CASTELL -ADDIATOR

Slide Rule 1/54 A

"System Darmstadt" III/54 A

for multiplication
division
addition
subtraction

INSTRUCTIONS

A.W. FABER - CASTELL, STEIN NEAR NUREMBERG

1/704 e

CASTELL - Addiator 1/54 A

System Darmstadt

FABER-CASTELL

NOTE.

The Slide Rule **CASTELL** **"System Darmstadt"**

resulted from the work of the

Mathematical Institute of the Technical University of Darmstadt

under the direction of Professor Walther

and was introduced by the firm of A. W. FABER **CASTELL**

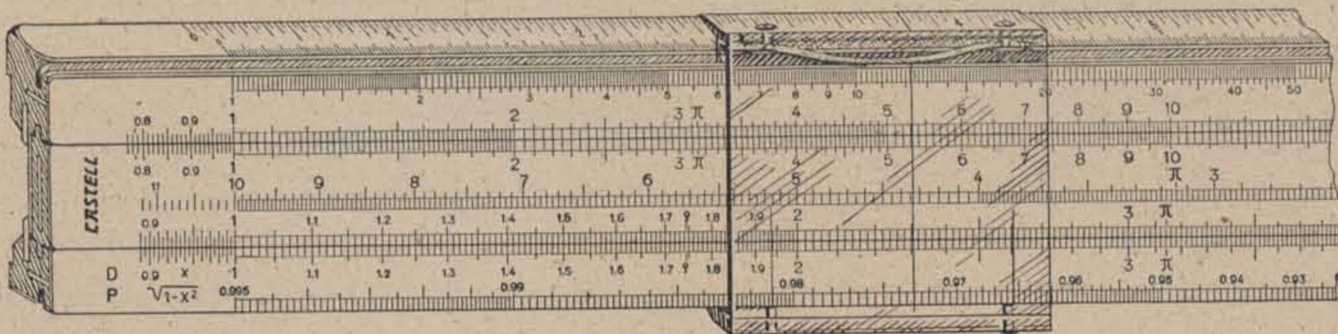
at the instance of Professor Walther.

Description of the Slide Rule.

The Slide Rule **CASTELL** **System Darmstadt** is a general purpose slide rule. Its logarithmic scales make possible all the calculations which are met with in mathematics and their practice. It carries no special scales such as would be required for commercial, nautical purposes, reinforced concrete, or any other narrow field of activity.

The scales of the System Slide Rules are grouped as follows:

1. The **Main Scales** A, B, C, D (x) und Cr.
2. The **Supplementary Scales** Cu, P ($\sqrt{1-x^2}$), L, the trigonometrical scales, and the log-log scales.



The Main Scales.

The Main Scales.

Even the simplest general slide rule has the upper scales, **A** and **B**, and the lower scales, **C** and **D**. Therefore, these are called the **Main Scales** of the rule.

Scales **A** and **B** are exactly alike, and extend from **1 to 100.** Scales **C** and **D** are also alike, and run from **1 to 10.** Scales **A** and **D** are on the body of the rule, and are, therefore, known as the **Rule Scales.** **B** and **C**, being on the slide, are known as the **Slide Scales.**

In addition to these four scales, there is a **reciprocal**, or reversed C, scale on the centre of the slide between B and C. This scale, **Cr**, runs from **10 to 1** (Fig. 1).

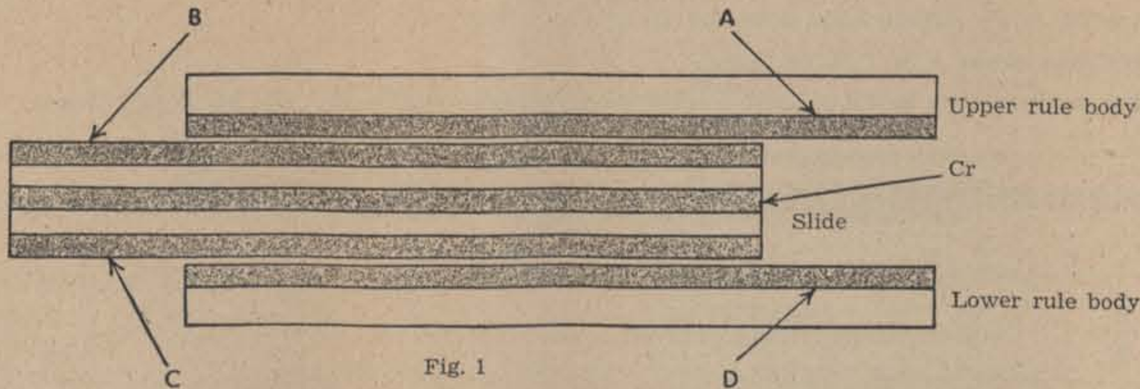


Fig. 1

These five scales are extended a short distance at each end, the extra graduations being a different colour to the main part of the scales.

For all calculations containing only multiplication and division the three scales C, D, and Cr should be used.

The Supplementary Scales.

Additional scales are provided to facilitate calculations other than multiplication, division, squares and square roots:

The **cube scale Cu** is on the rule face above A. It is graduated from **1 to 1000,** and is used with Scale D (Fig. 2).

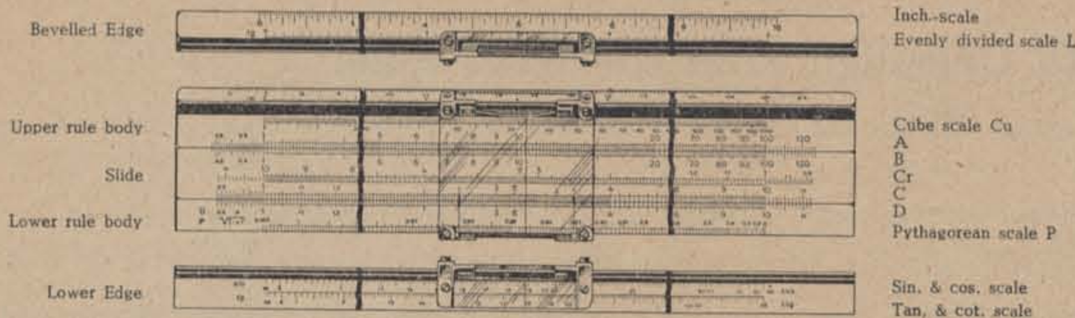


Fig. 2

The **evenly divided scale L** on the bevelled edge of the rule body is used in conjunction with Scale D for reading common logarithms.

The **Pythagorean scale P** ($\sqrt{1-x^2}$) is on the rule face below D, with which scale it is to be employed. Its uses will be explained later.

The **trigonometrical scales** will be found on the lower edge of the rule body.

Finally, there is a **log-log-scale,** graduated in three sections, **from 1·01 to 10^5**, on the back of the slide.

The cursor enables these scales to be employed in any combination. The long centre line is generally used, while the two short lines at the sides are provided for a special purpose, which will be explained later.

How To Calculate With The Slide Rule.

As the scales of the slide rule are tables of logarithms, their operation is based on logarithmic laws. It is well known that:

1. **Multiplication** of two factors is carried out by the **addition** of their logarithms.
2. **Division** is carried out by **subtracting** the logarithm of the divisor from the logarithm of the dividend.

The table of logarithms, therefore, replaces every method of calculating by a simpler operation, and the slide rule even avoids these simple operations, since they are graphically carried out. It follows, therefore, that on the slide rule:

Multiplication of two numbers is transformed into addition of two lengths.

Division of one number by another is changed to subtraction of one length from another.

Graphical calculation is best explained by using two millimetre scales. In Fig. 3, the addition 35 + 45 = 80 is worked.

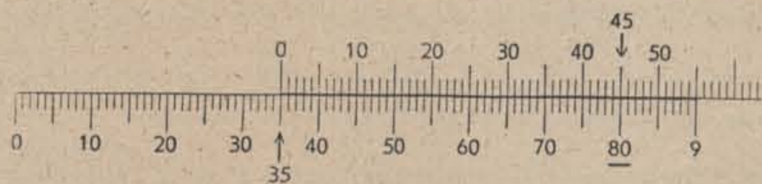


Fig. 3

Fig. 4 shows the subtraction 115 — 53 = 62.

Fig. 4

Now, the slide rule scale is a graphical representation of **logarithms**, as Fig. 5 shows. The number

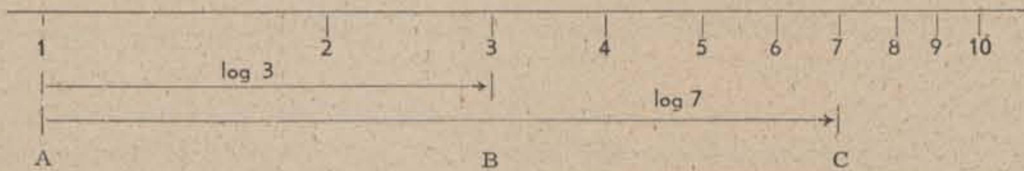


Fig. 5

3 stands at the extreme end of the length or section log 3; all logarithmic lengths are measured from the beginning of the scale, and this point is marked 1, because log 1 = 0.

When both the graphical calculations of Fig. 3 and Fig. 4 have been carried out on the logarithmic scale, the result is not the sum and difference of both numbers, but the **product** and **quotient**, as Figures 6 and 7 show.

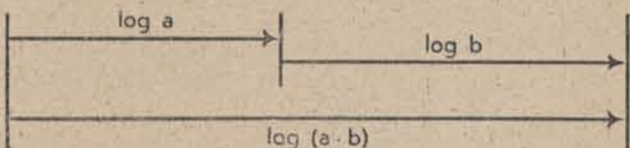


Fig. 6

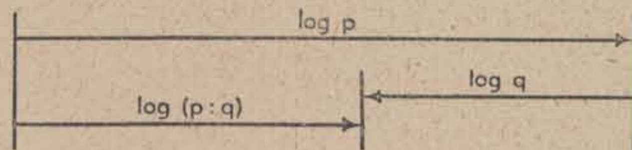


Fig. 7

All other uses of the logarithmic scales are only variations of these two fundamental problems.

If we wish to calculate on the rule in the same way as in Figures 6 and 7, we must "set" the given numbers on the scales. We must also know how to "read" the scales. Some practice is required before the value of the graduations is understood. It is necessary to glance over the whole range of the scales and to be specially careful not to confuse numbers like 3·04 and 3·4, or 2·14 and 2·18. When the graduations are thoroughly understood, it is time to practise the insertion and valuation of the last figure. It is a rule that the first two figures should be set or read with certainty, while the third has to be estimated. Only when the first figure is 1 the first three can be found without estimation.

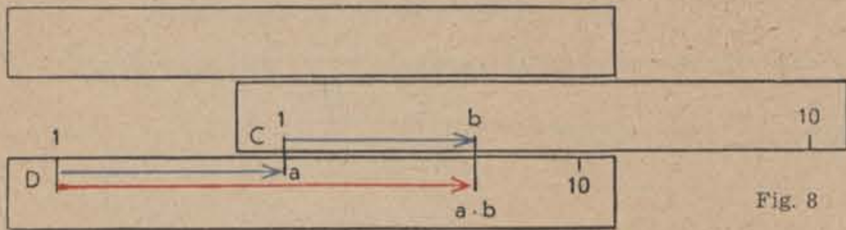
Experience over many years has shown that this degree of accuracy is quite sufficient for all applications in mathematics.

There is no decimal point in slide rule calculations. Therefore, numbers like 13·45; 0·1345; 1345; 1·345 are all read as a row of figures 1—3—4—5. Where the decimal point must be placed in the answer will usually be clear from the problem. But when this is not so, a rough estimate with round numbers will indicate the number of figures in the answer.

CALCULATING GRAPHICALLY.

Multiplication With The C And D Scales.

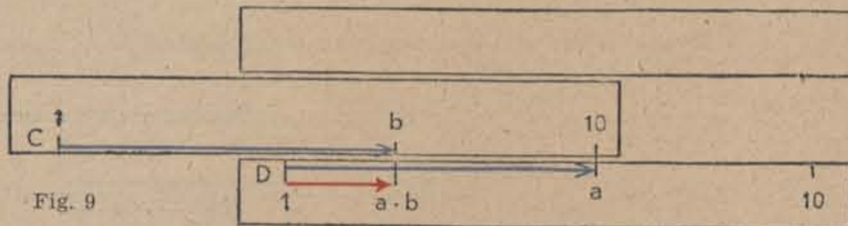
Either



a · b

Fig. 8

Or



a · b

Fig. 9

When the slide projects so much to the right that the factor b is not against the D scale, it is necessary to use the other end of the slide, as shown in Fig. 9.

The two scales form a **table**; Scale D is **b-times** the value of Scale C.

Division With The C And D Scales.

Either

$\frac{a}{b}$

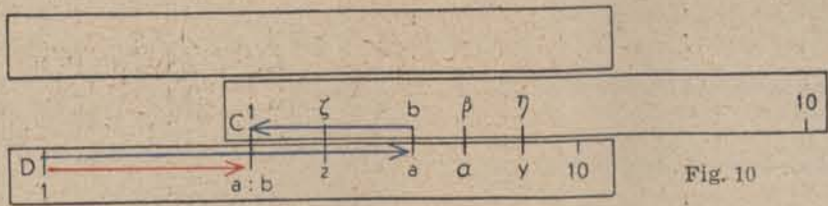


Fig. 10

Or

$\frac{a}{b}$

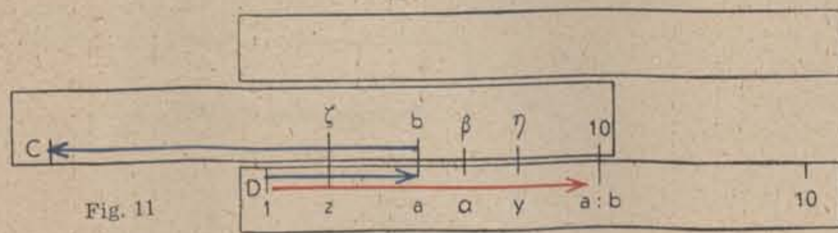


Fig. 11

The answer can only be read at **that** end of the C scale which is inside the rule body.

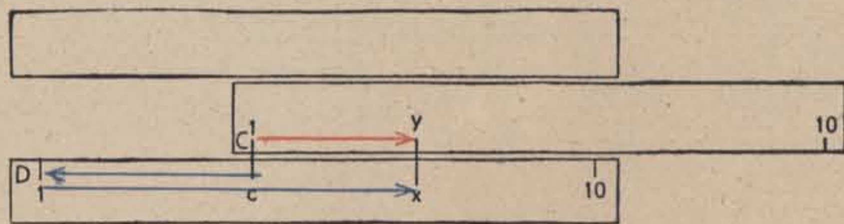
This setting produces a **table** of all pairs of numbers that have the ratio **a—b.**

$$\frac{\alpha}{\beta} = \frac{y}{\eta} = \frac{z}{\zeta} = \frac{a}{b}$$

In this manner all conversions requiring the fourth proportion can be solved, such as, for instance:

when metres on C are set to yards on D, read 75 m. = 82 yds.

If $y = \frac{x}{c}$ is to be solved for many values of x, the method shown in Fig. 12 will be the most convenient.



$$\mathbf{y} = \frac{\mathbf{x}}{\mathbf{c}}$$

Fig. 12

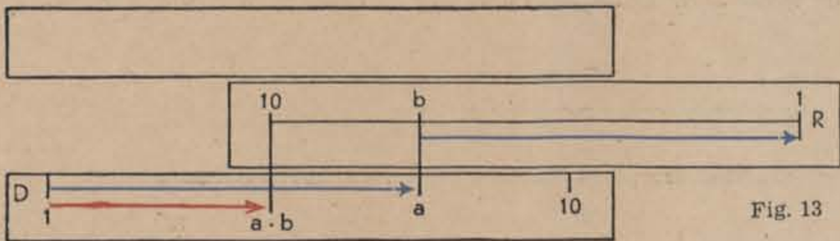
Multiplication And Division Using Scale Cr.

The reversed scale Cr (which requires care in reading) is a very important scale; it simplifies many calculations and makes others possible.

Between Scales **C and Cr** there is a **reciprocal relationship**, each graduation on C being the reciprocal of the one immediately above it on Cr, and vice versa. For instance, 30 and 0·0333, 2·5 and 0·4, 125 and 0·008.

If both factors a and b are set in line on D and Cr, with the help of the cursor, a very convenient method of multiplying is obtained. The answer is always found, either by reading to the left (Fig. 13) or to the right (Fig. 14).

a · b



Cr

Fig. 13

a · b

Cr

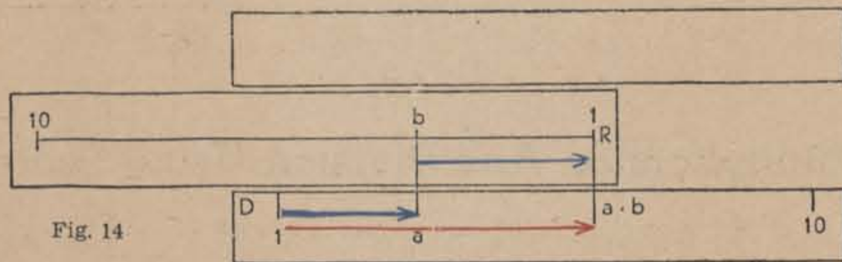
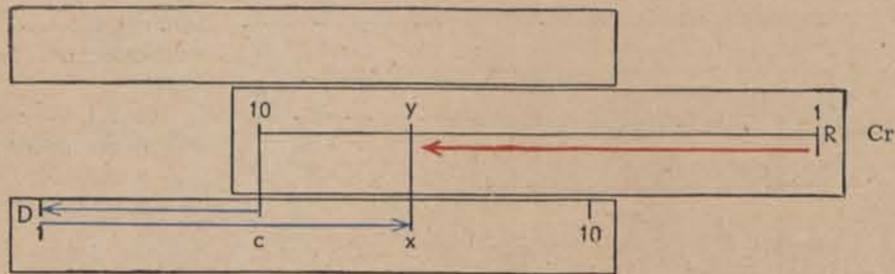


Fig. 14

If $y = \frac{c}{x}$ is to be solved for several values of x, the method shown in Fig. 15 should be employed. With this setting a table is formed which gives all pairs of numbers having c as their product **(inverse proportion).**



$$y = \frac{c}{x}$$

$$x \cdot y = c$$

Fig. 15

The rule is now set to give all possible factors of the number c that satisfy the quadratic equation

$$x^2 + s \times x + c = 0,$$

but the sum must be — s.

With the reverse scale, Cr, it is possible in most cases to find the product of three factors with one setting (Fig. 16). Reversing the procedure gives division by two divisors (Fig. 17).

$a \cdot b \cdot c$

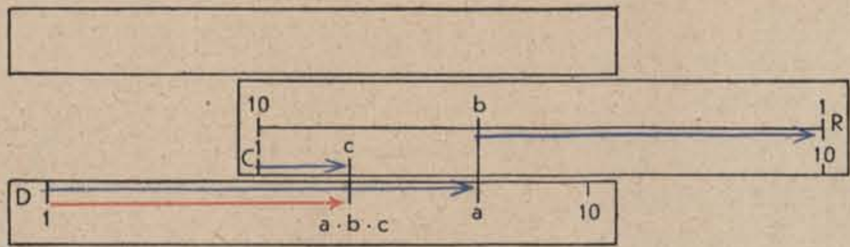


Fig. 16

Example:

What is the area of an ellipse with semi-axes of 15·4 inches and 6·2 inches?

$A = \pi\, a\, b = 15{\cdot}4 \times 6{\cdot}2 \times 3{\cdot}14 =$ 300 square inches.

$$\frac{p}{q \cdot r}$$

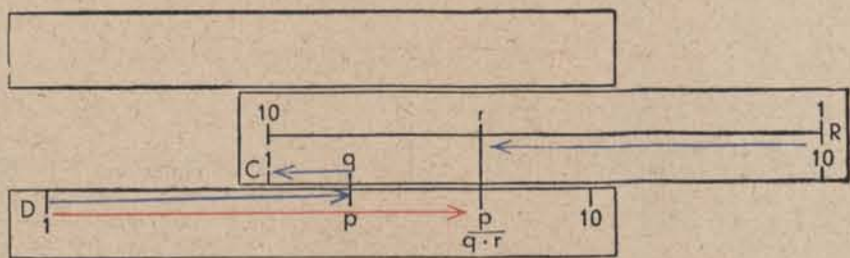


Fig. 17

Example:

An alternating current motor of 220 V has a power input of 2860 W with a current of 16 A; find the power factor (cos φ).

$$\cos\varphi = \frac{P}{VI} = \frac{2860}{220 \times 16} = 0 \cdot 812;$$

$(\varphi = 35 \cdot 7°;)$

Squares And Square Roots.

Both the upper scales are graduated to half length. The change over from D to A (or from C to B) gives the **square** of the number to which D (or C) has been set. **Square roots** are extracted by reversing this procedure (Fig. 18).

Example:
Given the side of a square (50 inches).
Find the area.

$$A = 50^2 = 2500 \text{ sq. in.}$$

Example:
What is the diameter of a shaft if P = 50 HP and V = 400 r.p.m?

$$d = 12 \times \sqrt[4]{\frac{P}{V}} = 12 \times \sqrt{\sqrt{\frac{50}{400}}} = 7 \cdot 138$$

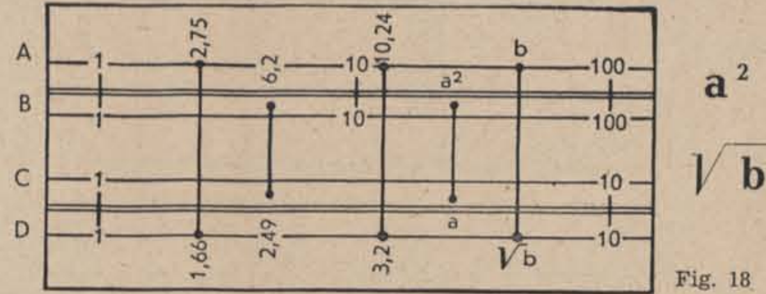


Fig. 18

In extracting a square root it is essential to set the number on the correct half of the upper scale for $\sqrt{x}$ and $\sqrt{10\,x}$ do not differ merely in the position of the decimal point. If the figures 6..2 be set to the left, the root of 6·2 appears below, while if they are set to the right the root of 62 is obtained. One must thus proceed in accordance with the numbers as shown (1...10...100). If the number lies outside the scale range 1 to 100, it should be factorised by hundreds to bring the significant figures within these limits.

Example: $\sqrt{1922} = \sqrt{100 \times 19{\cdot}22} = 10 \times \sqrt{19{\cdot}22} = 10 \times 4{\cdot}38 = \mathbf{43{\cdot}8}$

$\sqrt{0{\cdot}000\,071} = \sqrt{71 : 1\,000\,000} = \sqrt{71} : 1\,000 = 8{\cdot}43 : 1000 = 0{\cdot}008\,43.$

When both the upper and lower scales are used in conjunction many combined calculations are possible, as the following figures will show.

When the calculation contains a number of which the square has to be found, start the operation on the lower scales. The answer will then appear on one of the upper scales. Eight different types of calculation can be made.

$(a \cdot b^2)$

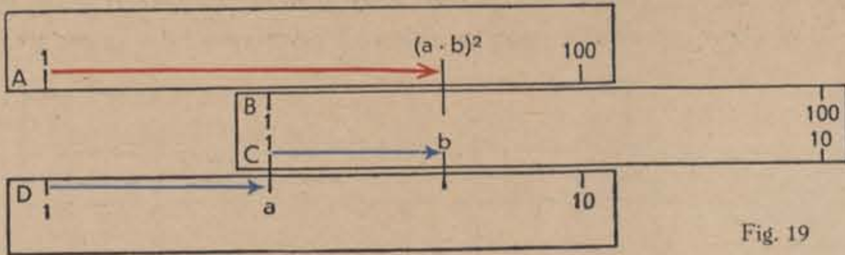


Fig. 19

Example:

Calculate the volume of a circular ring with a mean diameter of 18 inches and a thickness of 2 inches.

$$V = \frac{D \cdot \pi^2 \cdot d^2}{4} = 4.5 \times (2 \times \pi)^2$$
$$= 4.5 \times 39.4 = \mathbf{177} \text{ cub. inch.}$$

$\left(\frac{a}{b}\right)^2$

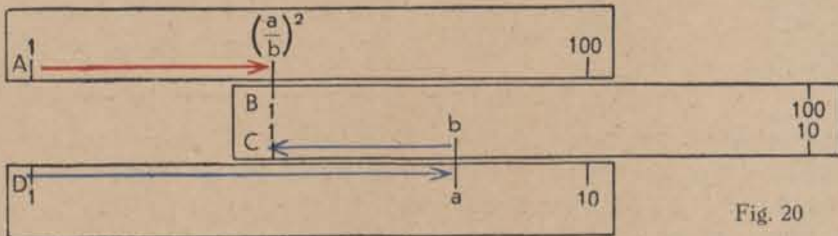


Fig. 20

Example:

The single period of oscillation of a pendulum 43·75 inches long is 6·34 sec. Find the acceleration due to fall.

$$g = \frac{\pi^2 \cdot l}{T^2} = l\left(\frac{\pi}{T}\right)^2 = \left(\frac{3.14}{6.34}\right)^2 \times 43.75$$
$$= 0.24522 \times 43.75 = \mathbf{10.735} \text{ yards/sec.}^2;$$

$a^2 \cdot b$

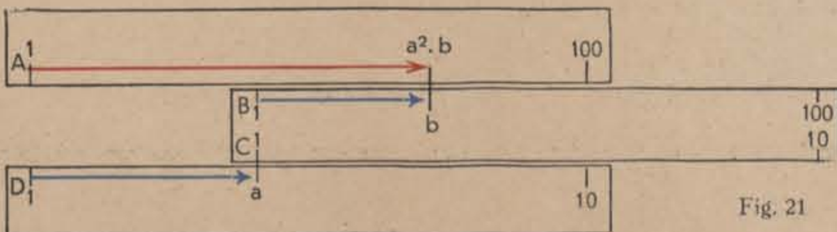


Fig. 21

Example:

Find the volume of a rectangular solid body with a base side of 4 yards and a height of 3 yards.

$$V = 4^2 \times 3 = 16 \times 3 = 48 \text{ yards}^3$$

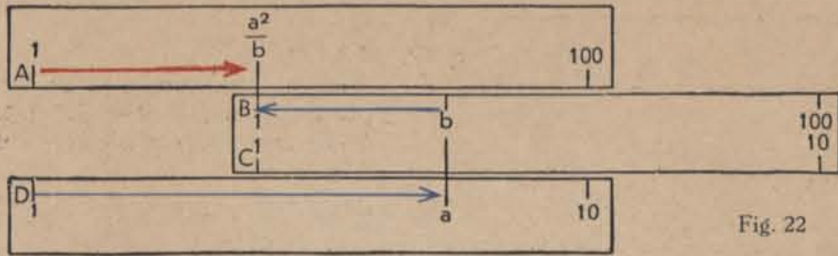


Fig. 22

Example:

What is the power in kW absorbed by a resistance of 40 Ω when connected to a supply of 220 V?

$$P = \frac{V^2}{R} = \frac{220^2}{40} = 1210\,\mathrm{W} = 1{\cdot}21\,\mathrm{kW}$$

$$\frac{a^2}{b}$$

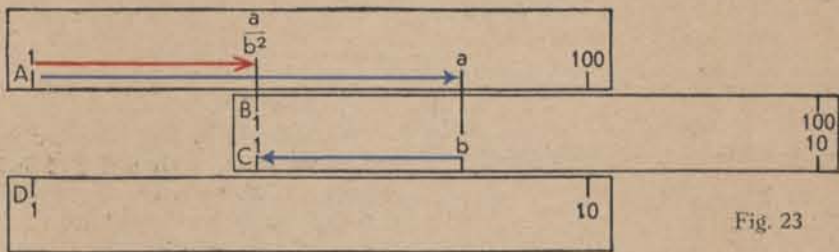


Fig. 23

Example:

Find the resistance of a winding which absorbs 1420 Watts with a current of 5·4 A.

$$R = \frac{P}{I^2} = \frac{1420}{5{\cdot}4^2} = 48{\cdot}7\,\Omega$$

$$\frac{a}{b^2}$$

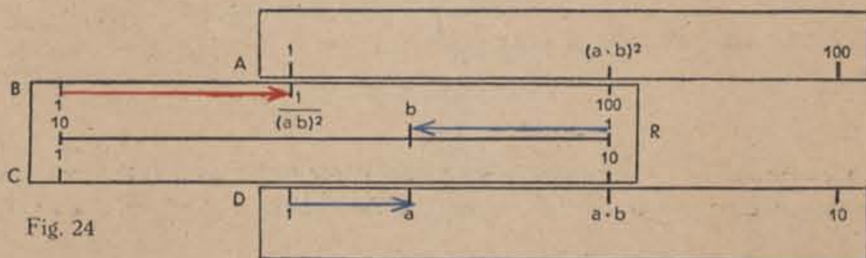


Fig. 24

Example:

A choke coil is connected in series to a condenser of 20 microfarads. Determine the self induction factor which the coil must have if voltage resonance is to be obtained at a frequency of 50.

$$L = \frac{1\,000\,000}{\omega^2 \times C} = \frac{1\,000\,000}{(2\pi \times f)^2 \times 20}$$

$$= 50\,000 \cdot \frac{1}{(6{\cdot}28 \times 50)^2} = \mathbf{0{\cdot}507\ H;}$$

$$\frac{1}{(a \cdot b)^2}$$

$\frac{1}{a^2}$

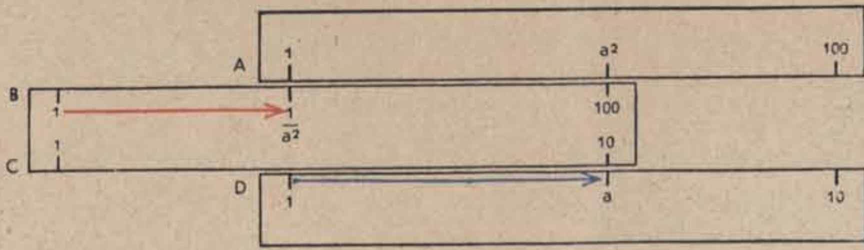


Fig. 25

Example:

Find the resistance R of an appliance having a power of 1320 Watts and drawing a current of 6 A.

$$R = P \times \frac{1}{J^2} = 1320 \times \frac{1}{6^2} = 36{\cdot}7\ \Omega$$

$\frac{1}{a^2 \cdot b}$

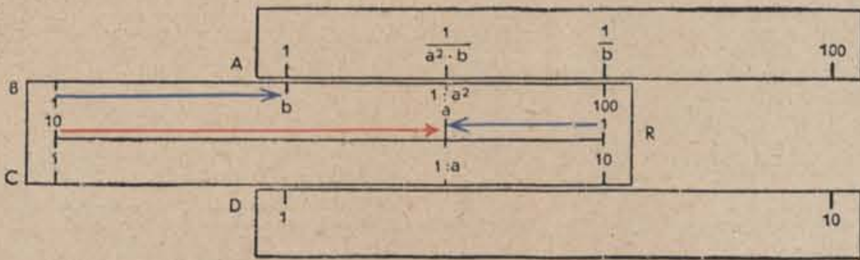


Fig. 26

Example:

A copper lead has a length of 1000 yards and a diameter of 0·16 inch. Conductivity of copper (1 yd.) at 60° F c = 32,700. Find the resistance.

$$R = 1000 \times \frac{1}{d^2 c} = \frac{1000}{0{\cdot}16^2 \times 32{,}700} = 1{\cdot}19\ \Omega.$$

If, however, the calculation involves the extraction of a **square root, start** on the **upper** scales, so that the root can be found on a lower scale. Care must be taken, in this case, to set the number on the correct half of scale A or B.

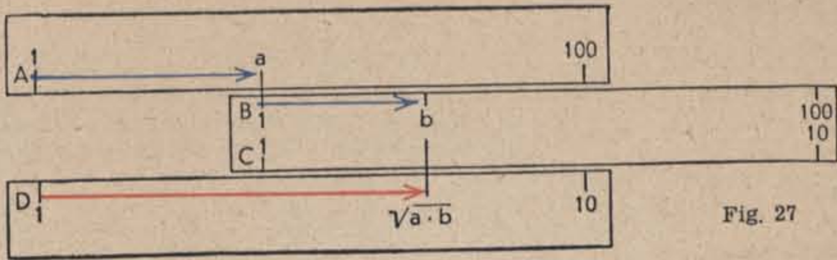


Fig. 27

Example:

Change a rectangle with sides of 2·55 and 6·6 yds. into a square.

$$s_q = \sqrt{2.55 \times 6.6} = 4.1 \text{ yds.};$$

$$\sqrt{a \cdot b}$$

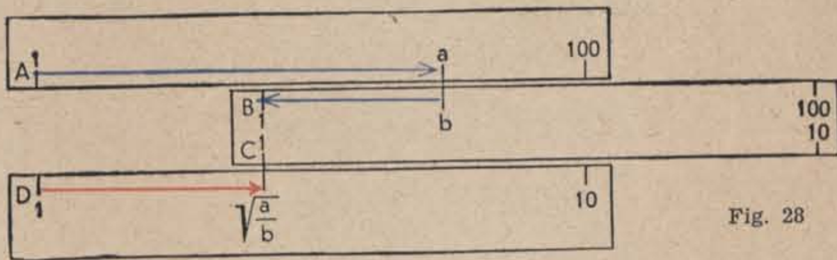


Fig. 28

Example:

Foucault's experiment was carried out with a pendulum with a length of 58 yds. What was the single period of oscillation, when the constant of gravitation was 10·735 $\frac{y}{sec.^2}$ at the place where the experiment took place?

$$T = \pi \cdot \sqrt{\frac{l}{g}} = \pi \sqrt{\frac{58}{10.735}} = 7.3 \text{ sec.}$$

$$\sqrt{\frac{a}{b}}$$

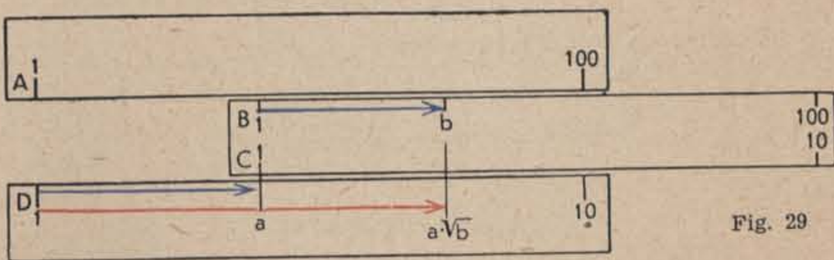


Fig. 29

Example:

The phase voltage of a star-connected system is 220 V. Find the total voltage.

$$V = V_p \times \sqrt{3} = 220 \times \sqrt{3} = 380 \text{ V};$$

$$a \cdot \sqrt{b}$$

$$\frac{a}{\sqrt{b}}$$

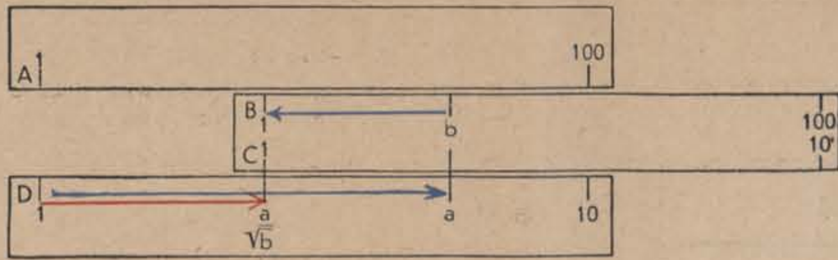


Fig. 30

Example:

Calculate the sections of the hypotenuse of a right-angled triangle, the height of which is 5·46 yds., one section of the hypotenuse being double of the other.

$$h^2 = p \times 2p = 2p^2;$$

$$p = \sqrt{\frac{h^2}{2}} = \frac{h}{\sqrt{2}} = 3{\cdot}86;\ q = 7{\cdot}72;$$

$$\frac{\sqrt{a}}{b}$$

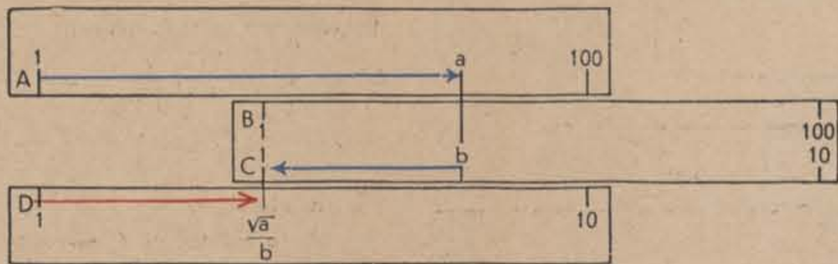


Fig. 31

Example:

A string of 27·6 inches in length, with a thickness of 0·08 inch. and a specific gravity of 0·02896 lb. / cub. inch. is bent by a power P = 13·2 lbs. Find the rate of vibration of its sound. g = 387 in./sec.

$$n = \frac{\sqrt{\dfrac{13{\cdot}2 \times 387}{3{\cdot}14 \times 0{\cdot}02896}}}{2 \times 27{\cdot}5 \times 0{\cdot}04} = \frac{\sqrt{56200}}{2{\cdot}2} = \frac{237}{2{\cdot}2} = 108.$$

$$\frac{1}{\sqrt{a \cdot b}}$$

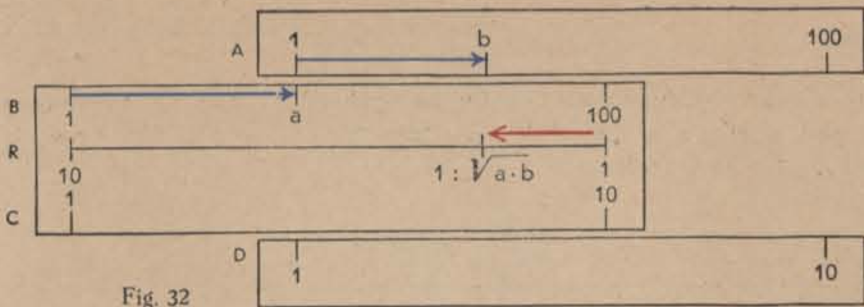


Fig. 32

Example:
A condenser of 2·3 microfarads and a choke coil of 5·8 henrys are connected in series. What angular velocity and what frequency are needed to obtain voltage resonance?

$$\omega = \frac{1}{\sqrt{L \cdot C}} \qquad \text{L = inductance} \quad \text{C = capacity}$$

$$\omega = 1000 \cdot \frac{1}{\sqrt{5{\cdot}8 \times 2{\cdot}3}} = 1000 \times 0{,}274 = 274;$$

$$f = \frac{\omega}{6{\cdot}28} = \frac{274}{6{\cdot}28} = 4{\cdot}36 \text{ cycles.}$$

$$\frac{1}{a \cdot \sqrt{b}}$$

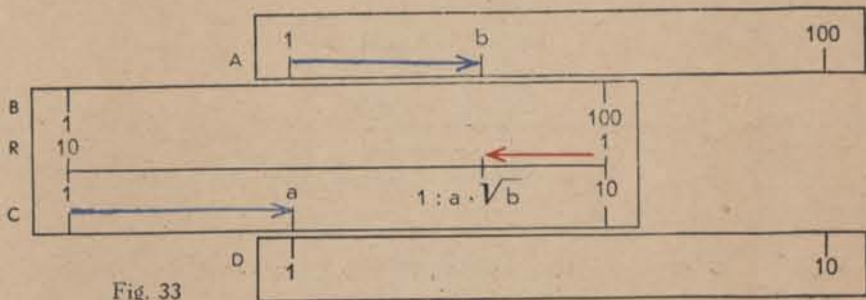


Fig. 33

Example:
The wattmeter of a three-phase current generator, the line voltage of which is 7 kV, indicates 5600 kW. cos φ = 0·83.
Find the strength of the current.

$$J = \frac{5600000}{\sqrt{3} \times 7000 \times 0{\cdot}83} = 800 \cdot \frac{1}{\sqrt{3} \times 0{\cdot}83} = 800 \times 0{\cdot}695 = \mathbf{556\ A};$$

$$\frac{1}{\sqrt{a}}$$

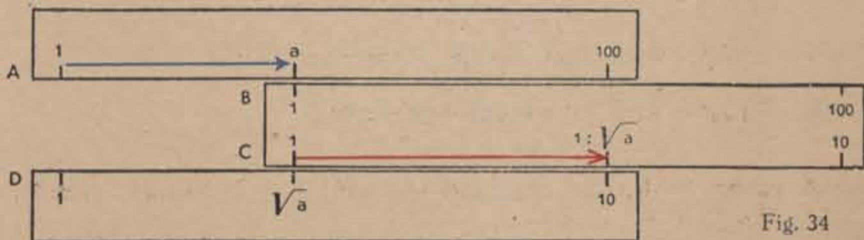


Fig. 34

Example:
Change a single phase alternating current of 120 V into direct current by means of a transformer.

$$Vd = \frac{2\,V}{\sqrt{2}} = \frac{1}{\sqrt{2}} \times 2V = \frac{1}{\sqrt{2}} \times 240 = 170\ V$$

Result above D 240 = 169·6 V or B 2 above D 240, under C 1 = 169·6 V.

Cubes And Cube Roots.

Scale Cu is graduated in the ratio 1:3. In passing over from Scale D to Cu the number is raised to the **third power,** while passing from Cu to D gives the **cube root,** as shown in Fig. 35. When setting the number for a cube root on Scale Cu it is necessary to watch the values 1 . . . 10 . . . 100 . . . 1000.

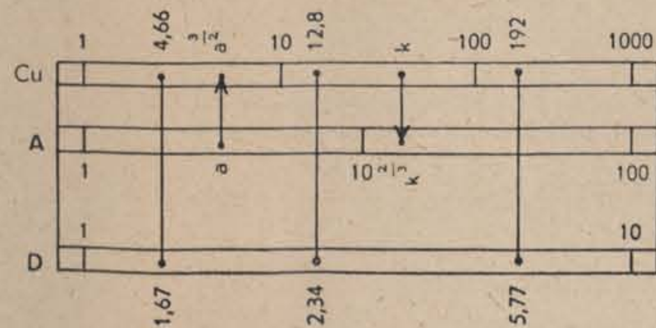


Fig. 35

If the number does not lie within the scale range 1 . . . 1000, it must be factorised by thousands to bring it within these limits.

Example: $\sqrt[3]{1,260,000} = \sqrt[3]{1,000^2 \times 1{\cdot}26} = 10^2 \times \sqrt[3]{1{\cdot}26} = 100 \times 1{\cdot}08 = 108$

$\sqrt[3]{0{\cdot}32} = \sqrt[3]{320 \div 1\,000} = \sqrt[3]{320} \div 10 = 6{\cdot}84 \div 10 = 0{\cdot}684$

If the cube scale be employed with Scale A, powers having the exponents $\frac{3}{2}$ and $\frac{2}{3}$ may be found (Fig. 35).

The Pythagorean Scale.

This scale represents the function $y = \sqrt{1-x^2}$. It is employed in combination with Scale D (= x), the latter having the range of values 0·1 to 1.

$\sqrt{1-x^2}$

Examples: x = 0·8   y = 0·6   (Fig. 36)

Sin α = 0·134,   cos α = 0·991

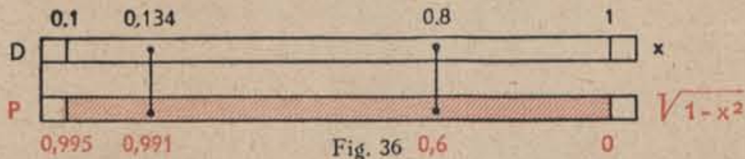


Fig. 36

Example:

Determine the effective current and wattless current of a circuit which absorbs 35 A at 220 V.

cos φ = 0·8

I_e = I × cos φ = 35 × 0·8 = 28 (A)

I_w = I × sin φ = 35 × 0·6 = 21 (A)

The Evenly Divided Scale.

This scale is used with Scale D for reading **common logarithms**, and may be used in place of a **three-figure table.** Naturally, it only gives the mantissae, the characteristic being found in the usual way.

lg x

Example: Log 52 = 1·716 (Fig. 37).

Log x = 3·574 x = 3750.

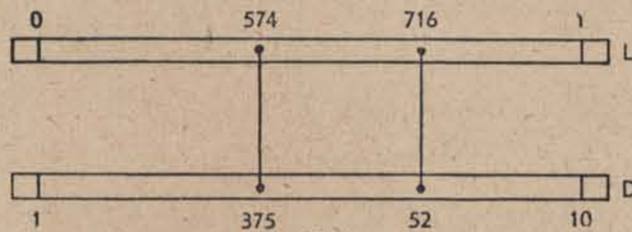


Fig. 37

The Cursor.

The three lines on the cursor can be employed as a scale. When the short right-hand line is set to a **diameter** on C or D, the centre line will give the **area** on B or A respectively (Fig. 38); and when the right-hand line is on any given **horse-power,** the left-hand line indicates the corresponding **kilowatts.**

HP—kW

d—A

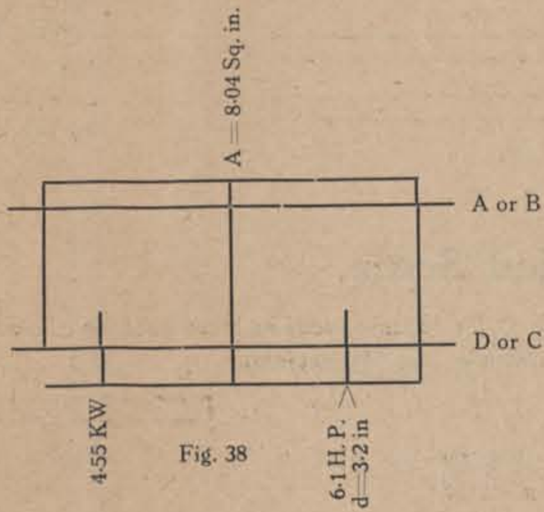


Fig. 38

The Trigonometrical Scales.

Use of the Scales as Tables.

Reading the sin-cos table from left to right, with the **Black Numbers,** we obtain a **Sine Table** on Scale D.

With large angles the reading becomes uncertain; in this case it is more accurate if the red numbers are used and read on Scale P. In Fig. 39, sin 76⁰ is given as 0·97 on Scale D, and as 0·9703 on P.

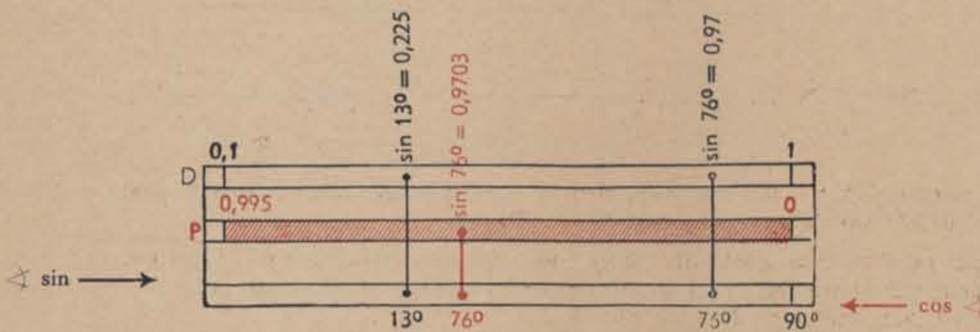


Fig. 39

sin *a*

cos *a*

Reading the **Red Numbers** from right to left, we obtain a **Cosine Table** on D.

With small angles the reading is not exact: it is more accurate if read on Scale P with the black numbers. In Fig. 40, cos 11° is shown on D as 0·982, and more accurately as 0·9816 on P.

sin *a*

cos *a*

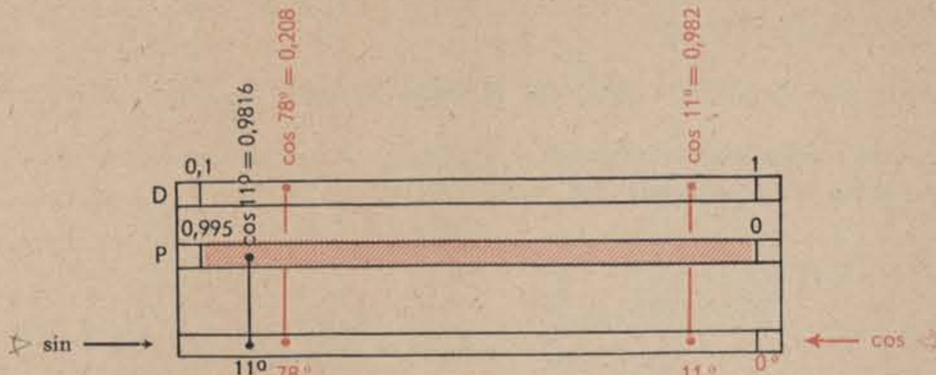


Fig. 40

Reading **black numbers** on the tan-cot scale from left to right, we obtain a **tangent table** on Scale D. Reading **red numbers** from right to left, we obtain a **cotangent table** on D.

It would appear as if only tangents of angles below 45° and cotangents of angles over 45° can be read. But as tangent and cotangent values are reciprocal, the use of Scale Cr permits all values to be read as shown in the examples of Fig. 41. The procedure is summarised in the following:

Tangents	under 45° over 45°	black numbers and D or C red numbers and Cr
Cotangents	under 45° over 45°	black numbers and Cr red numbers and D or C

Tan a

Cot a

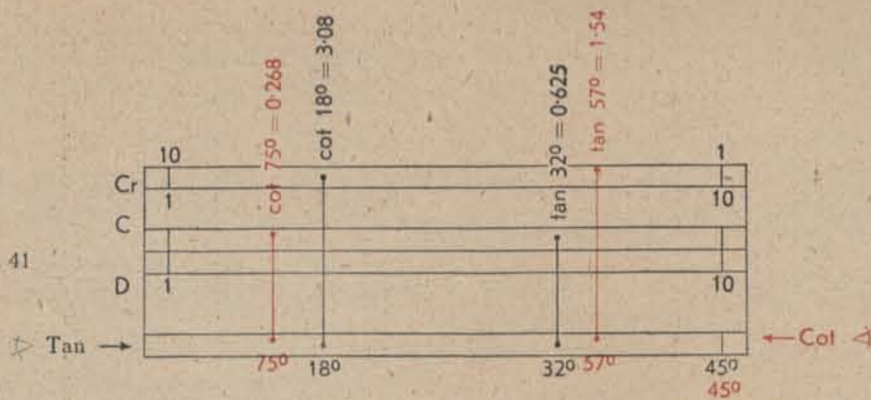


Fig. 41

The trigonometrical functions can be read in this way down to $5^{\circ}{\cdot}7$. Then, $\sin 5^{\circ}{\cdot}7 \approx \tan 5^{\circ}{\cdot}7 \approx 0{\cdot}1$. The following relationship can be used for still smaller angles: $\sin a \approx \tan \alpha \approx \text{arc}\ \alpha \approx 0{\cdot}01745\ \alpha^{\circ}$.

The mark ϱ has been placed at 1-7-4-5 on the C and D scales; it is employed as shown in Fig. 42. For instance, $\sin 3^{\circ} \approx \tan 3^{\circ} \approx \text{arc}\ 3^{\circ} \approx 0{\cdot}0524$. The error is less than $0{\cdot}25\%$.

(The symbol $\approx$ means "approximately equals".)

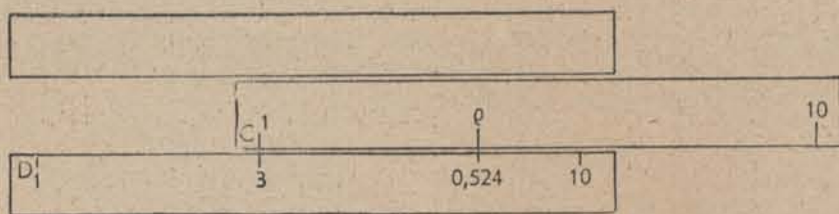


Fig. 42

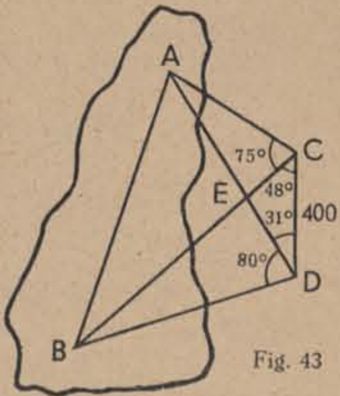


Fig. 43

Example: Find the distance between two points A and B, lying in a moor and therefore inaccessible. Outside the moor the distance CD = 400 yds. and the angles marked in the accompanying diagram are measured. (Fig. 43)

Calculation of I. EA:

$$EA = \frac{EC \times \sin 75^0}{\sin 26^0} = \frac{400 \times \sin 31^0 \times \sin 75^0}{\sin 101^0 \times \sin 26^0} = 462;$$

Calculation of II. EB:

$$EB = \frac{ED \cdot \sin 80^0}{\sin 21^0} = \frac{400 \times \sin 48^0 \times \sin 80^0}{\sin 101^0 \times \sin 21^0} = 833;$$

Calculation of III. AB:

$AB^2 = EA^2 + EB^2 - 2 \times EA \times EB \times \cos 101^0 = EA^2 + EB^2 + 2 \times EA \times EB \times \cos 79^0 = 213{,}444 + 693{,}889 + 769{,}692 \times 0{\cdot}191 = 907{,}333 + 147{,}011 = 1{,}054{,}344;$

$AB = \sqrt{1{,}054{,}344}$; **AB = 1,027 yds.**

Calculation with Trigonometrical Scales.

If it is desired to convert the sine of an angle to the cosine (or vice versa), the angle need not be read. On Scales D and P these pairs of values are shown one under the other. In converting from tangent to cotangent the reading of the angles is likewise dispensed with, as the corresponding values appear one under the other on C and Cr. It is only when converting sines or cosines to tangents or cotangents that the angle need be read. In converting sines and cosines to tangents and contangents it is of advantage to adopt the form

$$\operatorname{tg} x = \frac{\sin x}{\sqrt{1 - \sin^2 x}} \quad \text{and} \quad \cot x = \frac{\cos x}{\sqrt{1 - \cos^2 x}}$$

Example: Given the actual efficiency 32 kW and $\cos \varphi = 0{\cdot}81$.
Find the wattless efficiency.

Solution: $\text{ctg}\,\varphi = \frac{\cos\varphi}{\sqrt{1-\cos^2\varphi}}$. Set the cursor line over D 81 ($\cos\varphi = 0{\cdot}81$) and read on the P scale 0·587 ($\sqrt{1-\cos^2\varphi}$), then set C 587 over D 81 and read above C 1 the answer $\text{ctg}\,\varphi = 1{\cdot}38$ and over D 10 $\text{tg}\,\varphi = 0{\cdot}724$.

Now set the cursor line over D 32 and read on the C scale the result 23·2 kVA wattless efficiency.

As the functions can be found either on D or on Cr, further calculation (multiplication and division) can in many cases be carried out immediately. It is only when the value is read on P that it has to be transferred to the main scales.

The Log-Log-Scale (Exponential Scale).

This scale has manifold applications, but only the most important methods of calculating are given here. The following rule will be found helpful:

For a **single calculation** use the **slide** in **normal** position.

For a **series** of problems **invert** the **slide.**

When the slide is inverted the three sections of the log-log scale move between Scales A and D.

There is a tenth power relationship between each pair of adjacent sections of the log-log scale, which makes the reading of tenth roots and tenth powers extremely easy.

Example:	$1{\cdot}204^{10} = 6{\cdot}4$,	reading between 2nd and 3rd sections	a^{10}
	$1{\cdot}035^{10} = 1{\cdot}41$,	„ „ 1st „ 2nd „	
	$\sqrt[10]{75} = 1{\cdot}54$	„ „ 3rd „ 2nd „	$\sqrt[10]{a}$
	$\sqrt[10]{1{\cdot}248} = 1{\cdot}0224$	„ „ 2nd „ 1st „	

These examples show that, with the log-log scale, the position of the decimal point is definitely fixed.

Powers of e.

The **exponents** must be set **on** Scale **D.** If they are used in combination with the **lowest** section of the log-log scale, the graduations on D must be read as **1 to 10;** with the **middle** section they must be read as **0·1 to 1;** and with the **upper** section as **0·01 to 0·1.**

e^n

Normal Slide. / Inverted Slide.

Example: $e^{1 \cdot 61} = 5.$

Normal Slide: Set 1·61 on C to one end of D (say, to 1). Then, turn the rule over and read the answer, 5, on the lowest section under the left-hand index line.

Inverted Slide: Set the cursor line to 1·61 on D and read the answer, 5, on the lowest log-log section.

Example: $e^{0 \cdot 61} = 1 \cdot 84.$

Normal Slide: Set 61 on C (which has to be taken an 0·61) over either end of D (say, over 10). Turn the rule over and, at the right-hand end now, read 1·84 on the middle section.

Inverted Slide: Set the cursor line over 61 on D (0·61) and read 1·84 on the middle section.

Example: $e^{0 \cdot 029} = 1 \cdot 0294.$

Normal Slide: Set 29 on C (which is now 0·029) over either end of D (say, over 1), turn the rule over and read 1·0294, under the index line, on the upper section of the log-log scale.

Inverted Slide: Set the cursor over 29 (0·029) on D and read 1·0294 on the upper section.

If the power exponent is negative, adopt the form $e^{-n} = \frac{1}{e^n}$, first calculating with the positive n and then finding the reciprocal.

Example: Calculate the elastic force in the running on band of a band brake, of which the band passes round the drum twice.

Solution: $T_{running\ off} = 22$ lbs; $\alpha = 2 \times 360^0 = \text{arc}\ 4\pi = 12{\cdot}56$;

Coefficient of friction $\mu = 0{\cdot}18$;

$$T_{running\ on} = T_{running\ off} \times e^{\mu \times \alpha} = 22 \times e^{2{\cdot}261} = 22 \times 9{\cdot}60 = 211{\cdot}2 \text{ lbs.}$$

Roots of e.

$\sqrt[n]{e}$

Exemple: $\sqrt[4]{e} = e^{\frac{1}{4}} = e^{0{\cdot}25} = 1{\cdot}284.$

If the exponent of the root be changed to a power exponent, as in the above example, the solution is as in the foregoing. The conversion of the exponents is read from the reciprocal scales. It is, however, possible to find the root directly by means of Scale Cr.

Normal Slide.

Set 4 on Cr over either end (1, for instance) of D. Turn the rule over and read the answer, 1·284, under the left-hand index line on the middle section of the log-log scale.

Inverted Slide.

Set 4 on Cr under one of the index lines at the back of the rule (at the left-hand end, for instance). Then the answer 1·284, will be found on the middle section over 1 on D.

Hyperbolic Logarithms.

Hyperbolic logarithms are found by reading from the log-log scale to scale D or C.

$\log_e a$

Example: $\text{Log}_e 25 = 3{\cdot}22.$

Slide Normal.

Draw the slide to the right until 25 (on the lowest log-log section) appears under the index line. Turn the rule over and read 3-2-2 on C over 10 on D. As the lowest section was used, $\log_e 25 = 3 \cdot 22$.

With the slide to the left the reading is in exactly the same manner.

Slide Inverted.

Set the cursor line on 25 on the lowest log-log section and read $\log_e 25 = 3 \cdot 22$ on D.

With this setting we obtain a table of hyperbolic logarithms. There is no movement of the slide.

Example: Log_e 1·31 = 0·27.

Slide Normal.

Draw the slide to the left until 1·31 (on the middle section) appears under the index line. Turn the rule over and read the numbers 2 - 7 on C over 1 on D. Being on the middle section, it must be read as 0·27.

Proceed in the same way when reading to the right.

Slide Inverted.

Set the cursor line over 1·31 on the middle section and read the numbers 2 - 7 on D. This means 0·27, since it is on the middle section.

Example Log_e 1·0145 = 0·0144.

The procedure is exactly as before; the numbers 1-4-4 read on D, must be taken as 0·0144, as the uppermost section was used in setting.

Powers With Fractional Exponents.

a^n

Example: $3{\cdot}75^{2.96} = 50$

Bring 3·75 on the lowest section to the right-hand index line (Fig. 44a). Move the cursor over 1 on C (Fig. 44b) and bring 296 on C under the cursor line (Fig. 44c). Turn the rule over and read the answer, 50, under the index line (Fig. 44d).

Set 3·75 (on the lowest log-log section) over 1 on D, move the cursor line over 296 on D and read the answer, 50, directly above (Fig. 45).

a^n

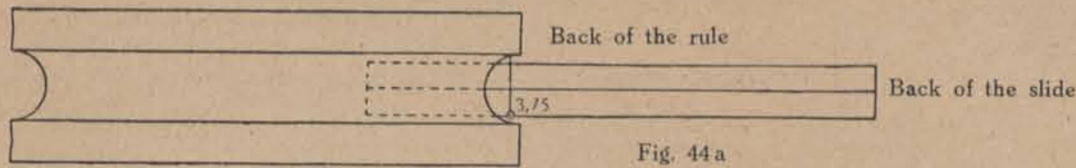


Fig. 44 a

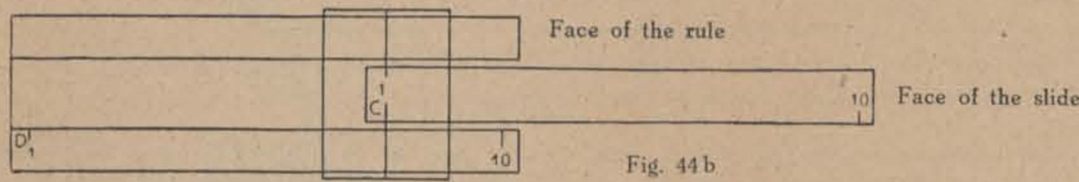


Fig. 44 b

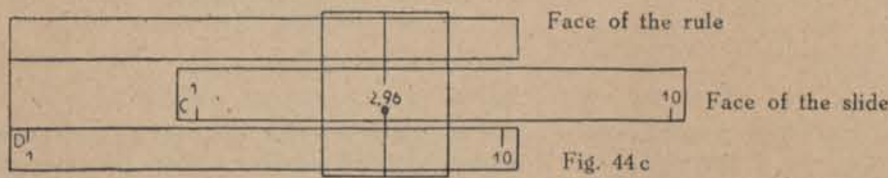


Fig. 44 c

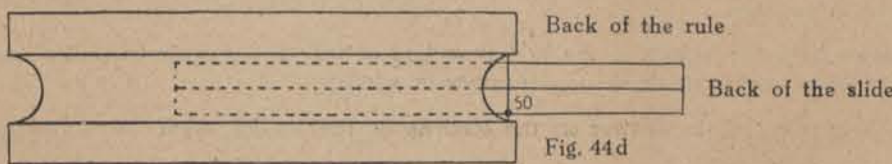


Fig. 44 d

Maximator Extension Scale.

This scale is intended for quick and accurate calculations by the logarithm method. By its use logarithms of numbers can be read quickly to the 4th or 5th place, thus avoiding long and tiresome references to logarithm tables and interpolated figures.

The scale is read upwards from the bottom to the top, the green portion giving the mantissa corresponding to the number itself on the white portion.

Any figures may be multiplied together by reading off their respective logarithms from the green scale opposite to their values on the white scale, and adding the logarithms together on the Addiator on the back of the slide rule.
The resultant is the log of the desired answer, of which the value can be read off on the white scale against the resultant log on the green scale.

Division is done by subtracting the log of the divisor from the log of the numerator and reconverting the resultant log in the same way. In this process no notice is to be taken of negative results as the right mantissa always shows up automatically in the round openings in the centre scale of the Addiator.

Example: $\boxed{4738} + \boxed{4144} \qquad \boxed{7908}$

$$\frac{29{,}775 \times 2{,}5965}{12{,}513} = 6{,}178$$

$$- \boxed{0974}$$

The framed figures are the mantissa corresponding to the numbers of the example.