Multiply 2 × 17.

STEP 1. Opposite 2 on D, set the LEFT index of C.
STEP 2. Opposite 17 on C read the answer (34) on D.

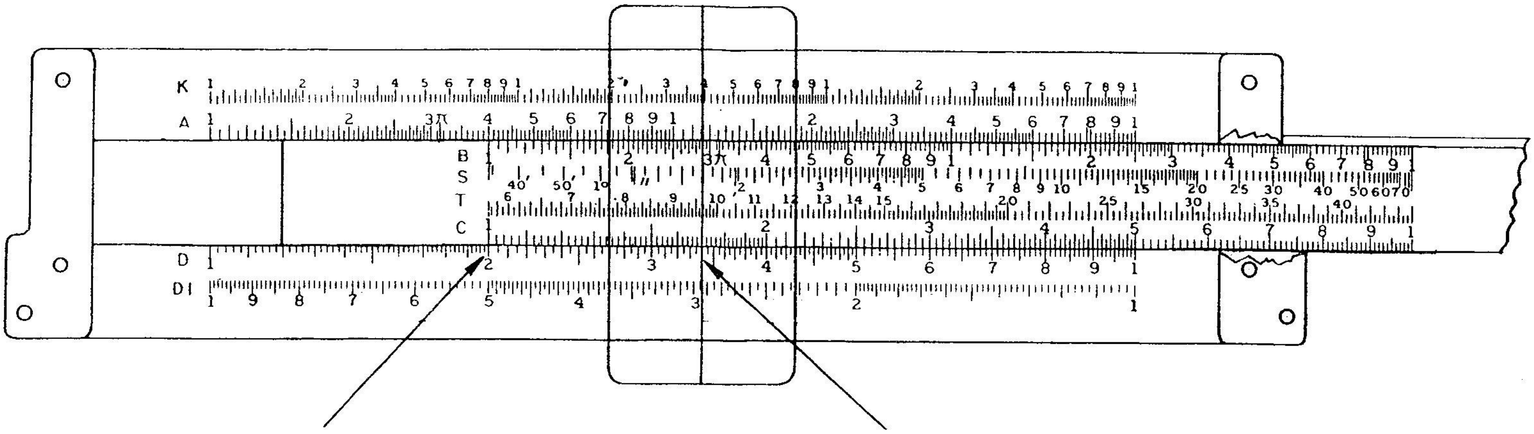


Fig. 5

Example 2. Multiply 5.4 × 0.25.

STEP 1. Opposite 54 on D, set the RIGHT index of C.
STEP 2. Opposite 25 on C, read 135 on D.

The decimal points have been disregarded in this operation. Rough mental calculation shows that the answer must be 1.35. Note in this case that the reading would have been "off scale" if the left index had been used.

These examples illustrate the general rule for multiplying two factors, namely:

STEP 1. Locate one of the factors on D and set either the right or left index of C over it.
STEP 2. Opposite the other factor on C, read the product on D.

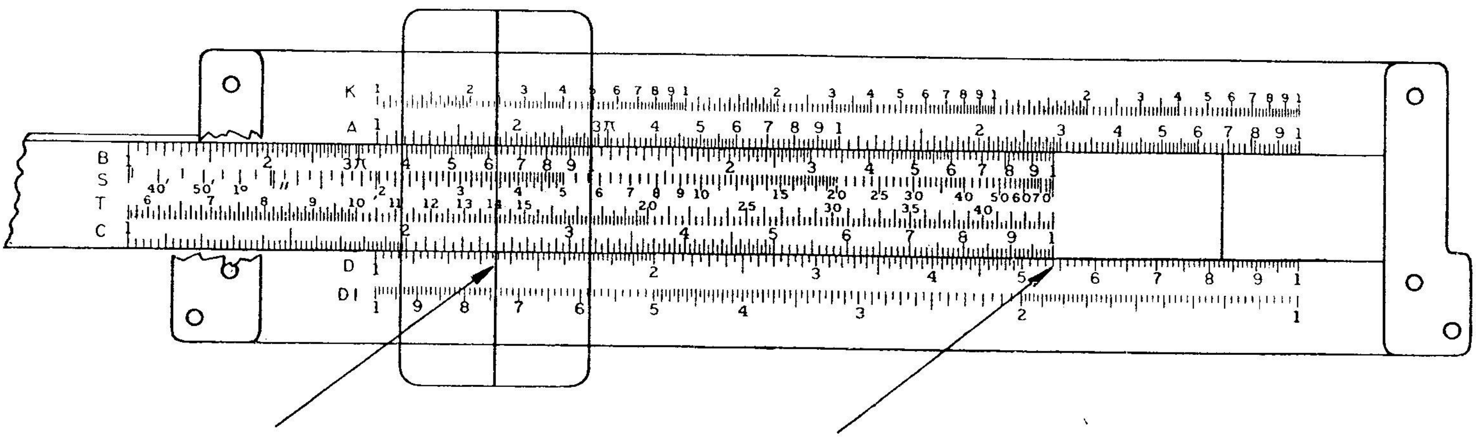


Fig. 6

12. Division using C and D. This operation is the inverse of multiplication, and the division of 34 by 17 is shown by Fig. 5. The steps are:

STEP 1. Opposite 34 on D, set 17 on C. *STEP 2. Opposite the index of C, read 2 on D.*

13. The Number of Digits in a Number. If a number is greater than 1 the number of digits in it is defined to be the number of figures to the left of the decimal point. If a (positive) number is less than 1 the number of digits in it is defined to be a negative number equal numerically to the number of zeros between the decimal point and the first significant figure.

Examples: 746.22 has 3 digits. 0.43 has 0 digits.
3.06 has 1 digit. 0.004 has −2 digits.

Rules can be given for keeping track of the decimal in multiplication and division in terms of the numbers of digits in the numbers but these will not be stressed here. An example is: When two numbers are multiplied using C and D as described in section 11, the number of digits in the product is equal to the sum of the numbers of digits in the factors if the slide projects to the left—and one less than this if it projects to the right.

14. Squares and Square Roots. Opposite any number on D, read its square on A.

Examples:
Opposite 3 on D, read 9 on A. Opposite 1.47 on D, read 2.16 on A.

Conversely, opposite any number on A, read its square root on D. Use the LEFT half of A if the number has an ODD number of digits and the RIGHT half if the number has an EVEN number of digits.

Examples:
Opposite 5 on A (left), read 2.24 on D. Opposite 64 on A (right), read 8 on D.

15. Cubes and Cube Roots. Opposite any number on D, read its cube on K.

Examples:
Opposite 2 on D, read 8 on K. Opposite 4.2 on D, read 74 on K.

Conversely, opposite any number on K, read its cube root on D. Use the right third of K if the number of digits in the number is a multiple of 3 (−3, 0, 3, 6, etc.); use the middle third if the number of digits is one less than a multiple of 3 (−1, 2, 5, 8, etc.); use the left third if the number of digits is two less than a multiple of 3 (−2, 1, 4, 7, etc.).

Examples:
Opposite 2 on K (left), read 1.26 on D.
Opposite 64 on K (middle), read 4 on D. Opposite 125 on K (right), read 5 on D.

16. Reciprocals. Opposite any number on D, read its reciprocal on DI.

Examples:
Opposite 2 on D, read $\frac{1}{2}$ = .5 on DI. Opposite 38.4 on D, read $\frac{1}{38.4} = 0.026$ on DI.

The decimal point is fixed by the rule that if a number which is not a power of 10 has x digits, its reciprocal has 1 − x digits. Thus 38.4 has two digits and its reciprocal has 1 − 2 = −1 digits.

17. Sine of an Angle. Opposite any angle between 34′ and 90° on S, read its sine on B (or A if rule is closed). If the sine is read on the RIGHT half of B the decimal point goes immediately before the first figure. If read on the LEFT half of B there is one zero between the decimal point and the first significant figure.

Examples:
Opposite 28° on S, read 0.47 on B. Opposite 3° 10′ on S, read 0.055 on B.

18. Tangent of an Angle. Opposite any angle between 5° 42′ and 45° on T, read its tangent on D. The decimal point goes before the first figure.

Example:
Opposite 20° 20′ on T, read 0.37 on D.

Note that $\cot 20° 20' = \frac{1}{\tan 20° 20'} = 2.7$ can be read on DI with the same setting.

This is true because the number on DI is the reciprocal of that on D and the cotangent is the reciprocal of the tangent.

To find the tangent of an angle between 45° and 84° 18′ use the fact that $\tan(90° - A) = \cot A = \frac{1}{\tan A}$. Thus we find tan 58° by reading cot 32° on DI.

Opposite 32° on T, read cot 32° = tan 58° = 1.6 on DI.

Since the tangent of an angle between 45° and 84° 18′ is between 1 and 10, the decimal point comes after the first figure.

If an angle is between 0 and 5° 42′ its sine may be used in place of its tangent. They are nearly equal for such small angles.

19. Multiplication using C_1, C_2 and D_1, D_2. As previously stated, C_1 and C_2 together constitute a standard 10-inch C scale. On C_1, which runs from 1 to $\sqrt{10}$ (= 3.1623), the numbers are printed in black; on C_2, which runs from $\sqrt{10}$ to 10, the numbers are printed in red. The 1 at the left end of C_1 is called the left (or BLACK) index of the C_1, C_2 combination, and the 1 at the right end of C_2 is called the right (or RED) index. Thus C_1 and C_2 are regarded as two parts of one 10-inch C scale having its left index at the left end of C_1 and its right index at the right end of C_2. Similarly for D_1 and D_2.

The procedure for multiplying two factors is essentially the same as with the ordinary C and D scales: Locate one of the factors on the D_1 D_2 scale and set an index of C_1 C_2, over it. Then, opposite the second factor on C_1 or C_2, read the product on the proper one of the D_1 D_2 scales.

Example: Multiply 12 × 2 and 12 × 4.

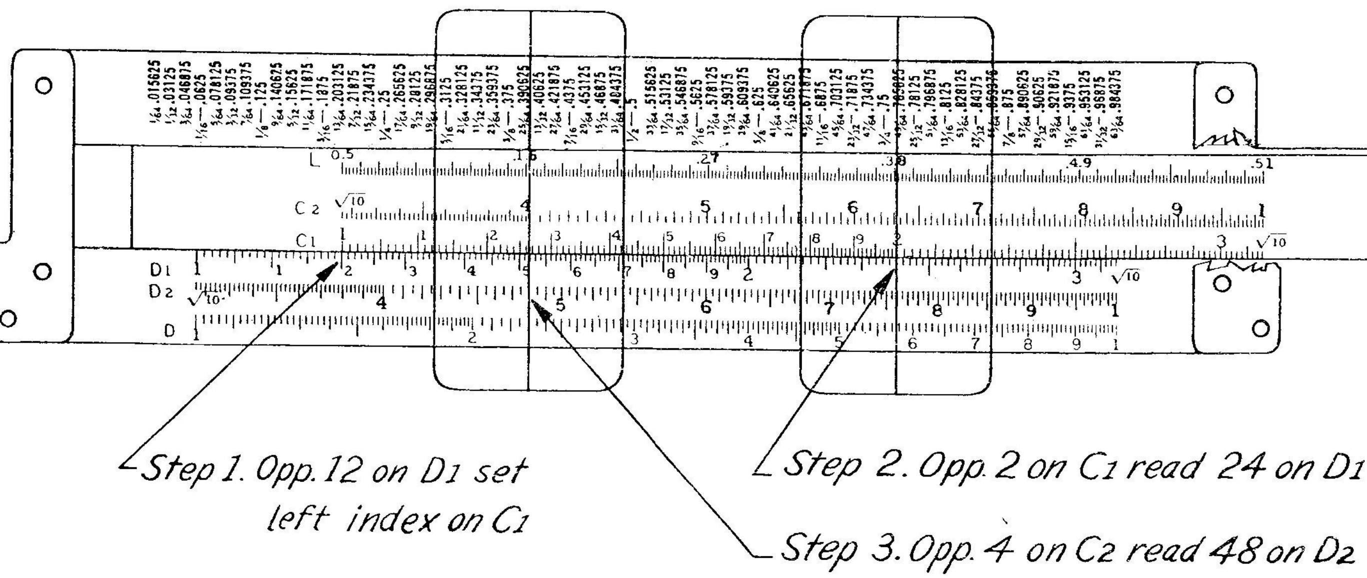


Fig.7

Note that in this case the index was set opposite a number of its own color (black index opposite black number); when this is the case the product is read on the scale having the SAME color as the second factor—black to black or red to red.

When either the left (black) index or the right (red) index of the C_1 C_2 combination is set over a number on D_1 or D_2 that has the OPPOSITE color, then the product is read on the scale whose color is OPPOSITE to that of the second factor.

Example: Multiply 24 × 2.

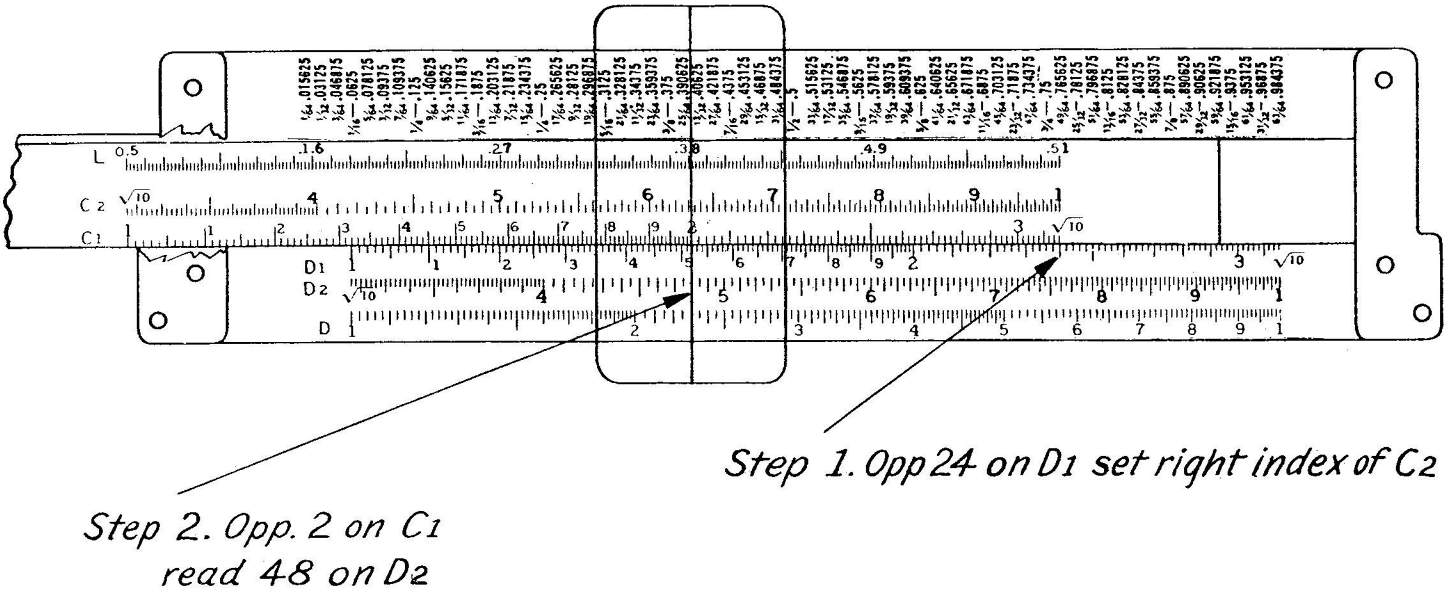


Fig. 8

20. Division using C_1 C_2 and D_1 D_2. To carry out the division $\frac{x}{y}$, locate x on the D_1 D_2 combination and pull y on C_1 or C_2 over it; read the quotient on the proper one of the D_1 D_2 scales opposite the index of C_1 C_2. If x and y are the SAME color, read answer on scale that has the SAME color as the index; if x and y are OPPOSITE in color, read answer on the scale that is OPPOSITE in color to the index.

Example 1. Divide 75 by 6.

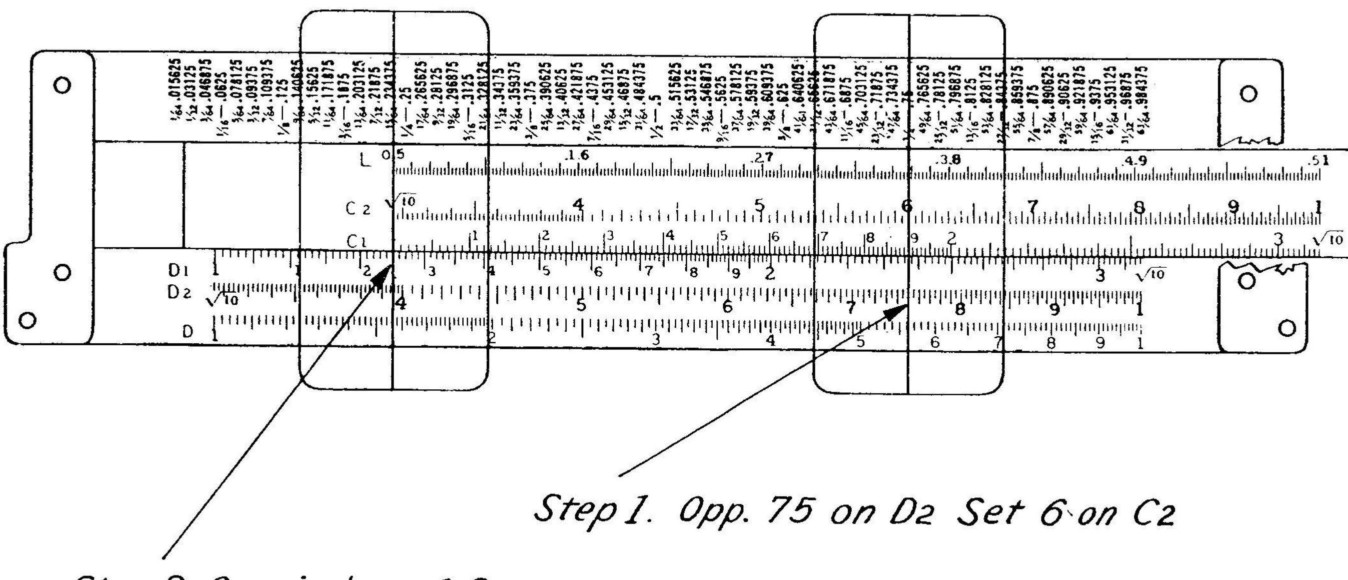


Fig. 9

Example 2. Divide 144 by 6.

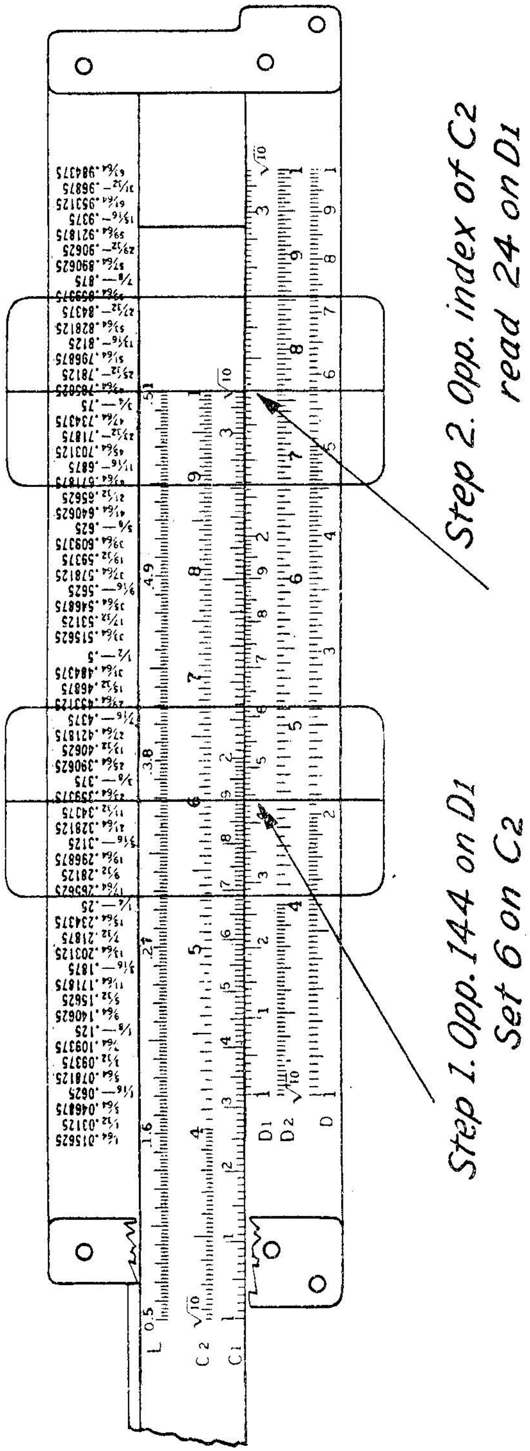


Fig. 10

21. Squares and Square Roots using D_1 D_2 and D. The D scale used here is the one under D_1 and D_2 on the back face.

Opposite any number on D_1 or D_2, read its square on D.

Examples:

Opposite 4 on D_2, read 16 on D.
Opposite 16 on D_1, read 256 on D.
Opposite 1.47 on D_1, read 2.16 on D.

Conversely, opposite any number on D read its square root on D_1 or D_2. Use D_1 if the number of digits in the number is odd and D_2 if even.

Examples:

Opposite 6 on D, read $\sqrt{6} = 2.45$ on D_1.
Opposite 60 on D, read $\sqrt{60} = 7.75$ on D_2.
Opposite 2530 on D, read $\sqrt{2530} = 50.3$ on D_2.
Opposite 0.0149 on D, read $\sqrt{0.0149} = 0.122$ on D_1.

22. Logarithms. Only the mantissa or decimal part of the common logarithm of a number is read from the slide rule. The characteristic is supplied mentally by the use of rules with which the reader is assumed to be familiar. The scales used are C_1 C_2 and L: Opposite a number N on C_1 or C_2, read the mantissa of its logarithm on L. Use the black numbers on L if the number is on C_1 (black) and the red numbers if the number is on C_2 (red).

Examples:

Opposite 15 on C_1 (black) read .176 on L.
Opposite 278 on C_1 (black) read .444 on L.
Opposite 375 on C_2 (red) read .574 on L.
Opposite 628 on C_2 (red) read .798 on L.

Determining the number N when its logarithm is known is of course the reverse operation.

Example: Find N if log N = 1.802.

STEP 1. Opposite the mantissa .802 on L, read 634 on C_2 (red).
STEP 2. Since the characteristic is 1 there are two figures to the left of the decimal point and N = 63.4.

PROPERTIES OF CIRCLES

Circumference = diameter × 3.1416.
Area = square of radius × 3.1416.
= square of diameter × 0.7854.
Side of inscribed square = diameter × 0.7071.
Side of inscribed hexagon = radius of circle.
Length of arc = number of degrees in angle × diameter × 0.008727.
Length of chord = diameter of circle × sine of ½ included angle.
Area of sector = length of arc × ½ of radius.
Area of segment = area of sector minus area of triangle.